Imperial College London

Mathematics Aptitude Test (MAT)

Sample Paper 1

[30 minutes]

There are **30 questions** in the MAT.
The solutions for each question is provided at the end of the question paper.

The questions are based on the standard A-level Maths syllabus. They are designed to test problem-solving abilities.

Instructions

1. You are not expected to answer all questions, unanswered questions are not penalised. Each incorrect answer deducts 25% of the marks available for this question.
2. The test consists of 30 questions — 20 are worth one mark, 10 are worth two marks. You will be able to see how many marks each question is worth during the test.
3. All questions are multiple choice. Only one is correct.
4. You are allowed to use plain A4 papers for working out your answers.
5. This is a closed-book examination, any access to resources and calculators are not allowed.

1. Solve the following: [1 mark]

$$\log_e 1 = \sin(x)$$

   A. 1
   B. $\frac{\pi}{2}$
   C. π
   D. $\arcsin(e)$

2. Simplify the following expression: [1 mark]

$$\log_{10}(8!)$$

   A. $7\log_{10} 2 + 2\log_{10} 3 + \log_{10} 35$
   B. $6\log_{10} 2 + \log_{10} 3 + \log_{10} 35$
   C. $5\log_{10} 2 + 3\log_{10} 3 + \log_{10} 35$
   D. None of the above

3. Solve the following simultaneous equations: [1 mark]

   1. $x + y - 4z = \frac{1}{2}(y^2 - xy)$
   2. $\ln(2) + \ln(z) = 0$
   3. $8y - 20z = 6$

   A. $x = 2,\ y = 1,\ z = 0$
   B. $x = 1,\ y = 2,\ z = 0$
   C. $x = 1,\ y = 2,\ z = \frac{1}{2}$
   D. $x = 2,\ y = 1,\ z = \frac{1}{2}$

4. Choose the answer that contains all the solutions to the following equation: [2 marks]

$$(\ln(x))^2 - \ln(x^2) - 63 = 0$$

   A. $3, -1$
   B. e^{-7}, e^2
   C. e^{-7}
   D. None of the above

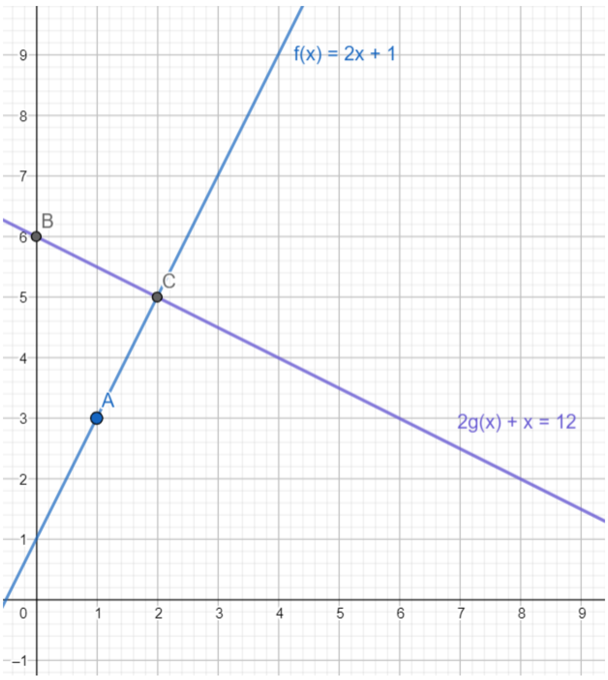
5. What is the x^3 coefficient in the series expansion of the following: [2 marks]

$$(x - \frac{1}{2})^7(x^2 + 1)^3$$

A. $\frac{161}{64}$
B. $\frac{161}{32}$
C. $\frac{161}{128}$
D. None of the above

6. $f(x)$ passes through point $A(1, 3)$. [1 mark]
$g(x)$ crosses the y-axis at point B.
$f(x)$ and $g(x)$ intersect at point C.

What is the area of ABC ?



A. $\frac{5}{2}$
B. $\frac{175}{32}$
C. 15
D. $\frac{5\sqrt{365}}{2}$

7. Given that the arc length of a curve is calculated using: [2 marks]

$$\int_a^b \sqrt{(\frac{dy}{dx})^2 + 1}\, dx$$

where $x \in [a, b]$ and

$$\frac{d}{dx}(\ln|\sec x + \tan x|) = \sec x$$

What is the arc length of the curve defined by

$$y = \ln|\sec x|$$

in the interval of $0 \leq x \leq \frac{\pi}{4}$?

