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With thanks to Susan Gill and Mark Walsh at Colchester Royal Grammar School, as well as those primary teachers who trialled these resources and gave valuable feedback.

Should you have any thoughts as to how we might improve future versions, or if there are other topics you would like us to cover, please email: outreach.engineering@imperial.ac.uk

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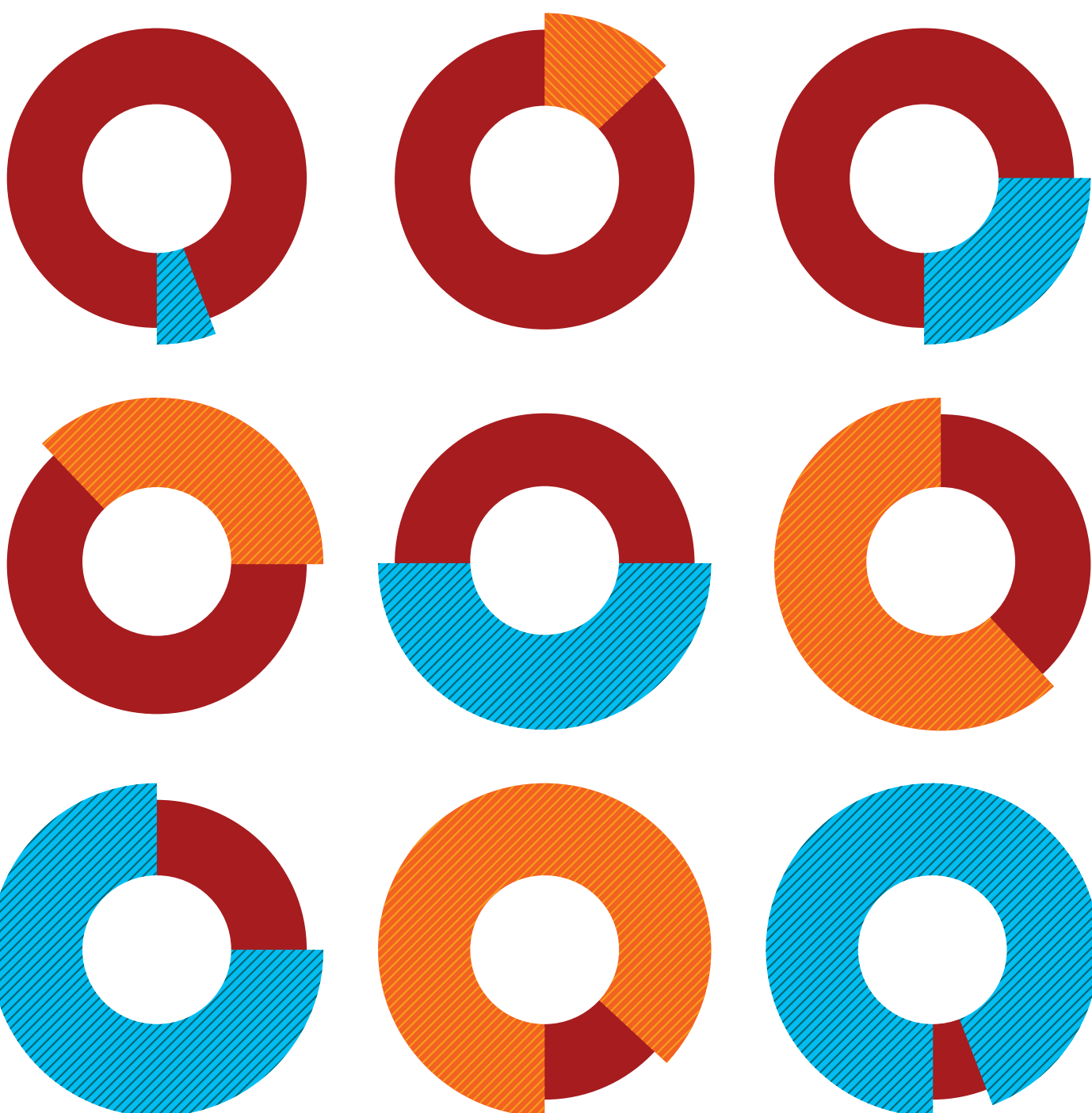


MATHS TEACHING RESOURCES

For teachers of high-achieving
students in KS2

1

PERCENTAGES



Welcome

These resources have been put together with you, the primary teacher, at the forefront of our thinking. At Imperial College London we recognise the importance of keeping high-attaining students engaged with maths and want to do everything we can to help teachers provide for all of their students. Maths is critical for us as a facilitating subject for further eventual study in engineering or science, as well as being fascinating in its own right.

