Certifying Multilevel Coherence in the Motional state of a Trapped Ion

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Certifying coherence

A defining feature of quantum mechanics is the ability for a system to be in a coherent superposition of states. One method of determining the number of states coherently superposed is state tomography. This however scales poorly with the size of the quantum system. It also relies on having confidence that the tomographic procedure can be performed accurately.

The existence of a superposition of two states can be verified if an interference fringe is produced in a Ramsey type experiment, without needing full tomography. For superpositions of k>2 states, it is possible to produce a certifier based on a generalized interference pattern that verifies that k-coherence is present [1].

If we consider a qubit that can be coupled to the state under study, we can produce an interference pattern by combining a period of free evolution of the state (providing a phase evolution) with a mapping operation, then measuring the qubit:

$$p(\phi) = \langle \chi | \hat{\mathcal{U}}_{\mathrm{m}} \hat{\mathcal{U}}_{\mathrm{f}}(\phi) \rho \, \hat{\mathcal{U}}_{\mathrm{f}}^{\dagger}(\phi) \hat{\mathcal{U}}_{\mathrm{m}}^{\dagger} | \chi \rangle$$

Our certifier is then defined in terms of the moments of this interference pattern:

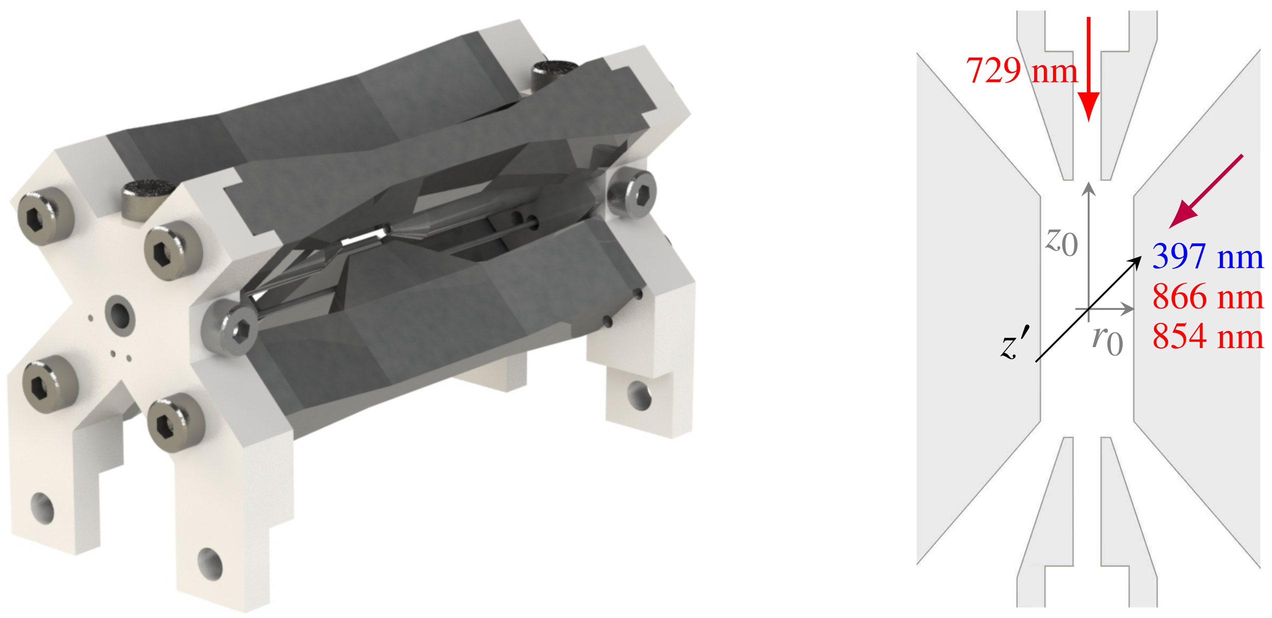
$$C = \frac{M_3}{M_1^2} \qquad M_n = \frac{1}{2\pi}\int_0^{2\pi} p(\phi)^n \, d\phi$$

It can be shown that C has the property that it can only be greater than 1.25 for a 3-coherent state and greater than 1.86 for a 4-coherent state.

