Imperial College
London

DEPARTMENT OF MATHEMATICS

GUIDE TO MODULES

THIRD YEAR (BSc/MSci)
2023-2024.

Notes and syllabus details of modules for Mathematics students in their Third Year. For degree programmes:

G100, G103	MATHEMATICS (BSc, MSci)
G104	MATHEMATICS WITH A YEAR ABROAD (MSci)
G102	MATHEMATICS WITH MATHEMATICAL COMPUTATION
G125	MATHEMATICS (PURE MATHEMATICS)
G1F3	MATHEMATICS WITH APPLIED MATHEMATICS/MATHEMATICAL PHYSICS
G1G3	MATHEMATICS WITH STATISTICS
G1GH	MATHEMATICS WITH STATISTICS FOR FINANCE
GG31	MATHEMATICS, OPTIMISATION AND STATISTICS

Information on the Mathematics modules available to third-year students on the Joint Mathematics and Computing Programmes can be found in this guide.

These notes should be read in conjunction with your undergraduate student handbook and the programme specifications for your year. Some of the information may be subject to alteration.

Updated information will be posted on the Maths Central Blackboard site.

Dr Christopher Hallsworth, Mathematics DUGS
August 2023.

Third Year Programmes

In the 3rd year of your Mathematics programme, you choose from a wide range of elective modules. There are no compulsory modules, but if you wish to graduate with a specialist degree title, then you need to choose a certain number of modules related to the specialism.

You are advised to read these notes carefully and discuss your choice of options with your Personal Tutor. An 'Option Fair' will take place after exams in the Summer Term; this will include presentations about the modules on offer from the Pure, Statistics and Applied Mathematics and Mathematical Physics sections of the Department.

Each 3rd year Mathematics module has a credit value of 7.5 ECTS and has a module code beginning with MATH6. In this guide, we will refer to these as MATH6 modules.

In your 3rd year, you take modules with a total credit value of 60 or 62.5 ECTS. This will consist of eight 7.5 ECTS modules or seven 7.5 ECTS modules and two 5 ECTS modules. You must take **at least seven MATH6 modules.** The remaining ECTS can come from 2nd year Mathematics electives not taken in year 2 (5 ECTS each), a further MATH6 module (7.5 ECTS), a module from another Department (subject to approval by DUGS) or certain CLCC or Business School BPES modules (5 ECTS). Where a very strong case can be made, the DUGS may allow some 3rd year students to take a small number of 4th year (level 7) Mathematics modules.

Examples

A total credit value of 60 or 62.5 ECTS allows you to take a combination of 7.5 and 5 ECTS modules in year 3. Some example of possible module combinations include:

Eight MATH6 modules;
Seven MATH6 modules and two MATH5 electives not taken previously;
Seven MATH6 modules, one MATH5 elective not taken previously, one approved CLCC/ Business School module;
Seven MATH6 modules, one MATH5 elective not taken previously, one Department of Computing module (5 ECTS);
Seven MATH6 modules, one Department of Computing module, one approved CLCC/ Business School module.

At most one of your modules can be from CLCC or BPES; not one of each. You may take two computing modules.

Lecturers will give advice on suitable books at the start of each module. You should contact lecturers if you require further details about module content in order to make your choice of options.

You will be asked to make a preliminary selection of your Mathematics modules before the start of the academic year. However, except for modules assessed solely by coursework, or modules outside the Department which will have separate registration deadlines, you will not be committed to your choice of optional modules until the completion of your exam entry information early in term 2.

CLCC / BPES

Registration for CLCC (Horizons) and Business School (BPES) modules opens in May and the Mathematics UG Office sends round an email detailing the registration process. At most one of your modules can be from CLCC or BPES; not one of each. The other 5 ECTS should come from a year 2 maths option or a computing module.

CLCC (Horizons) Information

In Mathematics, any one Horizons module may be taken for credit in Year 3 with the following exceptions: Language modules and the Change Makers Independent Project may only be taken for extra-credit. Horizons options are available here:

https://www.imperial.ac.uk/horizons/taking-a-module-for-credit/module-options-by-department/

Enrolment will take place on the Horizons Enrolment system:

https://www.imperial.ac.uk/horizons/enrolment/

Module descriptions can be found here:

https://www.imperial.ac.uk/horizons/module-options/

Some module descriptions are not currently available as they are being updated, but they will be ready before enrolment opens.

BPES information

You are only allowed to take either

- Managing Innovation - BUSI96004 (Autumn) or
- Finance & Financial Management - BUSI96002 (Spring)

for credit in Year 3. If you took one of these modules for i-Explore in year 2 you may not of course take it again for credit in year 3.

In addition, if you took Corporate Finance Online in year 2 for i-Explore, you may not register for Finance & Financial Management for credit next year, because of shared content between these modules. More information on BPES is available here:

https://www.imperial.ac.uk/business-school/programmes/undergraduate-study/bpes-programme/

Further remarks on non-MATH6 modules

Each year the Department of Computing makes a small number of its modules available to 'external students' (such as Mathematics students). These modules are usually advertised towards the start of the new academic year and the Mathematics UG Office will email you with details when available. Note that assessment for these modules is often at the end of term, when you will also have other coursework deadlines, so you will need to manage your workload carefully.

Students wishing to take a 4th year (level 7) module in their 3rd year require permission from the DUGS. Such requests should not normally be made until close to the start of the academic year.

Very occasionally the Department allows a student to take a module (other than those indicated above) from another Department. This requires discussion with, and approval from, the Mathematics DUGS and the agreement of the other Department. Such requests should not normally be made until close to the start of the academic year.

Computation of your year total

If you take 60 ECTS, consisting of 8 modules all of which are worth 7.5 ECTS, then these modules are weighted equally in the computation of your year 3 total.

If you take 62.5 ECTS, consisting of seven MATH6 modules and two 5 ECTS modules, then the required 52.5 ECTS of MATH6 modules will still count for 87.5 percent of the year total. The remaining 12.5 percent of the year total will come from the average of your marks for the two 5 ECTS modules. This ensures that the weighting on MATH6 modules in the year total is consistent across all students regardless of whether they are taking 60 or 62.5 ECTS.

Specialist BSc degree programmes

Choice of electives is dependent on Degree programme. Students on specialist programmes will be required to take a certain number of electives related to their programme. Students not meeting the requirements may be graduated with an alternative degree programme (such as G100).

G102: Select at least 3 modules from MATH60022 - MATH60027, MATH60050.

G125: Select at least 5 modules from MATH60028 - MATH60042, MATH60140, MATH60050.

G1F3: Select at least 5 modules from MATH60001 - MATH60027, MATH60137, MATH60141, MATH60050.

G1G3: Select at least 5 modules from MATH60026, MATH60043 - MATH60049, MATH60139, MATH60050.

G1GH: Select at least 6 modules from MATH60012, MATH60142, MATH60027, MATH60043 - MATH60049, MATH60130, MATH60138, MATH60050.

GG31: Select at least 5 modules from MATH60005, MATH60024, MATH60027, MATH60043 - MATH60049, MATH60130, MATH60138, MATH60139, MATH60141 MATH60050.

For the BSc project MATH60050 to be counted as one of the 'specialist' modules, it must be on a project related to the specialism.

Personal Tutors and other academic support

In Year 3 your Personal Tutor will continue to offer general support for your studies including discussing your module choices and future options. You will meet your Personal Tutor at least twice a term in terms 1 and 2 during Meet Your Personal Tutor Weeks. You are welcome to also contact your Personal Tutor at different times of the term with queries. You should discuss with your Personal Tutor your academic progress and any concerns and questions. Personal Tutors are also there to support you with any wellbeing concerns you may have and to sign-post you to Departmental or College support services if need be.