The content in this booklet was developed by teachers at Colchester Royal Grammar School in conjunction with Imperial. Based on initial feedback from primary teachers who trialled it, we have included specific guidance and examples to help teachers develop their own understanding of the material but we would welcome any further suggestions for improvement.

The material for the first three booklets has been loosely based on the previous 'Level 6' curriculum content and we are currently looking into developing some further assessment materials to sit alongside these in case schools wish to offer an informal certification.

I hope that you and your students find these resources useful and most of all enjoyable.



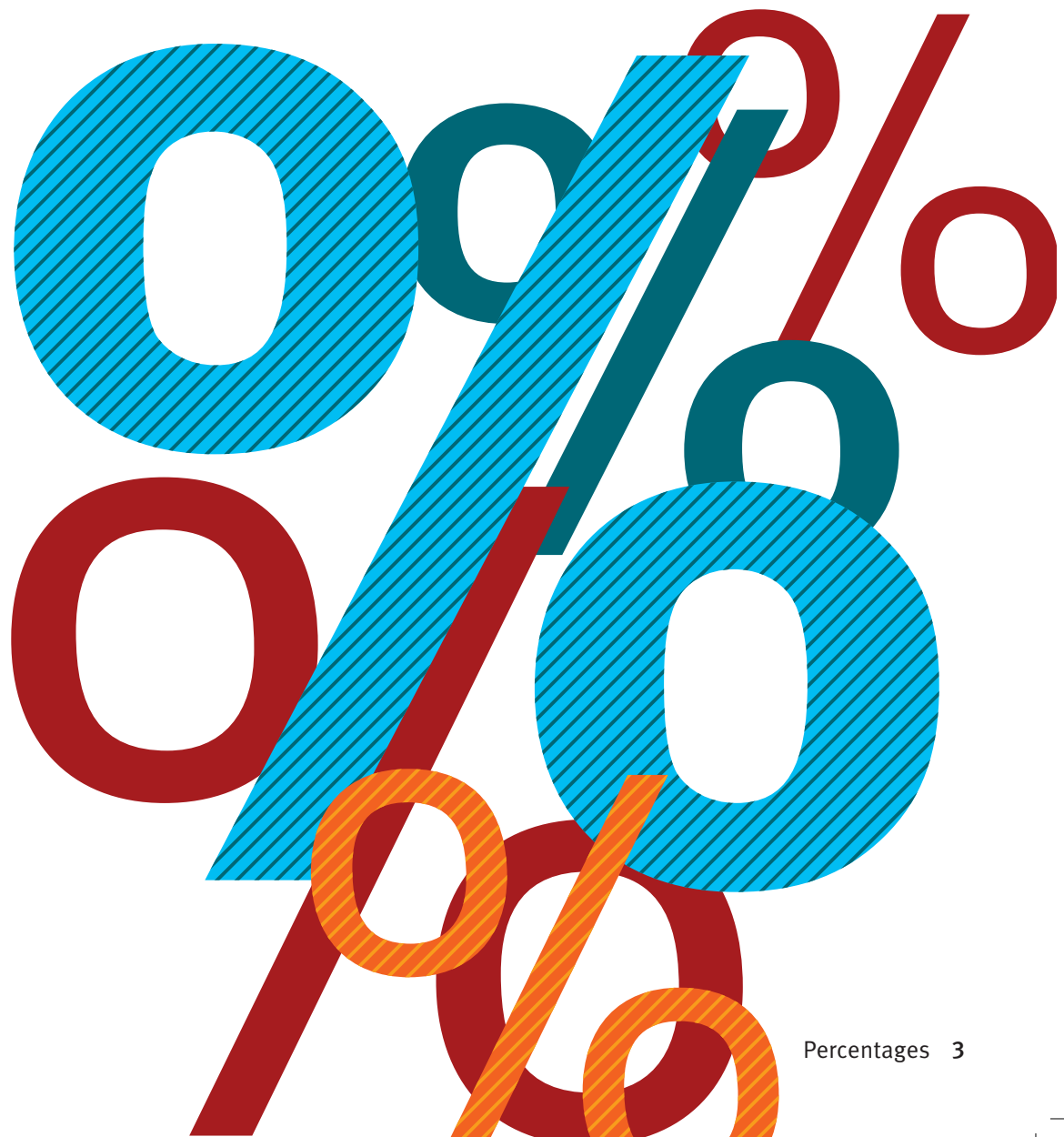
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Percentages are used to describe a change in terms of a fraction of an amount.

This means they can be used to compare proportional changes in different amounts.