A. $\ln|\sqrt{2} - 1|$

B. $\ln|\sqrt{2} + 1|$

C. $e^{\sqrt{2}}$

D. None of the above

8. What is the coefficient of the x^2 term in the series expansion of the following? [2 marks]

$$\frac{2x - 1}{\sqrt{x + 1}}$$

Hint:

$$(1 + x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + ...$$

where n is rational and $|x| < 1$

A. $\frac{3}{8}$

B. $\frac{-11}{8}$

C. -1

D. $\frac{-3}{8}$

9. Which statements are true about the curve $y = 2x^3 - 8x^2 - 2x + 8$? [1 mark]

1. the curve is concave downwards in the interval $[-1, 2]$
2. the curve has a point of inflection at $\left(\frac{4}{3}, \frac{112}{27}\right)$
3. the curve is concave upwards in the interval $[2, 5]$

A. none of the statements are true

B. 2 only

C. 3 only

D. all of the statements are true

10. What is the gradient of the normal to the curve $y = x^3 + 2x^2 - 2x + 3$ at the point where $x = -2$? [1 mark]

    A. 2

    B. $-\frac{1}{2}$

    C. -2

    D. $\frac{1}{2}$

11. Evaluate this integral: [2 marks]

$$\int_0^{\frac{\pi}{2}} \frac{2xe^{\sin x} - x^2 \cos(x) e^{\sin x}}{e^{2\sin x}} \, dx$$

    A. $\frac{\pi e}{2}$

    B. $\frac{\pi}{4e}$

    C. $\frac{\pi^2}{4e}$

    D. None of the above

12. Solve the following integral: [2 marks]

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{\tan x}{\ln(\cos(x))}\, dx$$

A. $\ln \frac{1}{2}$
B. $\ln 2$
C. 3
D. None of the above

13. Solve the following differential equation stating the general solution: [2 marks]

$$-\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2$$

A. $y = \ln|x + C|$
B. $y = \ln|Ax + C|$
C. $y = e^{Ax}$
D. None of the above

14. When $\theta = 5°$, what is the approximate value of the following? [1 mark]

$$\frac{2\tan(\theta)\sin(\theta) + 2\sin(\theta)\cos(\theta)}{4\cos(\theta) - 4}$$

A. $\frac{-(\pi+36)}{\pi}$
B. -1.2
C. $1 + \frac{36}{\pi}$
D. $\frac{-\pi-36}{36}$

15. Which of the following is equal to $12\sin\left(\theta - \frac{\pi}{6}\right)$? [1 mark]

A. $3\sqrt{3}\sin(2\theta) + 6\sin^2(\theta) - 6$
B. $6\sin(2\theta) + 3\sqrt{3}\sin^2(\theta) - 3\sqrt{3}$
C. $72\sin^2(\theta) + 36$
D. None of the above

16. Solve the following in the range $-90° \leq \theta \leq 90°$. [1 mark]

$$\tan^2(\theta) - 1 = \sec(\theta)$$

A. $\pm 60°$
B. $\pm 90°, 30°$
C. $0°$
D. $\pm 90°, -30°$

17. Which of the following is equal to $\cos^4(\theta)$? [1 mark]

Hint:

$$\cos(a \pm b) = \cos(a)\cos(b) \mp \sin(a)\sin(b)$$

A. $\cos(4\theta) + 4\cos(2\theta) + 3$

B. $\frac{1}{8}\cos(4\theta) + \frac{1}{2}\cos(2\theta) + \frac{3}{8}$

C. $\cos(4\theta) + 2\cos(2\theta) + 2$

D. $8\cos(4\theta) - 4\cos(2\theta) - 3$

18. [1 mark]

$$f(n) = \frac{(2n)!}{2^n n!}$$

Which of the following is $f(n)$ equivalent to?

A. $f(n) = (2n-1)(2n-3)(2n-5)...1$

B. $f(n) = (2n+1)(2n-1)(2n-3)...1$

C. $f(n) = (2n)(2n-1)(2n-2)...1$

D. $f(n) = (2n)(2n-2)(2n-4)...1$

19. Two points A and B are defined as: [1 mark]

$$A = \mathbf{i} + 2\mathbf{j}$$

$$B = 3\mathbf{i} + 4\mathbf{j}$$

What is the bearing of A from B?

A. 225°

B. 045°

C. 135°

D. 315°

20. The angle between a vector $\mathbf{a} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ and the positive y-axis can be found using: [1 mark]

$$\cos(\theta_y) = \frac{y}{|\,\mathbf{a}\,|}$$

What is the angle between the positive y-axis and the vector:

$$\mathbf{b} = \begin{bmatrix} \sqrt{8} \\ 3 \\ -1 \end{bmatrix}$$

A. $\cos^{-1}(\frac{2}{3})$

B. 30°

C. 60°

D. 45°

21. Which of the following describes the following curve's domain best? [1 mark]

$$x = 2\cot^2(t), y = 3\text{cosec}^2(t), 0 < t \leq \frac{\pi}{4}$$

A. $x \in R$

B. $x > 0$

C. $x \geq 0$

D. $x \neq 0$

22. $\lfloor x \rfloor$ represents the floor function: this rounds the input down to the nearest integer. e.g. $\lfloor 0.9 \rfloor = 0$. Solve the following integral: [2 marks]