Trapped ion realisation

We use these methods to certify the existence of three coherence in the motional state of a single trapped calcium ion.

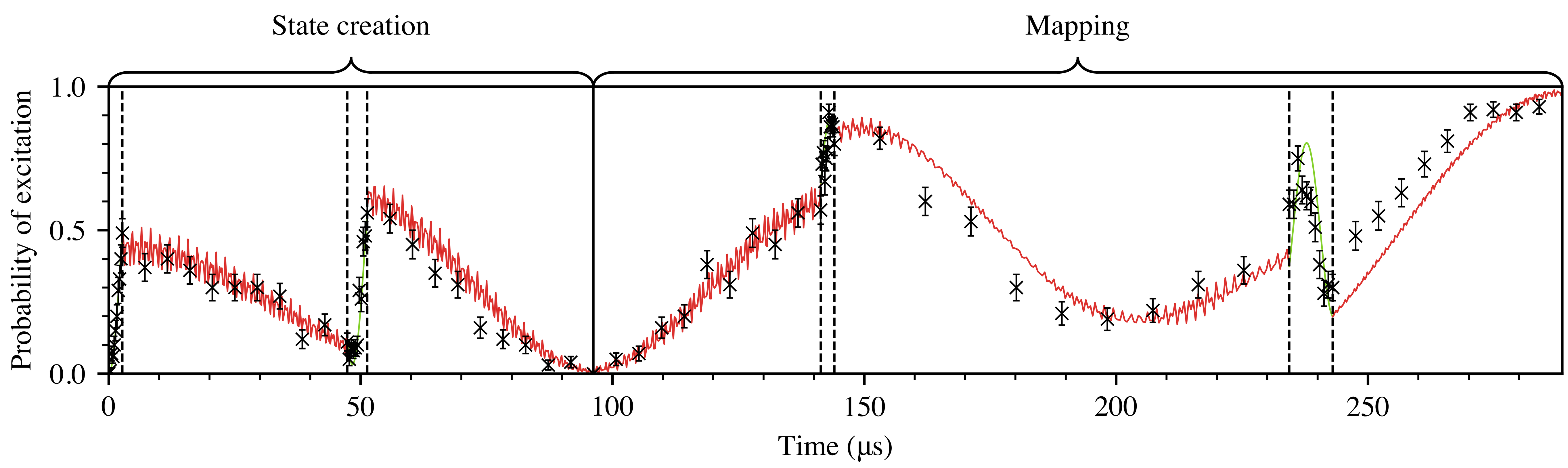
An optical qubit can be defined between a Zeeman substate in the $S_{1/2}$ ground state and a substate in the metastable $D_{5/2}$ state. A laser is then used to drive either carrier transitions, which affect only the qubit state, or sideband transitions, which also affect the motional state. Since the ion is in the Lamb-Dicke regime, only 1st order sideband transitions are available.