We have chosen to begin with increases, then decreases and then 'reverse' calculations but these can be learnt in any order. The same method can be applied to all three situations.



Percentage increase: adding on

A percentage increase means adding on a proportion of a number to itself, this allows us to compare increases.



A house has increased its value by 17% in the past year.

If it was valued at £250,000 a year ago what is its value now?

The value of the house a year ago (£250,000) represents 100%.

There are three ways of increasing this amount:

TWO-STAGE METHOD

Calculate 17% and add it on to £250,000:

17% of £250,000

OR

$17\% = 10\% + 7 \times 1\%$

First find 10% by dividing the original amount by 10, then divide by 10 again for one hundredth which is 1%

$10\% = £250,000 \div 10 = £25,000$

OR

$1\% = £25,000 \div 10 = £2,500$

$17\% = £25,000 + 7 \times £2,500$

$= £25,000 + £17,500$

$= £42,500$

$117\% = £250,000 + £42,500$

$= £292,500$

OR

$17\% = £250,000 \times 0.17$

$= £42,500$

$117\% = £250,000 + £42,500$

$= £292,500$

ONE-STAGE METHOD

Calculate 117% of £250,000:

$117\% = 100\% + 10\% + 7 \times 1\%$

$= £250,000 + £25,000$

$+ 7 \times £2,500$

$= £292,500$

MULTIPLIER METHOD

Convert 117% to a decimal:

$117\% = £250,000 \times 1.17$

$= £292,500$

Percentage increase card sort

A house valued at £250,000 is increased by 20%.
Calculate the new value of the house.

Put the cards in the correct order and carry out the calculation:

TYPE ONE
Work out 10% of
£250,000
by dividing by 10

TYPE TWO
Convert 'add
20%' into a
multiplier

TYPE ONE
Add on to
£250,000

TYPE TWO
Multiply
£250,000
by 1.2

TYPE ONE
Multiply by 2
to obtain 20%

Percentage increase questions



Try to answer questions 1–5 without a calculator:

- 1** The price of food has increased by 10% between 2012 and 2013. A wholemeal loaf of bread cost 90p in 2012. How much did it cost in 2013?
- 2** Energy bills rose by 5% over the past year. Mrs Allen's bill last year was £520. How much will she pay this year?
- 3** A baby calf increases its weight by 125% in the first three months of its life. What weight would you expect a newly born 18kg calf to weigh at three months?
- 4** Brenda is paid £1,500 a month. Her employer increases her wage by 4%. Calculate the new wage Brenda is paid each month.
- 5** A drink contains 270ml. The size is increased by 15%. How much drink does it now contain?



You may wish to use a calculator for the following questions:

- 6** Calculate these percentage increases:
 - A** increase £320 by 11%
 - B** increase £525 by 20%
 - C** increase £250 by 105%
 - D** increase £480 by 50%
 - E** increase 640ml by 15%
 - F** increase 760m by 26%
 - G** increase 850km by 23%
 - H** increase 245 litres by 30%
 - I** increase 360m by 46%
 - J** increase £1,200 by 30%
 - K** increase £985 by 18%
- 7** A scale model needs to be tripled in length. What percentage increase is this?

Percentage decrease: subtracting

What if something is getting smaller rather than bigger?
The method here is very similar to that for increases.



A car has decreased its value by 21% in the past year.

If it was valued at £18,000 a year ago what is its value now?

The value of the car a year ago (£18,000) represents 100%.

There are three ways of decreasing this amount:

TWO-STAGE METHOD:

Calculate 21% and subtract this from £18,000

21% of £18,000

OR

21% = 10% + 10% + 1% or 2 x 10% + 1%

First find 10% by dividing the original amount by 10, then divide by 10 again for one hundredth which is 1%

= £18,000 ÷ 10 = £1,800:

1% = £18,000 ÷ 100 = £180

21% = £1,800 x 2 + £180 = £3,780

A decrease of 21%:

100% - 21% = £18,000 - £3,780 = £14,220

The value of the car is now £14,220

21% = £18,000 x 0.21 = £3,780

100% - 21% = £18,000 - £3,780

= £14,220 (100% - 21% = 79%)

ONE-STAGE METHOD:

A decrease of 21% is the same as finding 79% of the amount

79% = 50% + 2 x 10% + 9 x 1%
= £9,000 + 2 x £1,800 + 9 x £180

= £14,220

MULTIPLIER METHOD:

Convert 79% to a decimal

79% = £18,000 x 0.79

= £14,220

Percentage decrease card sort

A car valued at £12,000 has depreciated by 15%.
Calculate the new value of the car.