In Year 3, Academic support is provided through problems classes and office hours/ question and answer sessions/ discussion boards. The frequency of problems classes is generally less than in previous years: around 5 classes per module. Students are strongly encouraged to set up their own small study groups to support each other by working together on lecture notes and non-assessed problem sheets.

Graduation and progression requirements

Graduation Requirements for three-year degrees G100, G102, G125, G1F3, G1G3, G1GH and GG31:
In order to graduate, you must have passed all modules in the current level of study at first attempt, at resit or by an allowed (dependent on degree programme) compensated pass. In year 3 of BSc programmes, the Exam Board may award compensated passes in up to 15 ECTS of modules where the module mark is below the pass mark but at least 30%. The overall weighted average for the year must be at least 40%, including where a module(s) has been compensated, in order for you to graduate.

MSci Degrees (G103 and G104**):** The MSci is an undergraduate ‘Masters’ degree with a final year at the level of a taught postgraduate MSc programme. On successful completion, a degree title on the lines of ‘Master in Science (incorporating Bachelor’s level study)’ is awarded.

Very occasionally, circumstances may require the Department to graduate an MSci student with a BSc.

Progression Requirements for G103 and G104:
G103: In order to progress to the next level of study, you must have passed all modules (equivalent to 60 ECTS) in the current level of study at first attempt, at resit or by a compensated pass. In year 3 of MSci programmes, the Exam Board may award compensated passes in up to 7.5 ECTS of modules where the module mark is below the pass mark but at least 30%. Additionally, the overall weighted average mark for the year, including where a module(s) has been compensated, must normally be at least 58%.

A student who has been permitted to remain on G103 in year 3 despite not having achieved a year total of 60% from year 2 may be expected to obtain a year total higher than 58% in year 3.

G104: A student who is not permitted to remain on G104 for year 3 will be transferred to a BSc or MSci Mathematics degree.

Full progression information is available in the Programme Specifications:
http://www.imperial.ac.uk/staff/tools-and-reference/quality-assurance-enhancement/programme-information/programme-specifications/

MSci (G103, G104) year 4

In year 4 of the MSci you will normally take six 4th year Mathematics modules (MATH7 modules) and complete a year-long research project

Most MATH6 modules are also available to 4th year and MSc students in an extended MATH7 format. Typically this will involve additional material ('mastery material') for self-study and an additional question on the main exam paper. There are also some modules which are only normally available to 4th year and MSc students. The MATH7 taught modules are 7.5 ECTS and have a pass mark of 50%.

The MSci project is worth 15 ECTS. It is a supervised project in some area of Mathematics. The main aim is to develop a deep understanding of a particular area or topic by means of a supervised project in some area of mathematics. The project may be theoretical and/or computational and the topic for each student is chosen in consultation with the Department. Arrangements for the project are made in the summer term of the 3rd year, after the main examinations. Work on the project should continue throughout all three terms of the Fourth Year and the project is submitted shortly after the Fourth Year examinations.

Module assessment and examinations

Most MATH6 modules are examined by one written examination of 2 hours in length.

Most of the modules have an assessed coursework/progress test element, limited in most cases to 10% of overall module marks. Some modules have a more substantial coursework component (for example, 25 percent) and others are assessed entirely by coursework. Details can be found in the tables below. Precise details of the number and nature of coursework assignments will be provided at the start of each module.

Students should bear in mind that single-term modules assessed by projects require considerable extra time-commitment during that term. You should note that, in principle, 7.5 ECTS represents 187.5 hours of effort on a module and completing this in a single term is a substantial task. For this reason, students may take at most one such module in a term.,.

Note: Students who take a module which is wholly assessed by coursework will be deemed to be officially registered on the module after the submission of a specified number of pieces of assessed work for that module. Thus, once a certain point is reached in one of these modules, a student will be committed to completing it. In contrast, students only become committed to modules with summer examinations when they enter for the examinations in February. For modules outside Mathematics which have earlier examinations, you will be committed to the module once you register for the examination or at any earlier point specified by the host Department.

The BSc project module MATH60050 is examined by a research project; an oral element forms part of the assessment.

Degree Classification

The total of marks for examinations, assessed coursework, progress tests, assignments and projects, with the appropriate year weightings, is calculated and presented at the Examiners' Meeting (normally held at the end of June) for consideration by the Academic Staff and External Examiners. Degree classifications are determined according to the criteria given in the borderline classification algorithm, which can be found on Maths Central. You may also wish to consult the programme specification:

https://www.imperial.ac.uk/mathematics/undergraduate/course-structure-and-content/

Reassessment

Students who do not obtain a pass or compensated pass in examinations at the first attempt will be expected to attend resit examinations where appropriate. In determining whether a Mathematics module has been passed at a resit attempt, if a resit exam has been passed, then the Board of Examiners may discount any module marks obtained from coursework and award an overall pass mark for the module. This will apply to 1st and 2nd year modules and any other modules with a final exam worth at least 90 percent of the module mark. In cases where a student has not achieved the required amount of credit and no further resit attempts are permitted, the Board will graduate the student with an appropriate exit award, as detailed in the programme specifications and regulations.

Students who fail a module will normally be provided with a reassessment opportunity prior to the next academic year. At the discretion of the Board of Examiners, reassessment may be offered in an alternative format, such as an oral examination.

Students who have not achieved the required Passes by the beginning of the new academic year are required by College to spend a year out of attendance. During this time they are not considered College students. This may create a number of issues and hold visa implications.

Students who are required to take a year out due to failed examinations or who take an interruption are not normally permitted to resubmit any coursework previously submitted during their year out.

Resit examinations are for Pass credit only – a maximum mark of 40 percent will be credited. Once a Pass is achieved, no further attempts are permitted.

Criteria for Degree Classification at Borderlines

The discussion here refers to candidates at the first/upper-second borderline. Similar considerations apply to the other grade boundaries.

For all courses, a first is automatic for a programme total P of 69.5 or more. For candidates with $68 \leq P < 69.5$, the classification is determined by considering the uplift criteria as described below.

BSc courses

Classification is based on two uplift criteria: candidates with $68 \leq P < 69.5$ will be promoted to the first class if either of the criteria below is satisfied.

(a) Year 3 Total ≥ 69.5

(b) 30 ECTS in year 3 Mathematics modules at the higher classification (≥ 70 marks). External modules with a high mathematical content may be included, at the discretion of the board of examiners.

MSci courses

Classification is based on three uplift criteria: candidates with $68 \leq P < 69.5$ will be promoted to first class if any two of the criteria below are satisfied.

(a) Year 4 Total ≥ 69.5

(b) MSci project at the higher classification (≥ 70 marks)

(c) 22.5 ECTS in year 4 Mathematics electives at the higher classification (≥ 70 marks). External modules with a high mathematical content may be included, at the discretion of the board of examiners.

Mitigating Circumstances

Candidates with accepted mitigating circumstances for modules where no mitigation has been applied may be uplifted for $65 \leq P < 69.5$. In this case, the uplift criteria above may be modified to account for the circumstances of individual candidates. The rationale for any uplift should nevertheless make reference to the criteria above, e.g. noting the evidence for performance at the higher class in any components of the programme unaffected by mitigating circumstances. Care should be given to ensuring that mitigating circumstances are fully taken into account, without giving unfair advantage to the student, or subjecting them to more demanding requirements.

Degree Changes

Students are able to change between three-year mathematics degree programmes (or dropping down from a four-year to a three-year programme) by completing a Degree Change form and (if appropriate) ensuring that they comply with the requirements for any specialist coding module options.