$$\int_{\lfloor e \rfloor}^{\lfloor \pi \rfloor} \lfloor 3x \rfloor dx$$

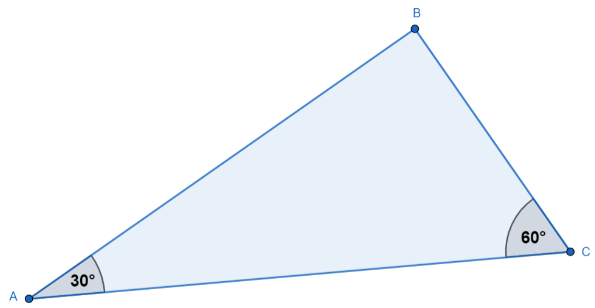
A. 7

B. 8

C. 9

D. None of the above

23. In triangle ABC, $\overrightarrow{AB} = 3\mathbf{i} + 4\mathbf{j}$. [1 mark]
What is the length of side BC?



A. $5\sqrt{3}$

B. $\frac{5}{\sqrt{3}}$

C. $\frac{\sqrt{3}}{5}$

D. $\frac{1}{5\sqrt{3}}$

24. What is the inverse of the function below? [1 mark]

$$f(x) = \frac{1}{x^2 - 2}, \{x \in R, x \geq 0\}$$

A. $f^{-1}(x) = \sqrt{1 + \frac{2}{x}}$

B. $f^{-1}(x) = \sqrt{\frac{1+2x}{x}}$

C. $f^{-1}(x) = \sqrt{\frac{1}{x+2}}$

D. $f^{-1}(x) = \frac{\sqrt{1+2x}}{x}$

25. The first term of a geometric series is a and the common ratio is r. $S_4 = 80$ and $S_\infty = 81$, where S_n is the sum of the first n terms in the series. What is r? [1 mark]

A. $\frac{1}{3}$

B. $\pm\frac{1}{3}$

C. $\pm\frac{1}{\sqrt{3}}$

D. $\frac{1}{\sqrt{81}}$

26. The first term of a geometric series is a and the common ratio is $\frac{1}{\sqrt{2}}$. What is the sum of the first ten terms? [1 mark]

A. $\frac{31}{64}a(1 + \frac{1}{\sqrt{2}})$

B. $\frac{33}{16}a(1 - \frac{1}{\sqrt{2}})$

C. $\frac{33}{64}a(1 - \frac{1}{\sqrt{2}})$

D. $\frac{31}{16}a(1 + \frac{1}{\sqrt{2}})$

27. Which of the following series converge? [1 mark]

1. $$\sum_{n=1}^{\infty} e^{-n}$$

2. $$\sum_{n=1}^{\infty} (-\ln(3))^n$$

3. $$\sum_{n=1}^{\infty} \left(e^{-\ln(2)}\right)^n$$

A. 1 only

B. 1 and 3 only

C. 2 and 3 only

D. all series diverge

28. Given that $y = \sin x$, evaluate the following sum at $x = \frac{\pi}{3}$: [2 marks]

$$y + \sum_{n=1}^{99} \frac{d^n y}{dx^n}$$

A. 0

B. $\frac{1}{\sqrt{2}}$

C. $\frac{\sqrt{3}}{2}$

D. None of the above

29. Consider the following definitions: [1 mark]

- A function is EVEN when $f(x) = f(-x)$
- A function is ODD when $f(-x) = -f(x)$

Which of the following statements are true?

1. The product of 2 odd functions is always an odd function
2. The sum of 2 even functions is always an even function
3. The sum of 2 odd functions is always an odd function
4. The product of 2 even functions is always an even function

A. Only 1 is true

B. 2,3,4 are true

C. All statements are true

D. None are true

30. The integral L is defined as [2 marks]

$$L = \int_0^a e^{-st} f''(t) dt$$

where

$$f''(t) = \frac{d^2 f(t)}{dt^2}$$

Given the boundary conditions,

$$f(a) = f'(a) = 0$$

find integral L in terms of I, where I is defined as

$$I = \int_0^a e^{-st} f(t) dt$$

A. $s^2 I - sf(0) - f'(0)$
B. $-s^2 I - sf(0) - f'(0)$
C. $-s^2 I + sf(0) - f'(0)$
D. $s^2 I + sf(0) - f'(0)$

Question	Points	Answer
1	1	C
2	1	A
3	1	C
4	2	D
5	2	A
6	1	A
7	2	B
8	2	B
9	1	C
10	1	B
11	2	C
12	2	A
13	2	B
14	1	A
15	1	D
16	1	A
17	1	B
18	1	A
19	1	A
20	1	D
21	1	B
22	2	A
23	1	B
24	1	B
25	1	B
26	1	D
27	1	B
28	2	A
29	1	B
30	2	A

End of MAT