Put the cards in the correct order and carry out the calculation:

TYPE ONE
Work out 10% of £12,000 by dividing by 10

TYPE TWO
Find 100%–15%

TYPE THREE
Find 100%–15%

TYPE ONE
Work out 5% of £12,000

TYPE TWO
Find 50% and 5% of £12,000

TYPE THREE
Convert 85% to a decimal

TYPE ONE
Add these values together

TYPE TWO
Find 35% of £12,000

TYPE THREE
Find 85% of £12,000

TYPE ONE
Take away from £12,000

TYPE TWO
Add these values together

Reverse percentages questions



Try to answer questions 1–5 without a calculator:

- 1** A holiday costs £820 but it has been reduced by 15%.
Calculate the new cost of the holiday.
- 2** By paying the electricity bill in full, the supplier reduces it by 8%.
Mr Jill's bill last quarter was £300. He paid the bill in full and received the reduction.
How much did he have to pay?
- 3** 'Clothes are Us' are reducing all of their prices by 15% in their annual sale.
A dress cost £36 before the sale.
How much would you expect to pay for it in the sale?
- 4** The price of a TV has decreased by 32% over the past two years.
Two years ago a TV was priced at £850.
What would be the price today?
- 5** A packet of cereal contains 900g.
Calculate the size if it is decreased by 15%.



You may wish to use a calculator for the following questions:

- 6** Calculate these percentage decreases:
 - A** decrease £230 by 12%
 - B** decrease £45 by 8%
 - C** decrease £350 by 85%
 - D** decrease £120 by 40%
 - E** decrease 520ml by 30%
 - F** decrease 65m by 15%
 - G** decrease 750km by 30%
 - H** decrease 556 litres by 25%
 - I** decrease 820m by 42%
 - J** decrease £3500 by 35%
 - K** decrease £75 by 10%

Reverse percentages

Reverse percentage questions involve working backwards from the final amount to find the original amount.

What if we were given the final amount and asked to find the original?



Ashley was given a pay rise. Her new salary is now £22,000 after a 10% rise.

Calculate her salary before her pay rise.

100% represents her salary before the pay rise.

A pay rise of 10% means that 110% represents her new salary. Therefore, $110\% = £22,000$

We need to work out 1% then multiply this by 100 to obtain 100%

OR

we can work out 10% and then multiply this by 10

$$\begin{aligned} 1\% &= £22,000 \div 110 = £200 \\ 100\% &= 100 \times £200 = £20,000 \end{aligned}$$

OR

Her salary before the pay rise was £20,000

$$\begin{aligned} £22,000 \div 110 \times 100 \\ = £20,000 \end{aligned}$$



The number of workers in a factory fell by 15% to 255.

How many workers were there originally?

100% represents the number of workers that were there originally.

There are now 85% of the workforce, 255 people

$$\text{Find } 1\% = 255 \div 85 = 3$$

$$100\% = 100 \times 3$$

$$= 300$$

Originally, there were 300 workers

$$255 \div 85 \times 100 = 300$$

OR

$$255 \div 0.85 = 300$$

Reverse percentages card sort

A shopping bill has increased by 12% to £224 per week for the Harrison family. Calculate the shopping bill before the increase.

Put the cards in the correct order and carry out the calculation:

TYPE ONE
Find 100% by multiplying by 100 (the bill before the increase)

TYPE ONE
Find 1% of £224 by dividing by 112

TYPE ONE
Add 12 to 100% (this represents £224)

A coat was reduced by 30% in a sale to £35. Calculate the price of the coat before the sale.

Put the cards in the correct order and carry out the calculation:

TYPE ONE
Subtract 30% from 100%. This represents £35 (the price of the coat in the sale)

TYPE TWO
Work out 1% of £35 by dividing by 70

TYPE ONE
Multiply by 100 to get 100% (the price of the coat before the sale)

Reverse percentages questions



Try to answer questions 1–5 without a calculator:

- 1** A holiday cost £850 after it has been reduced by 15%.
Calculate the original cost of the holiday.
- 2** By paying the electricity bill in full, the supplier reduces it by 8%.
Mr Black's bill last quarter after the reduction was £184.
What was the bill originally?
- 3** 'Clothes are Us' reduced all of their prices by 25% in their annual sale. A dress cost £18.75 in the sale.
Calculate the cost of the dress before the sale.
- 4** A washing machine has been reduced by 20% to £320.
Calculate the original price of the washing machine.
- 5** 45% of plastic bottles in a factory are blue and the remaining 165 are green.
How many plastic bottles are blue?