Students wishing to move to the G103 programme (after the second year) must be able to comply with the Year 2 and 3 mark requirements.

International students who require a visa are advised to consult the International Student Support Office prior to making ANY degree change as you may be required to apply for a new visa (outside of the UK).

To request a degree change, students must complete a Degree Change form which can be found on Blackboard Maths Central. The form should be returned to the Undergraduate Office.

All degree transfer requests should normally be made by 31st of March in the year of the transfer, but transfers can in principle be made in any of years 1, 2 or 3.

If you change your degree coding and are registered on a module outside the Department, including BPES or CLCC, please ensure that you let the host Department know, so that you are not removed from their module lists.

Third Year Module List

Note that not all of the individual modules listed below are offered every session and the Department reserves the right to cancel a particular module if, for example, the number of students wishing to attend that module does not make it viable. Similarly, some modules are occasionally run as ‘Reading Modules’. In general, we recommend that you consider having a small number of backup options in case a module changes, or if you decide that you no longer wish to take a module.

Modules marked * are also available in MATH7 form for Fourth Year MSci students (which typically involves taking a longer Examination). When a module is offered, it is usually, but not always, available in both forms. **No student may take both the 3rd year and 4th year forms of a module.**

If you take the module MATH60049 you will not be allowed to select the Machine Learning component of the 4th year Advanced Topics in Statistics module.

The following notes relate to the tables on optional modules for Year 3 as below:

The indicated lecturers are provisional; TBC indicates 'to be confirmed'.

All MATH6 modules are equally weighted and have a credit value of 7.5 ECTS

The % Exam indicates a written exam, unless otherwise indicated.The % CW indicates any coursework that is completed for the module. This may include in-class tests, projects, or problem sets to be turned in.

APPLIED MATHEMATICS/MATHEMATICAL PHYSICS/NUMERICAL ANALYSIS

College Module Code	Module Titles	Terms	Lecturer	% Exam	% CW
FLUIDS					
MATH60001*	Fluid Dynamics 1	1	Professor X. Wu	90	10
MATH60002*	Fluid Dynamics 2	2	Professor J. Mestel	90	10
MATH60003*	Introduction to Geophysical Fluid Dynamics	2	Dr P. Berloff	90	10
MATHEMATICAL METHODS					
MATH60004*	Asymptotic Methods	1	Dr G. Peng	90	10
MATH60005*	Optimisation	1	Dr D. Kalise	90	10
MATH60006*	Applied Complex Analysis	1	Dr S. Brzezicki	90	10
MATH60141*	Introduction to Game Theory	1	Dr S. Brzezicki	90	10
DYNAMICS					
MATH60007*	Dynamics of Learning and Iterated Games	1	Professor S.V. Strien	40 (Oral)	60
MATH60008*	Dynamical Systems	1	Professor J. Lamb	90	10
MATH60009*	Bifurcation Theory	2	Dr D. Li	90	10
MATH60143*	Dynamics, Symmetry and Integrability	2	Professor D. Holm	90	10
MATH60011*	Classical Dynamics	1	Dr B. Walter	90	10

FINANCE

MATH60012*	Mathematical Finance: An Introduction to Option Pricing	1	Dr P. Siorpaes	90	10
MATH60142*	Mathematics of Business & Economics	2	Mr M. Autenrieth	90	10
MATH60130*	Stochastic Differential Equations in Financial Modelling	2	Professor D. Brigo	90	10
MATH60138*	Rough Paths and Applications to Machine Learning	2	Dr C. Salvi	90	10

BIOLOGY

MATH60014*	Mathematical Biology	1	Dr E. Keaveny	90	10
MATH60137*	Mathematical Biology 2: Systems Biology	2	Dr O. Karin,	90	10

MATHEMATICAL PHYSICS

MATH60015*	Quantum Mechanics I	1	Dr E-M. Graefe	90	10
MATH60016*	Special Relativity and Electromagnetism	1	Dr G. Pruessner	90	10
MATH60017*	Tensor Calculus and General Relativity	2	Dr C. Ford	90	10
MATH60018*	Quantum Mechanics 2	2	Dr R. Barnett	90	10

APPLIED PDEs, NUMERICAL ANALYSIS and COMPUTATION

MATH60019*	Theory of Partial Differential Equations	1	Mr. V Navarro Fernandez	90	10
MATH60020*	Function Spaces and Applications	1	Professor P. Germain	90	10
MATH60021*	Advanced Topics in Partial Differential equations	2	Dr A. Menegaki	90	10
MATH60022*	Finite Elements: Numerical Analysis and Implementation	2	Professor C. Cotter & Dr D. Ham	50	50
MATH60023*	Numerical Solution of Ordinary Differential Equations	1	Dr D. Ruiz I Balet	0	100
MATH60024*	Computational Linear Algebra	1	Professor C. Cotter	50	50
MATH60025*	Computational Partial Differential Equations	2	Dr S. Mughal	0	100
MATH60026*	Methods for Data Science	2	Professor M Barahona and Dr B. Bravi	0	100
MATH60027*	Scientific Computation	1	Dr P. Ray	0	100

PURE MATHEMATICS

College Module Code	Module Titles	Terms	Lecturer	% Exam	% CW
ANALYSIS					
MATH60028*	Probability Theory	1	Dr I. Krasovsky	90	10
MATH60029*	Functional Analysis	2	Dr P. Rodriguez	90	10
MATH60030*	Fourier Analysis and the Theory of Distributions	2	Dr I. Krasovsky	90	10
MATH60031*	Markov Processes	1	Dr A. Chandra	90	10
GEOMETRY					
MATH60032*	Geometry of Curves and Surfaces	2	Dr M. Taylor	90	10
MATH60033*	Algebraic Curves	1	Dr S. Sivek	90	10
MATH60034*	Algebraic Topology	2	Y.A.Chen	90	10
MATH60140*	Geometric Complex Analysis	2	Dr D. Cheraghi	90	10
ALGEBRA AND DISCRETE MATHEMATICS					
MATH60035*	Algebra 3	1	Dr N. Porcelli	90	10
MATH60036*	Group Theory	1	Professor M. Liebeck	90	10
MATH60037*	Galois Theory	1	Professor T. Schedler	90	10
MATH60038*	Graph Theory	1	Dr M. Zordan	90	10
MATH60039*	Group Representation Theory	2	Professor T.Schedler	90	10
MATH60040*	Formalising Mathematics	2	Professor K. Buzzard	0	100
MATH60132*	Mathematical Logic	1	Professor D. Evans	90	10
NUMBER THEORY					
MATH60041*	Number Theory	1	Mr B. Sempliner	90	10
MATH60042*	Algebraic Number Theory	2	Dr G. A. Boxer	90	10

STATISTICS

College Module Code	Module Titles	Terms	Lecturer	% exam	% CW
MATH60043*	Statistical Theory	2	Dr K. Ray	90	10
MATH60044*	Applied Statistical Inference	2	Dr C. Hallsworth	75	25
MATH60045*	Applied Probability	1	Dr N. Kantas	90	10
MATH60046*	Time Series Analysis	1	Dr E. Cohen	90	10
MATH60047*	Stochastic Simulation	1	Dr D. Akyildiz	75	25
MATH60048*	Survival Models	2	Dr H. Battey	90	10

MATH60049*	Introduction to Statistical Learning	2	Professor G. Nason	90	10
MATH60139*	Spatial Statistics	2	Dr A. Sykulski	90	10

PROJECT (Only Available to Final Year BSc Students)

College Module Code	Module Titles	Terms	Lecturer	% Exam	% CW
MATH60050	Research Project in Mathematics	2 + 3	Dr T. Bertrand	0	100

MATH60001 Fluid Dynamics I

Brief Description

Fluid dynamics investigates motions of both liquids and gases. Being a major branch of continuum mechanics, it does not deal with individual molecules, but with an ‘averaged' motion of the medium (i.e. collections of molecules). The aim is to predict the velocity, pressure and temperature fields in flows arising in nature and engineering applications. In this module, the equations governing fluid flows are derived by applying fundamental physical laws to the continuum. This is followed by descriptions of various techniques to simplify and solve the equations with the purpose of describing the motion of fluids under different conditions.

