MATHEMATICS SUMMER REVISION 2023

1.0 Introduction

Over the last few years, our incoming students have had varying degrees of disruption, affecting thier last 2-3 years of secondary education, regardless of which country they were in. This applies to our A-level students, but equally to many others. Whatever the situation, we felt they would benefit from revising a bit of Mathematics in the summer before beginning year one.

The results have been surprisingly good: there is a direct correlation between engagement with the summer course in September and exam marks in January/May. Of course, correlation is not causation, as we all know. The most motivated students may come to the summer course and this will be reflected in their exam marks. However, engagement is key, and if you begin in September, the hope is you will stay engaged all year! So we're going to continue offering the Mathematics Summer Revision Course, and this is your invitation.

We have a programme of self-study set out for you, which will be supported online. You can choose to do it entirely on your own, or you can choose to join us online for as much as you want. See below for the detailed programme of study.

The online courses are **for free**, anyone can sign up. They were written by Dr Philip Ramdsen from the Mathematics Department at Imperial, and are on the edX platform: edX A-level Mathematics. Enrol now, and have a look! You will need to create an account, but it's not complicated.

Here's how you should proceed now:

- Below are all the topics from A-level Mathematics, divided into four parts: Year 12, parts 1 and 2, and Year 13, parts 1 and 2, corresponding to the four courses on edX. Some topics are marked optional and no detail is given: these are not essential preparation for our year one, and we list them only for completeness.
- Each table has links to all the exercises for that course, with topics ordered into 7 modules. Begin at the top, with the earlier, less challenging topics, and go through some of the exercises for each topic. Each set has an easy, a medium and a more difficult section. If you can do the difficult exercises without hesitation, move on to the next set. Especially for some of the basic topics, you may find them a bit easy.
- The first two courses are year 12 A-level Mathematics, so you should have done the material in 2021-22, which may have had some disruption, Even if you've caught up since then, it's a good idea to look at the exercises, just to warm up the thinking muscles, so to speak.
- Once you find you cannot do some of the exercises without having to put in some effort, or maybe you feel unsure about some of the answers, this is the moment to start your edX course. If you haven't already done so, enrol now in the relevant course, find the correct module and proceed.
- On edX, each module includes several lecture videos with explanations and examples. At the end of each short lecture, there's a challenge problem for you to solve before watching the next video. Try these challenges, the solution is usually at the beginning of the next video. Often there are excellent interactive applets allowing you to visualize some of the ideas. At the end of each section, there's a set of exercises. If you do the exercises online, there are hints, you get immediate marks, and next, you can access the solutions. They are the same exercises as linked in the tables, below.

  After each set of exercises, make sure that you complete the "check-point" to unlock the solutions. All the solutions are available as a pdf, but some of them are also done in video recordings. The last column in each table shows which solutions are available as video.
- For most of the topics, there will be additional, challenging problems, for those of you that like being stretched! These are from the STEP support programme at Cambridge and *some* of them are listed at the end, by topic.
- For some topics there are links to HELM (Helping Everyone Learn Mathematics) modules for some extra material we would like you to look at.

- Finally, last but definitely not least, is Complex Numbers. This is not in the A-level syllabus, but we think it's important enough to include here. It's the first thing we do in Mathematics in October, so the HELM module (or equivalent, see section 1.5) will be excellent preparation.

Carry this as far as you want to go. We will be happy to answer your questions and everyone will benefit. The main thing is to work your way through the A-level topics and refamiliarize yourself with the material, almost as if you had an exam coming.

1.1 Year 12, Part 1

Module	Topic	Other Info	Solution Recordings
1 Indices & Surds	Indices		1b, 1f, 2b,3b
	Surds		1c, 3, 4d, 5c
	A-level style questions		1,2,5
2 Inequalities	Solving Inequalities		1f,i,j, 2f,i,j,3c,d, 4,9
	Graphical Representation of Inequalities		2,4,6
	A-level style questions		all
3 Factorization & Algebra	Factor Theorem		1b,3,5
	Algebraic Division		1c,3b,4d,5b
	Partial Fractions	HELM	
1,2 and 3	Assessment 1		
4 Circles	Equation		1c,2,3
	Geometry		1,3
5 Curve Sketching	Transformations of Graphs		1,2,3,5
	Standard Graphs		1,3a
	A-level questions		
4 and 5	Assessment 2		
6 Mechanics	Introduction	optional	
7 Statistics	Introduction	optional	

Most of this first part is warm-up, but pay special attention to module 5 on Curve sketching. Also very important is the HELM material on Partial Fractions: it's not in the A-level syllabus, and we have noticed that it often needs refreshing.

Click to enrol on the edX course: Year 12, Part 1. where you will find all 7 topics: videos, animations, challenges, solutions.