You may wish to use a calculator for the following questions:

- 6** From the final amount and the percentage increase or decrease calculate the original amount:
 - A** £350 after an increase of 10%
 - B** £240 after a decrease of 15%
 - C** 650ml after an increase of 5%
 - D** £120 after a decrease of 30%
 - E** 520ml after a decrease of 13%
 - F** 155m after an increase of 5%
- 7** Bob is paid a salary of £25,000, but 20% tax is deducted on all income over £10,600.
How much does he receive overall?

Percentage increase

CARD SORT

Two sets are provided, demonstrating different routes to the same answer.

Ideas for differentiation:

- **All students:** Cards as above
- **Most students:** Add some cards with blank spaces, students to fill in the missing steps
- **Some students:** Write their own cards

You might also consider adding some incorrect cards using the template available at: www.imperial.ac.uk/schools

QUESTIONS

We have assumed that teachers might be able to provide additional basic numerical examples if these are required, and have chosen to focus on specific contexts in these questions.

Note that most questions require specific units for a fully correct answer – this is not only good practice but also allows students to begin demonstrating awareness of the context of their answers.

SOLUTIONS TO QUESTIONS

- 1 99p
- 2 £546
- 3 40.5kg
- 4 £1560
- 5 310.5ml
- 6 Most students will probably prefer to use a calculator for these, and we believe this is absolutely fine (though watch out for any who discover the % key!). We have left it to the teacher to decide and discuss what an appropriate degree of accuracy is.

A £355.20	G 1045.5km
B £630.00	H 318.5 litres
C £512.50	I 525.6m
D £720.00	J £1,560
E 736ml	K £1,162.30
F 957.6m	
- 7 200%. Note that we have deliberately avoided more sophisticated questions such as “a model is doubled in length, how much more material is needed?” – i.e. those which mix length, area and volume because of the added complexity of linear/area/volume scale factors. This might provide an interesting area for extension though.

Percentage decrease

This need not be taught as a separate topic after percentage increases, and indeed our intention is to show that the methods are identical. We like the idea of inviting students to figure out how to adapt their chosen method to 'subtraction'/decreases themselves.

CARD SORT

Three sets are provided, demonstrating different routes to the same answer.

Ideas for differentiation:

- **All students:** Cards as above
- **Most students:** Add some cards with blank spaces, students to fill in the missing steps
- **Some students:** Write their own cards

You might also consider adding some incorrect cards using the template available at: www.imperial.ac.uk/schools

SOLUTIONS TO QUESTIONS

- 1 £697
- 2 £276
- 3 £30.60
- 4 £578. **Note:** the wording of this question is typical of those found in secondary curricula – the TV has decreased in price by 32% overall, not by 32% each year. A common mistake would be to work out a decrease of $2 \times 32 = 64\%$
- 5 765g
- 6 Try to encourage students to try a combination of methods throughout these otherwise straightforward questions.
 - A £202.40
 - B £41.40
 - C £52.50
 - D £72
 - E 364ml
 - F 55.25m
 - G 525km
 - H 417%
 - I 475.6m
 - J £2,275. The apparent (but not relevant) connection between the numbers 3,500 and 35 may cause mistakes here.
 - K £67.50

Reverse percentages

This may also lend itself to being integrated with the previous topics since it really concerns an only slightly different way of working which allows students to respond to more complex and more realistic-feeling questions.

The second example on page 10 (workers in a factory) is worth highlighting and observing that a 'fall of 15%' cannot be calculated by thinking about 115% \rightarrow 100%, which is why the guidance specifically emphasises that the 'original' (old) figure should be thought of as 100%.

CARD SORT

Here the two sets relate to separate questions since we didn't feel multiple methods were particularly beneficial.

Ideas for differentiation:

- **All students:** Cards as above
- **Most students:** Add some cards with blank spaces, students to fill in the missing steps
- **Some students:** Write their own cards

You might also consider adding some incorrect cards using the template available at: www.imperial.ac.uk/schools

SOLUTIONS TO QUESTIONS

In general, the numbers in the first few questions lend themselves to 'neat' solutions when the correct method is used, though the level of challenge increases quite quickly here.

- 1 £1,000
- 2 £200
- 3 £25.00
- 4 £400
- 5 135. This question requires at least two steps – firstly to work out the total number (300) based on 165 bottles being 55% of that, and then finding 45% of 300.
- 6 All solutions given to two decimal places, but this could be reduced.
 - A £318.18
 - B £282.35
 - C 619.05ml
 - D £171.43
 - E 597.70ml
 - F 147.62m
 - G £25,000 – £10,600 = £14,400
 - H 20% of £14,400 is £2,880
 - I £25,000 – £2,880 = £22,120