Learning Outcomes

On successful completion of this module you will be able to

- state the underlying assumptions of the continuum hypothesis;

- compare and contrast the different frameworks that can be used to describe fluid motion and to identify the connections between them;

- derive exact solutions of the Navier-Stokes equations and justify physical and mathematical assumptions made in obtaining them;

- perform simplifications arising under the assumption of inviscid flow which permit the integration of the Euler equations, leading to results such as Bernoulli's equation and Kelvin's circulation theorem;

- demonstrate a sound understanding of the method of conformal mappings and be able to use this method to analyse various two-dimensional inviscid flows.

Module Content

The module is composed of the following sections:

Introduction: The continuum hypothesis. Knudsen number. The notion of fluid particle. Kinematics of the flow field. Lagrangian and Eulerian frameworks. Streamlines and pathlines. Strain rate tensor. Vorticity and circulation. Helmholtz's first theorem. Streamfunction.

Governing Equations: Continuity equation. Stress tensor and symmetry, Constitutive relation. The Navier-Stokes equations.

Exact Solutions of the Navier-Stokes Equations: Couette and Poiseuille flows. The flow between two coaxial cylinders. The flow over an impulsively started plate. Diffusion of a potential vortex.

Inviscid Flow Theory: Integrals of motion. Kelvin's circulation theorem. Potential flows. Bernoulli's equation. Cauchy-Bernoulli integral for unsteady flows. Two-dimensional flows. Complex potential. Vortex, source, dipole and the flow past a circular cylinder. Adjoint mass. Conformal mapping. Joukovskii transformation. Flows past aerofoils. Lift force. The theory of separated flows. Kirchhoff and Chaplygin models.

MATH60002 Fluid Dynamics 2

Brief Description

In this module, we deal with a wide class of realistic problems by seeking asymptotic solutions of the governing Navier-Stokes equations in various limits. We shall start with the "slow, small or sticky" case, when the Reynolds number is low and we obtain the linear Stokes equations. Then we consider the lubrication limit, and show how a thin layer of fluid is able to exert enormous pressures and prevent moving solid bodies from touching. Next we shall consider the "fast and vast" limit of high Reynolds number, which is characteristic of most flows we encounter in everyday life. In the final part of the module we consider a mixture of advanced topics, including flight, bio-fluid-dynamics and an introduction to flow stability.

Learning Outcomes

On successful completion of this module you will be able to:

- simplify and solve the governing Navier-Stokes equations in situations where there is a short lengthscale in one of the coordinate directions;

- apply the general properties of low Reynolds number flows to predict the drag on slow-moving bodies, like a solid sphere or spherical bubble, and appreciate the causes of the 'Stokes paradox';

- analyse lubrication-like flows in thin layers;

- derive the boundary-layer equations and identify self-similar solutions for flows at large Reynolds number;

- determine stability criteria for various fundamental flows;

- model animal locomotion at low and high Reynolds numbers.

Module Content

The module is composed of the following sections:

I – Low-Reynolds-number flows

Dynamic Similarity. Properties of the Stokes equations. Uniqueness and minimal dissipation theorems. The analysis of the flow past a solid sphere and spherical bubble. Stokes paradox.

II – Lubrication Theory

Derivation of Reynolds' lubrication equation and examples. Hele-Shaw and thin film flows.

III – High-Reynolds-number flows; Boundary-layer theory

The notion of singular perturbations. Derivation of boundary-layer equations. Blasius flow, Falkner-Skan solutions and applications. Von Mises variables and their application to periodic boundary layers. Prandtl-Batchelor Theorem for flows with closed streamlines.

IV – Introduction to hydrodynamic stability

Importance of stability. Rayleigh-Taylor and Kelvin-Helmholtz instabilities. Circular flow stability criterion.

V – Swimming and Flight; Animal locomotion

Scallop theorem. Resistive Force Theory. Introduction to 3D-aerofoil theory. Flight strategies.

MATH60003 Introduction to Geophysical Fluid Dynamics

Brief Description

This is an advanced-level fluid-dynamics course with geophysical flavours. The lectures target upper-level undergraduate and graduate students interested in the mathematics of planet Earth, and in the variety of motions and phenomena occurring in planetary atmospheres and oceans. The lectures provide a mix of theory and applications.

Learning Outcomes

On successful completion of this module you will be able to:

- demonstrate a deep understanding of the foundations of geophysical fluid dynamics;

- model a broad range of natural phenomena associated with the atmosphere and ocean;

- appreciate the main concepts and terminology used in the field;

- derive the boundary layer equations for flow in a rotating frame and justify the relative importance of various terms in the equations of motion;

- describe, select appropriately and apply a range of methods and techniques for solving practical problems.

Module Content

The module is composed of the following sections:

I - Introduction and basics;

II - Governing equations (continuity of mass, material tracer, momentum equations, equation of state, thermodynamic equation, spherical coordinates, basic approximations);

III - Geostrophic dynamics (shallow-water model, potential vorticity conservation law, Rossby number expansion, geostrophic and hydrostatic balances, ageostrophic continuity, vorticity equation);

IV - Quasigeostrophic theory (two-layer model, potential vorticity conservation, continuous stratification, planetary geostrophy);

V - Ekman layers (boundary-layer analysis, Ekman pumping);

VI - Rossby waves (general properties of waves, physical mechanism, energetics, reflections, mean-flow effect, two-layer and continuously stratified models);

VII - Hydrodynamic instabilities (barotropic and baroclinic instabilities, necessary conditions, physical mechanisms, energy conversions, Eady and Phillips models);

VIII - Ageostrophic motions (linearized shallow-water model, Poincare and Kelvin waves, equatorial waves, ENSO "delayed oscillator", geostrophic adjustment, deep-water and stratified gravity waves);

IX - Transport phenomena (Stokes drift, turbulent diffusion);

X - Nonlinear dynamics and wave-mean flow interactions (closure problem and eddy parameterization, triad interactions, Reynolds decomposition, integrals of motion, enstrophy equations, classical 3D turbulence, 2D turbulence, transformed Eulerian mean, Eliassen-Palm flux).

MATH60004 Asymptotic Methods

Brief Description

This advanced course presents a systematical introduction to asymptotic methods, which form one of the cornerstones of modern applied mathematics. The foundation of asymptotic approximations is laid down first. The key ideas and techniques for deriving asymptotic representations of integrals, and for constructing appropriate solutions to differential equations will be explained. The techniques introduced find wide applications in engineering and natural sciences.

Learning Outcomes

On successful completion of this module, you will be able to:

- appreciate the foundation upon which asymptotic approximations are based;

- describe a variety of asymptotic methods and for each method acquire a thorough understanding of the key ideas involved and their mathematical nature;

- demonstrate basic skills in applying each of these methods to solve classical problems;

- combine, modify and extend methods to unfamiliar problems, such as those that emerge from research topics or practical applications;

- outline how asymptotic methods can in principle be applied to a wide variety of problems.

Module Content

I - Asymptotic approximations (fundamentals)

Order notation. Diverging series, asymptotic expansions. Parameter expansions, overlap regions, distinguished limits and uniform approximations.