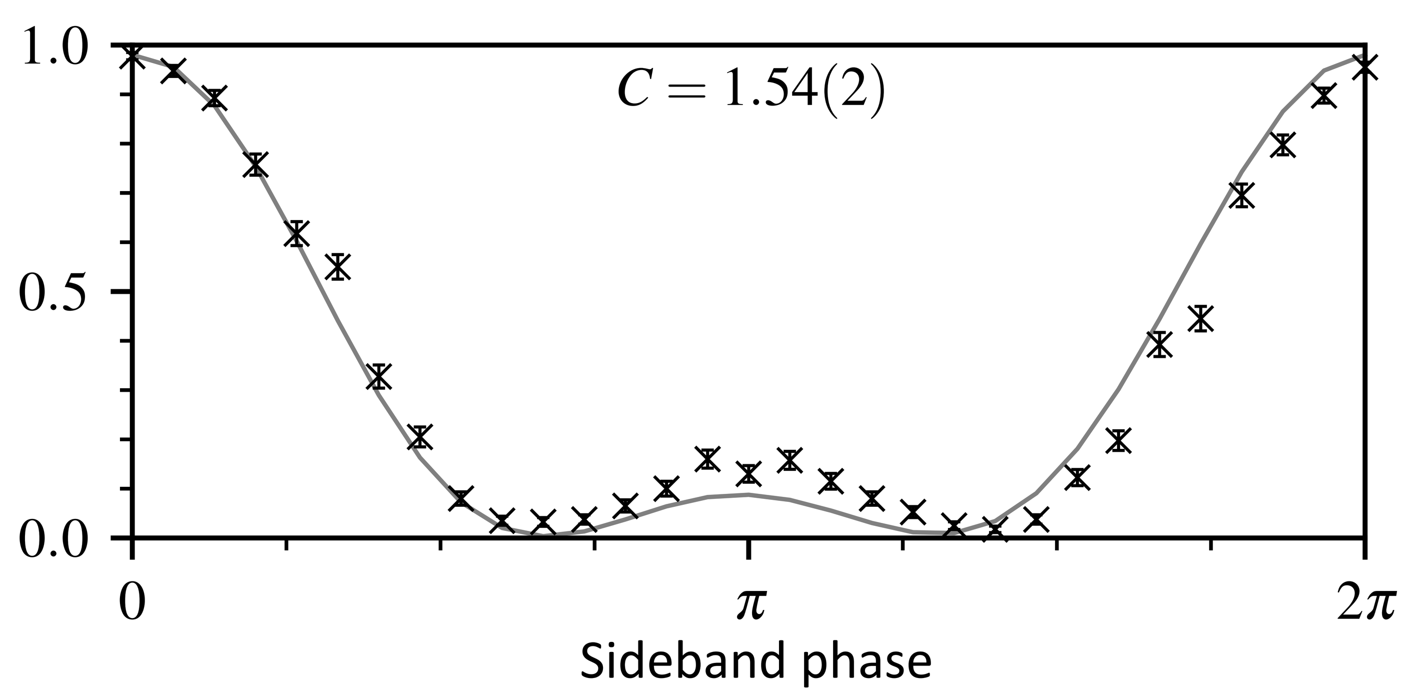
We use a macroscopic linear Paul trap, holding a single $^{40}Ca^{+}$ ion. Hollow endcaps allow a 729nm laser, used to address the qubit transition, to be coupled to only the axial mode of motion. Other lasers fields are used for Doppler cooling, state preparation, and detection. The ion is imaged from above onto a PMT for detection of the qubit state.

After sideband cooling the ion to initialize it to the motional ground state, the state to be studied is prepared using carrier and sideband pulses. This sequence is designed to leave the qubit in the ground state at its conclusion.

Results



A series of four pulses are used to prepare the state $(|g,0\rangle + |g,1\rangle + |g,2\rangle)/\sqrt{3}$. To certify this state, five pulses are then used to implement the mapping operation before the qubit state is measured. Rather than using a period of free evolution, equivalently a phase offset is applied to all the sideband pulses during the mapping stage. The optimal mapping operation is found using numerical methods. The red line is a simulation of the process - the high frequency oscillations are due to off-resonant carrier excitation during the sideband pulses



As the phase of the sideband pulses is changed, an interference pattern is produced. From this, a value for the certifier of 1.54 is obtained. Since this is greater than 1.25, the state in question must have been in a superposition of (at least) three basis states.

Conclusion

Multilevel coherence can be certified efficiently using an interference-type experiment.

The certification theory makes very few assumptions about the processes used to manipulate the state, meaning it is reliable even in the case of imperfect or unreliable control, unlike state tomography.

We demonstrate this in a trapped ion system, certifying that a given quantum state must be in a superposition of three motional Fock states.

[1] Dive, Benjamin, et al. "Characterization of multilevel quantum coherence without ideal measurements." Physical Review Research **2**, 013220 (2020).

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