1.2 Year 12, Part 2

Module	Topic	Other Info	Solution Recordings
1 Differentiation I	Introduction		1d, 2d, 3d,4d,5
	Rational Powers		1d,2d,3d4d,5,7
2 Exponential Function	Exponential Function ,	Do the Investigation!	3,4,5, Investigation
	Logarithms		2,5,8,10
	Growth and Decay		4,7
3 Differentiation II	Expressions ,		
	Stationary Points		
	Tangents and Normals		2c,3c,5,8
1,2 and 3	Assessment 1		
4 Integration I	Indefinite Integration		1,2,3,5
	Definite Integration		1,2,3,5
5 Newton's Laws of Motion		optional	
4 and 5	Assessment 2	Do Integration questions	
6 Probability and Statistics I		optional	
7 Probability and Statistics II	Hypothesis Tesrting	optional	

Most of this second part is still basics, but be sure to get **very** familiar with everything involving the exponential function and differentiation for stationary points and tangents. Click to enrol: Year 12, part 2 for the full course. At the end of modules 3 and 5, try the Assessments.

1.3 Year 13, Part 1

We're assuming that you covered most of the year 12 material in 2018-19, without disruption, so the material from year 13 should really be your central focus - after warming up your mathematical thinking muscles, so to speak! Many of these topics, we will apply repeatedly in the Mathematics modules in year one.

Module	Topic	Other Info	Solution Recordings
1 Algebra & Functions	Domain and Range		2,3a,c,e,f,4b,5,6
	Composite functions		
	Inverse functions		
2 Sequence and Series I	Introduction		1,2e,f,g,4g,h,8,9,13,14
	Arithmetic & Geometric Series	convergence	4,5,6
3 Sequence and Series II	Binomial Expansion	Sigma notation	3,4,6
	Binomial Expansion: rational powers	convergence/divergence	1c,d,2,3,4
1,2 and 3	Assessment 1		
4 Trigonometry I	Radians, Arc Length, Sector Area		3,5,7,9
	Small angle approximations and graphs		2,4,5,7c,
5 Trigonometry II	Reciprocal and Inverse functions		1c,2,5,8,9
	Trigonometric Equations		1c,2a,3c,e,f
6 Numerical Methods I	Numerical Integration	optional	
7 Numercial Methods II	Newton-Raphson, Fixed-point iteration	optiona	

In this first part, the sections on Trigonometry are crucial: make sure you know this stuff inside out! Likewise for Geometric Series and the Binomial expansion. The extensions on convergence, divergence, etc, are excellent! Enrol in the full course here: Year 13, Part 1

1.4 Year 13, Part 2

Module	Topic	Other Info	Solution Recordings
1 Mechanics	Kinematics and Projectile motion	optional	
2 Mechanics	Friction/Moments/Equilibrium	optional	
3 Statistics	Normal distribution	optional	
4 Vector Algebra	Basics		2,3,5,6
	Problem Solving		1,2,3
	Scalar & Vector Product, Lines,Planes	HELM	
5 Differentiation III	Chain, product, quotient rules		1c,2c,3d,4f,i,6
	Trigonometric functions		1d,2c,4,6b
	Implicit and parametric		2d,4,6
4 and 5	Assessment 2		
6 Integration II	By inspection		4,5,6
	By substitution		3,5,8
	Integration by parts/partial fractions		3,6,7
	Integration by parts/partial fractions	HELM	
7 Differential Equations	First-order separable ODE		1b,d, 2c,3b
	Modelling/Problem Solving		1c,3,6
6 and 7	Assessment 3		

In this last part we concentrate on differentiation and integration methods, as well as simple differential equations. All of these are covered in year-one Mathematics in your degree, but we go over the basics quickly and extend them, so it's a very good idea to revise this as well.

The HELM workbooks are for you to go further in: (a) Vector Algebra: scalar product, vector product, equation of line, equation of plane, and (b) Integration by parts and integration using partial fractions. There's no need to do everything in these workbooks, as they do overlap with the edX course to some extent.

Here's the fourth edX course to enrol: Year 13, Part 2

1.5 Complex Numbers

The edX courses don't include this last topic. There are many very good ways to study complex numbers on your own. My own particular favourite is the HELM workbook: HELM10: Complex Numbers. Or get Stroud: "Engineering Mathematics", a classic text.

If you want a person explaining it, here are two excellent ones:

Khan Academy: Complex numbers
Mario's Math Tutoring: Complex Numbers

More formally, from one of the A-level exam boards: OCR/MEI: FP1 and OCR/MEI: FP2 for the relevant sections of the Further Maths A-level. Look for the videos on complex numbers.

Whichever method you use for the theory, go back and do the exercises in the HELM workbook for practice.

1.6 Challenging Problems

Challenging, extension work, from the Cambdrige STEP programme. Very good if you find everything above a bit too easy! These are for you to enjoy, we don't want you to think of them as a chore, so don't feel obliged to do these in any way.

STEP1: Surds, Algebra ,

STEP2: Algebra, Inequalities

STEP4: Algebra, Inequalities, Curve Sketching

STEP5: Trigonometry

STEP7: roots of polynomials, curve sketching

STEP8: circles, algebra, AM/GM

STEP9: sketching cubics

and so on....there are many, many more: STEP support

Not all the questions in each exercise are relevant for our year one, but many are. If a question involves a topic included in the above tables, it's relevant.