II - Introduction to perturbation methods

Asymptotic solution of algebraic equations with a small parameter. Regular vs. singular perturbations. Method of dominant balance. Local analysis of ordinary differential equations.

III - Asymptotic analysis of integrals

Method of integration by parts. Integrals of Laplace type: Laplace's method, Watson's Lemma. Integrals of Fourier type: method of stationary phase. Integral in the complex plane: method of steepest descent. Method of splitting the range of integration.

IV - Matched asymptotic expansion

Inner and outer expansions, matching principles, notions of 'boundary layer' and interior layer. Composite approximation. Application to relaxation oscillations.

V - Methods of multiple scales

WKB approximations including turning-point problems and eigenvalue quantisation. Secular terms and solvability conditions. Poincare-Lindstedt method for periodic solutions. Multiscale method for quasi-periodic solutions. Application to weakly perturbed oscillators, nonlinear resonance, parametric resonance.

MATH60005 Optimisation

Brief Description

This module is an introduction to the theory and practice of mathematical optimization and its many applications in mathematics, data science, and engineering. The module aims at endowing students with the necessary mathematical background and a thorough methodological toolbox to formulate optimization problems and developing an algorithmic approach to its solution.The module is structured into five parts: (i) formulation and classification of problems; (ii) unconstrained optimization; (iii) stochastic and nature-inspired optimization; (iv) convex optimization; (v) introduction to optimal control and dynamic optimization. The assessed coursework for this module involves a series of computational tasks, to be completed in groups of two or three

Learning Outcomes

On successful completion of this module you will be able to

- formulate a mathematical optimization problem by identifying a suitable objective and constraints;

- identify the mathematical structure of an optimization problem and, based on this classification, choose an appropriate methodological approach;

- develop a mathematical and computational appreciation of convexity as a fundamental feature in optimization;

- implement different computational optimization algorithms such as gradient descent and related variants;

- analyse the results of a computational optimization method in terms of optimality guarantees, sensitivities, and performance.

Module Content

1. Mathematical preliminaries
2. Unconstrained optimization
3. Gradient descent methods
4. Linear and non-linear least squares problems
5. Stochastic gradient descent
6. Nature-inspired optimization
7. Convex sets and functions
8. Convex optimization problems and stationarity
9. KKT conditions
10. Duality
11. Introduction to dynamic optimization and optimal control.

MATH60006 Applied Complex Analysis

Brief Description

The aim of this module is to learn tools and techniques from complex analysis and the theory of orthogonal polynomials that can be used in mathematical physics. The course will focus on mathematical techniques, though will also discuss relevant physical applications, such as electrostatic potential theory. The course incorporates computational techniques in the lectures.

Learning Outcomes

On successful completion of this module, you will be able to:

- apply the technique of contour deformation for calculating integrals;

- appreciate the connection that exists between computational tools such as quadrature and orthogonal polynomials and complex analysis;

- evaluate singular integral equations with Cauchy and logarithmic kernels;

- use the Wiener-Hopf method to solve a class of integral equations;

- compute matrix functions using contour inegration.

Module Content

This module covers the following topics:

Revision of complex analysis: complex integration, Cauchy's theorem and residue calculus;

Singular integrals: Cauchy, Hilbert, and log kernel transforms;

Potential theory: Laplace's equation, electrostatic potentials, distribution of charges in a well;

Riemann–Hilbert problems: Plemelj formulae, additive and multiplicative Riemann–Hilbert problems;

Orthogonal polynomials: recurrence relationships, solving differential equations, calculating singular

integrals;

Integral equations: integral equations on the whole and half line, Fourier transforms, Laplace transforms;

Wiener–Hopf method: direct solution, solution via Riemann–Hilbert methods.

MATH60141 Introduction to Game Theory

Brief Description

This module will give students an insight into the wide variety of mathematics and its many applications within the area of game theory. The module aims to promote an active learning style, involving many classroom games as well as games to be played as homework.

The module will cover the classical theory of games involving concepts of dominance, best response and equilibria, where we will prove Nash's Theorem on the existence of equilibria in games. We will see the concept of when a game is termed zero-sum and prove the related Von Neumann’s Minimax Theorem. We will briefly discuss cooperation in games and investigate the interesting Nash bargaining solution which arises beautifully from reasonable bargaining axioms.

Broadening our scope, we will look at the area of combinatorial game theory, building up our intuition through investigating the classical game of Nim in detail. We will also see the concept of a congestion game, often applied to situations involving traffic flow, where we will see the counter intuitive Braess paradox emerge and prove Nash’s theorem in another context.

The module will finish with a small tour through some other areas and applications of game theory.

Learning Outcomes

On successful completion of this module you will be able to:

- define the concepts of dominance, best-response and equilibria in a variety of competitive scenarios (games);

- solve (determine all equilibria or find optimal strategies) small games via a variety of techniques: iterated deletion of dominated strategies, finding equaliser strategies, use of subgames;

- determine when a game may be termed zero-sum, and be able to recognise, find and apply minimax and maximin strategies in these games;

- apply game theory to traffic flow or flow of information through networks, appreciating the differences and importance of optimal societal routing as compared with selfish individual routing;

- calculate bargaining solutions in simple co-operative games;

- determine whether communication is beneficial or not in different strategic situations;

Indicative Module Content

1. Recap of some basic notions in probability, calculus and analysis, some recap of induction in a game theoretic context.

2. Motivational/illustrative classroom games.

3. Dominance, best-response and equilibria.

4. Nash's theorem on equilibria in games.

5. Zero-sum games and Von Neumann's minimax theorem.

6. Subgame solutions as extensions to full game solutions.

7. Cooperative games, the Nash arbitration procedure and bargaining solutions.

8. Congestion games; Braess paradox, selfish routing vs optimal societal routing, existence of equilibria.

9. Combinatorial games; Nim, Nim sums and Nim values, sums of games.

MATH60007 Dynamics of Learning and Iterated Games

Brief Description

Recently there has been considerable interest in modelling learning. The settings to which these models are applied is wide-ranging. Examples include optimization of strategies of populations in ecology and biology, iterated strategies of people in a competitive environment and learning models used by technology companies such as Google.

This module is aimed at discussing a number of such models in which learning evolves over time and which have a game theoretic background. The module will use tools from the theory of dynamical systems and will aim to be rigorous. Topics will include replicator systems, best response dynamics and fictitious games, reinforcement learning and no-regret learning.

Learning Outcomes

On successful completion of this module, you will be able to:

- analyse 2D replicators systems for one and two player games;

- work comfortably with the notions of Nash, Correlated Equilibrium, Course Correlated Equilibrium and Evolutionary Stable Strategies;

- explain the notion of reciprocity in relation to Iterated Prisoner Dilemma games;

- appreciate the connection between Reinforcement Learning and replicator systems;

- outline the idea behind no regret learning models and the Blackwell approachability theorem.

Module Content

The module will cover the following topics:

- Replicator systems;

- Rock-paper-scissor games;

- Iterated prisoner dilemma games;

- Best response dynamics;

- Two player games;

- Fictitious games as a learning model;

- Reinforcement learning;

- No regret learning.

MATH60008 Dynamical Systems

Brief Description

The theory of Dynamical Systems is an important area of mathematics which aims at describing objects whose state changes over time. For instance, the solar system comprising the sun and all planets is a dynamical system, and dynamical systems can be found in many other areas such as finance, physics, biology and social sciences. This course provides a rigorous treatment of the foundations of discrete-time dynamical systems.

Learning Outcomes

On successful completion of this module, you will be able to:

- demonstrate a familiarity with the basic concepts of topological dynamics;

- provide an outline of the ergodic theory of dynamical systems;

- appreciate the concept of symbolic dynamics through which topological and probabilistic dynamical properties can be understood;

- demonstrate an understanding of precise mathematical characterisations of chaotic dynamics;

- apply the above context in a number of one-dimensional settings, in particular in the context of piecewise affine expanding maps.

Module Content

The module covers the following topics:

- Introduction: orbits, periodic orbits and their local stability;

- Topological dynamics: invariant sets and limit sets, coding and sequence spaces, topological conjugacy, transitivity and mixing;

- Chaotic dynamics: sensitive dependence, topological entropy, topological Markov chains;

- Ergodic theory: sigma-algebras and measures, invariant measures, Poincaré recurrence, ergodicity and Birkhoff's Ergodic Theorem, Markov measures and metric entropy.

MATH60009 Bifurcation Theory

Brief Description

This module serves as an introduction to bifurcation theory, concerning the study of how the behaviour of dynamical systems such as ODEs and maps changes when parameters are varied.The goal is to acquaint the students with the foundations of the theory, its main discoveries and the universal methods behind this theory that extend beyond its remit.

Learning Outcomes

On successful completion of this module, you will be able to:

- exploit basic dimension reduction methods (invariant manifold and invariant foliations);

- apply the method of normal forms;

- demonstrate a sound knowledge of the basics of stability theory;

- appreciate the role of control parameters and to construct bifurcation diagrams;

- describe the mathematical framework associated with classical local and global bifurcations.

Module Content

The following topics will be covered:

1) Bifurcations on a line and on a plane;

2) Centre manifold theorem; local bifurcations of equilibrium states;

3) Local bifurcations of periodic orbits – folds and cusps;

4) Homoclinic loops: cases with simple dynamics, Shilnikov chaos, Lorenz attractor;

5) Saddle-node bifurcations: destruction of a torus, intermittency, blue-sky catastrophe;

6) Routes to chaos and homoclinic tangency.

Dynamics, Symmetry and Integrability

Brief Description

This module on Dynamics, Symmetry and Integrability is a friendly and fast-paced introduction to the geometric approach to proving integrability of classical Hamiltonian systems, at the level suitable for advanced undergraduates and first-year graduate students in mathematics. It fills a gap between traditional classical mechanics texts and advanced mathematical treatments of the geometric approach to integrability. The key idea is to use the momentum maps (e.g. from Noether's theorem) to find enough conservation laws to prove integrability. The main examples of integrable PDEs discussed are those that model shallow water waves, particularly the Korteweg-de Vries and Camassa-Holm equations.

Learning Outcomes

On successful completion of this module, you will be able to:

- describe Hamiltonian motion on a smooth finite-dimensional manifold and demonstrate familiarity with the cotangent bundle (T*M, phase space) and the definition of canonical Poisson brackets, as well as Hamiltonian vector fields, symplectic forms, symplectic transformations and solutions as characteristic flows of Hamiltonian vector fields on T*M;

- define Liouville integrability for finite-dimensional Hamiltonian dynamical systems and appreciate that Liouville integrability requires a sufficient number of functionally independent conservation laws in involution;

- select and use several other methods (introduced via worked examples) for acquiring the conservation laws necessary to prove integrability including: reduction to elliptic curves, isospectral reformulation in Lax pair form using covariant derivatives with zero curvature and transformation of variables to the momentum maps which arise in Noether's theorem;

- determine momaps for Hamiltonian systems with symmetry for a variety of classic finite-dimensional problems including: rigid body motion in Rn, coupled nonlinear oscillators in C2 and C3 and the reduction of the CH equation to finite dimensions which results from a singular momap;

- interpret the Lax pair form of isospectral dynamics as coadjoint motion of a cotangent lift momentum map leading to the Lie-Poisson bracket which features widely in establishing the integrability of Hamiltonian systems.

Module Content

The module is composed of the following sections:

I - Dynamics

The main ideas of the course are illuminated by considering cases when the solution dynamics on the configuration manifold may be lifted to a (non-Abelian) Lie group symmetry of the Hamiltonian. With an emphasis on applications in ODEs of finite-dimensional mechanical systems, such as the rigid body SO(3) and coupled resonant oscillations U(2), and PDEs of nonlinear waves, such as the KdV and CH equation in infinite dimensions, the properties and results for integrability which are inherited from the geometrical formulation of dynamics induced by Lie group actions are discussed.

II - Symmetry

Symmetries of the Hamiltonian under Lie group transformations and their associated momentum maps are emphasised, both for reducing the number of independent degrees of freedom and in finding conservation laws by Noether's theorem.

III - Integrability

Definition: According to Liouville, a Hamiltonian system on a 2N-dimensional symplectic manifold M2N

is completely integrable, if it possesses N functionally independent conservation laws which

mutually commute under canonical Poisson brackets. What makes a dynamical system integrable, then? Enough conservation laws!

The course develops a series of geometrical methods for finding mutually Poisson-commuting conservation laws and thereby solving a sequence of integrable Hamiltonian problems ranging from rigid body motion to nonlinear wave PDEs. These methods include isospectral Lax pair formulations and algebraic geometry of elliptic curves for rigid body motion, as well as Lax equations and isospectrality principles due to bi-Hamiltonian structures for the KdV and CH water wave equations. In developing the solvability algorithms for this sequence of problems, the momentum map for the cotangent lift action of a Lie group on a manifold M plays a central role in representing the equations, their solutions and the analysis of their solution behaviour.

MATH60011 Classical Dynamics

Brief Description

Classical dynamics is developed through variational principles rather than Newtonian force laws. Lagrangian and Hamiltonian formulations are considered. The methods are applied to a variety of problems including pendulums, the Kepler problem, rigid bodies and motion of a charged particle in a magnetic field. The role of conserved quantities is emphasised. An introduction to more advanced ideas including Hamilton-Jacobi theory and action-angle variables is given.

Learning Outcomes

On successful completion of this module, you will be able to:

- reformulate Newton's laws through variational principles;

- construct Lagrangians or Hamiltonians for dynamics problems in any coordinate system;

- solve the equations of motion for a wide variety of problems in dynamics;

- identify and exploit constants of the motion in solving dynamics problems;

- apply Lagrangian and Hamiltonian methods to a problems in a variety of fields (e.g. Statistical Mechanics, Quantum Mechanics and Geometric Mechanics).

Module Content

This module will cover the following topics:

1. Review of the Calculus of Variations.

2. Newtonian Mechanics: momentum, angular momentum, conservative forces.

3. Lagrangian Mechanics: Hamilton's Principle, Lagrangians for conservative and non-conservative systems, generalised coordinates and momenta, cyclic coordinates, Noether's theorem (conservation of angular momentum as an example).

4. Hamiltonian Mechanics: Phase Space, Hamilton's equations, Poisson brackets, canonical transformations, generating functions, Hamilton-Jacobi theory, action-angle variables, integrability, application of Hamiltonian mechanics to rigid bodies.

MATH60012 Mathematical Finance: An Introduction to Option Pricing

Brief Description

The mathematical modelling of derivatives securities, initiated by Bachelier in 1900 and developed by Black, Scholes and Merton in the 1970s, focuses on the pricing and hedging of options, futures and other derivatives, using a probabilistic representation of market uncertainty. This module is a mathematical introduction to this theory, in a discrete-time setting. We will mostly focus on the no-arbitrage theory in market models described by trees; eventually we will take the continuous-time limit of a binomial tree to obtain the celebrated Black-Scholes model and pricing formula.

Learning Outcomes

On successful completion of this module, you will be able to:

- appreciate the fundamental principles involved in pricing derivatives;

- describe and critically analyse simple market models and explore their qualitative properties;

- confidently perform calculations involving pricing and hedging in discrete market models;

- demonstrate a familiarity with some key concepts in modern probability theory and apply them to perform computations;

- outline a mathematical formulation describing the behaviour of a number of financial derivatives.

Module Content

The module will cover the following topics:

financial derivatives, arbitrage, no-arbitrage pricing, self-financing portfolios, non-anticipative trading strategies, hedging of derivatives, domination property, complete markets, ‘risk-neutral’ probabilities, the fundamental theorems of asset pricing, conditional probability and expectation, filtrations, Markov processes, martingales, change of measure.

MATH60130 Stochastic Differential Equations in Financial Modelling

Brief Description

To deal with valuation, hedging and risk management of financial options, we briefly introduce stochastic differential equations using a Riemann-Stiltjes approach to stochastic integration. We introduce no-arbitrage theory in continuous time based on replicating portfolios, self-financing conditions and Ito's formula. We derive prices as risk neutral expectations. We derive the Black Scholes model and introduce volatility smile models. We illustrate valuation of different options and introduce risk measures like Value at Risk and Expected Shortfall, motivating them with the financial crises.

Learning Outcomes

On successful completion of this module you will be able to

- work comfortably with stochastic differential equations commonly encountered in finance

- explain what is meant by no-arbitrage markets and why no-arbitrage is important operationally;

- connect no-arbitrage by replication to the existence of a risk neutral measure;

- price and hedge several types of financial options with several SDE models;

- calculate risk measures such as Value at Risk and Expected Shortfall;

- write code to price options according to SDE models covered in the module.

Module Content

1. Recap of key tools from probability theory

2. Brownian motion

3. Ito and Stratonovich stochastic integrals

4. Ito and Stratonovich stochastic differential equations (SDEs)

5. No-arbitrage through replication

6. No arbitrage though risk neutral measure

7. Derivation of the Black Scholes formula

8. Introduction of a few volatility smile models

9. Pricing of several types of options

10. Introduction to crises and risk measures

11. The Barings collapse and the introduction of value at risk (VaR)

12. Problems of VaR and an alternative: expected shortfall (ES)

13. Numerical examples and problems with risk measures, including software code.

MATH60138 Rough Paths

Brief Description

The theory of rough paths provides a mathematical language to describe the effects a stream can generate when interacting with non-linear dynamical systems. It has had a significant impact on several areas of stochastic analysis, notably on the development of Hairer's Fields medal winning work on regularity structures for singular stochastic PDEs. In recent years, rough path theory has played a key role in the design of state-of-the-art machine learning algorithms for processing irregularly sampled time series data. This first half of this module will focus on the mathematics of signatures and signature kernels and applications to machine learning. The second half will focus on constructing rough integration, establish solutions to rough differential equations and their consistency with stochastic differential equations, and finally on the interplay between rough paths and modern deep learning models dubbed neural differential equations, which incorporate neural networks as vector fields of classical differential equations.

Learning Outcomes

On completion of this module students will be able to display mastery of a complex and specialised area of knowledge and skills in rough paths and applications to machine learning. In particular students who attended the course will be able to:

• Use the basic properties of the signatures and signature kernels and present their utility in machine learning.

• Understand the basics of rough integration, rough differential equations and their consistency with classical stochastic integration.

• Use neural differential equations models (Neural ODEs, CDEs, SDEs, RDES).

• Implement in Python some of the above models for synthetic and real-world examples

Indicative Module Content

1. Analytic and algebraic properties of the signature

2. Functional analysis, topology and probability on unparameterised paths

3. Signature kernels and associated reproducing kernel Hilbert spaces

4. Universality and characteristicness of signature kernels

5. Computing signature kernels as solutions to PDEs

6. Applications to machine learning

7. Geometric and controlled rough paths

8. From Young integration to rough integration

9. From stochastic to rough differential equations

10. Neural ODEs, SDEs, RDEs and the log-ODE method

Prerequisites in stochastic calculus, the theory and numerical analysis of ODEs and SDEs are assumed.

MATH60142 Mathematics of Business & Economics

Brief Description

This module gives a broad mathematical introduction to both microeconomics and macroeconomics, with a particular emphasis on the former. We consider the motivations and optimal behaviours of firms and consumers in the marketplace (profit and utility maximisation, respectively), and show how this leads to the widely observed laws of supply and demand. We look at the interaction of firms and consumers in markets of varying levels of competition. In the final section, we discuss the interplay of firms, households and the government on a macroeconomic scale.

Learning Outcomes

On successful completion of this module, you will be able to:

- Give a mathematical description of the problems of profit maximisation and utility maximisation.

- Apply the concepts of preference relations and utility functions as well as their interaction.

- Determine optimal behaviour by solving the profit or utility maximisation problem for stylised examples.

- Describe the change in demand with respect to a change in price or income.

- Characterise the equilibrium price or quantity as the maximiser of the community surplus, and discuss how deviations lead to a deadweight loss.

- Describe the interplay of firms, households, government, the financial sector and the oversees sector in a stylised way.

- State and apply the definition of Gross Domestic Product (GDP) as a measure of the economic activity.

- Discuss shortcomings of GDP as a measure of social welfare and how to overcome these shortcomings.

Module Content

An indicative list of sections and topics is:

- Theory of the firm: Profit maximisation for a competitive firm. Cost minimisation. Geometry of costs. Profit maximisation for a non-competitive firm.

- Theory of the consumer: Consumer preferences and utility maximisation. The Slutsky equation.

- Levels of competition in a market: Consumers' and Producers' surplus. Deadweight loss.

- Macroeconomic theory: Circular flow of income. Gross Domestic Product. Social welfare and allocation of income.

MATH60014 Mathematical Biology

Brief Description

Mathematical biology entails the use of mathematics to model biological phenomena in order to understand these systems, as well as predict their behaviour. It is an incredibly diverse field utilising the complete mathematical toolbox to ascertain insight into many areas of biology and medicine including population dynamics, physiology, epidemiology, cell biology, biochemical reactions, and neurology. This module aims to provide a foundational course in the subject area relying primarily on tools from applied dynamical systems, applied PDEs, asymptotic analysis and stochastic processes.

Learning Outcomes

On successful completion of this module you will be able to:

- translate biological phenomena into the language of mathematics;

- appreciate canonical problems in epidemiology, ecology, biochemistry and physiology;

- critically analyse sets of ordinary differential equations especially in the non-linear setting;

- critically analyse sets of partial differential equations especially when either travelling-wave solutions or pattern forming phenomena might emerge;

- utilise the concept of stochastic population processes for exact and approximate solutions;

- use the techniques of order-of-magnitude reasoning and dimensional analysis.

Module Content

Examples and topics include:

1) One-dimensional systems: existence and uniqueness; fixed points and their stability; bifurcations; logistic growth; SIS epidemic model; spruce budworm model; law of mass action; Michaelis-Menten enzyme dynamics.

2) Multidimensional systems: existence, uniqueness, fixed point stability; two-dimensional systems; SIS model for two populations; genetic control systems; population competition models; predator-prey dynamics and the Lotka-Volterra model.

3) Oscillations and bifurcations: Poincaré-Bendixson Theorem; oscillations in predator-prey models; relaxation oscillators; Fitzhugh-Nagumo model; fixed point bifurcations; Hopf bifurcations and limit cycles.

4) Spatial dynamics: reaction-diffusion equations; Fisher-Kolmogorov equation; travelling waves in predator-prey systems; spatial SIS model; spread of rabies in a fox population; Turing instabilities; pattern formation in one and two dimensions.

5) Stochastic processes: continuous-time Markov chains; simple birth and death processes; stationary probability distributions; logistic growth process; branching processes and drug resistance; multivariate processes; stochastic enzyme dynamics; stochastic predator-prey dynamics.

Jupyter notebooks containing codes written in Python will be utilised throughout the course and a working knowledge of, or a willingness to learn and use Python, is expected.

MATH60137 Mathematical Biology 2

Brief Description

This module will provide an introduction to the interdisciplinary field of mathematical systems biology. Drawing on analogies between biological and engineered systems, we will learn about mathematical approaches to model functional aspects of biological systems. We will discuss a wide range of topics, including control, memory, and computation in biological systems. Each topic will be discussed in the context of specific experimental systems.

Learning Outcomes

On successful completion of this module you will:

- be able to describe the major concepts and principles of systems biology, including design principles and emergent properties in biological systems

- be able to develop mathematical theories on functional aspects of biological systems, including biochemical and cellular circuits

- appreciate the role of feedback regulation in biological systems, and acquire tools to analyze feedback systems

- critically evaluate the relation between theory and experiment in systems biology

- acquire an understanding of a range of mathematical and computational motifs that play an important functional role in a wide range of biological systems

- develop an appreciation for the complexity and diversity of biological systems, and an understanding of the role of interdisciplinary approaches in advancing our understanding of these systems

Indicative Module Content

1. Introduction to biological circuits

2. Negative and positive feedback (responses, oscillations, memory, differentiation)

3. Integral feedback (adaptation, scale invariance)

4. Gradients: sampling and optimization

5. Bifurcations and feedback tuning to bifurcations

6. Hopfield networks

7. Optimal control and learning

MATH60015 Quantum Mechanics I

Brief Description

Quantum mechanics is one of the most successful theories in modern physics and has an exceptionally beautiful underlying mathematical structure. It provides the basis for many areas of contemporary physics, including atomic and molecular, condensed matter, high-energy particle physics, quantum information theory, and quantum cosmology, and has led to countless technological applications. This module aims to provide an introduction to quantum phenomena and their mathematical description. We will use tools and concepts from various areas of mathematics and physics, such as classical mechanics, linear algebra, probability theory, numerical methods, analysis and geometry.

Learning Outcomes

On successful completion of this module, you will be able to

- appreciate Schrödinger's formulation of quantum mechanics, wave functions and wave equations;

- construct the mathematical framework of quantum mechanics, including the 4 postulates of quantum mechanics and the Dirac notation;

- solve the eigenvalue problem for basic one-dimensional quantum systems;

- exploit the method of stationary states to deduce the time-evolved quantum state from the initial state of a system;

- communicate fluently using the Dirac notation.

Module Content

The module will cover the following topics:

- Hamiltonian dynamics;

- Schrödinger equation and wave functions;

- stationary states of one-dimensional systems;

- mathematical foundations of quantum mechanics;

- quantum dynamics;

- angular momentum.

A selection of topics among the following additional optional topics will be covered depending on students interests:

- approximation techniques;

- explicitly time-dependent systems;

- geometric phases;

- numerical techniques;

- many-particle systems;

- cold atoms;

- entanglement and quantum information.

MATH60016 Special Relativity and Electromagnetism

Brief Description

This module presents a beautiful mathematical description of a physical theory of great historical, theoretical and technological importance. It demonstrates how advances in modern theoretical physics are being made and gives a glimpse of how other theories (say quantum chromodynamics) proceed. This module does not follow the classical presentation of special relativity by following its historical development, but takes the field theoretic route of postulating an action and determining the consequences. The lectures follow closely the famous textbook on the classical theory of fields by Landau and Lifshitz.

Learning Outcomes

On successful completion of this module you will be able to

- demonstrate an understanding of the relation between space and time and apply Lorentz transforms;

- appreciate the structure of special relativity as derived from the principle of least action;

- determine relativistic particle trajectories;

- derive Maxwell's equations from first principles and apply them to a variety of interactions of charges and fields;

- critically analyse various solutions of the electromagnetic wave equations.

Module Content

This course follows closely the following book: L.D. Landau and E.M. Lifschitz, Course on Theoretical Physics Volume 2: Classical Theory of Fields.

Special relativity: Einstein’s postulates, Lorentz transformation and its consequences, four vectors, dynamics of a particle, mass-energy equivalence, collisions, conserved quantities.

Electromagnetism: Magnetic and electric fields, their transformations and invariants, Maxwell’s equations, conserved quantities, wave equation.

MATH60017 Tensor Calculus and General Relativity

Brief Description

This module provides an introduction to General Relativity. Starting with the rather simple Mathematics of Special Relativity the goal is to provide you with the mathematical tools to formulate General Relativity. Some examples, including the Schwarzschild space-time are considered in detail.

Learning Outcomes

On successful completion of this module, you will be able to:

- appreciate the application of tensors in special relativity;

- demonstrate a working knowledge of tensor calculus;

- explain the concepts of parallel transport and curvature;

- formulate and solve the geodesic equation for a given space-time metric;

- derive Einstein's field equations and analyse Schwarzschild's solution.

Module Content

This module will cover the following topics:

1. Special Relativity

2. Tensors in Special Relativity

3. Tensors in General Coordinates Systems 4. Parallel Transport and Curvature

5. General Relativity

6. The Schwarzschild Spacetime

7. Variational Methods

MATH60018 Quantum Mechanics 2

Brief Description

Quantum mechanics (QM) is one of the most successful theories in modern physics and has an exceptionally beautiful underlying mathematical structure. Assuming some prior exposure to the subject (such as Quantum Mechanics I), this module aims to provide an intermediate/advanced treatment of quantum phenomena and their mathematical description. Quantum theory combines tools and concepts from various areas of mathematics and physics, such as classical mechanics, linear algebra, probability theory, numerical methods, analysis and geometry.

Learning Outcomes

On successful completion of this module, you will be able to:

- outline key aspects of quantum mechanics at the intermediate/advanced level;

- harness the power of symmetry in understanding quantum mechanics;

- describe many-particle quantum mechanical systems, and demonstrate familiarity with the formalism of second quantisation;

- solve complex quantum mechanical problems using the machinery introduced in this module.

Module Content

This module will cover the following core topics:

- quantum mechanics in the momentum basis;

- the Heisenberg picture;

- the use of symmetry and general transformations in quantum mechanics;

- Elements of quantum computation;

- perturbation theory;

- adiabatic processes;

- second quantisation;

- introduction to many-particle systems;

- Fermi and Bose statistics.

Additional topics may include: WKB theory, the Feynman path integral, and quantum magnetism.

MATH60019 Theory of Partial Differential Equations

Brief Description

In this module, students are exposed to different phenomena which are modelled by partial differential equations. The course emphasizes the mathematical analysis of these models and briefly introduces some numerical methods.

Learning Outcomes

On successful completion of this module you will be able to:

- appreciate how to formally differentiate complicated finite dimensional functionals and simple infinite dimensional functionals;

- describe, select and use a variety of methods for solving partial differential equations;

- outline how various partial differential equations respect conservation laws;

- utilize energy methods to critically analyse the stability of solutions to PDEs.

Module Content

The module is composed of the following sections:

1. Introduction to PDEs

   1.1. Basic Concepts

   1.2. Gauss Theorem

2. Method of Characteristics

   2.1. Linear and Quasilinear first order PDEs in two independent variables.

   2.2. Scalar Conservation Laws

3. Diffusion

   3.1. Heat equation. Maximum principle

   3.2. Separation of variables. Fourier Series.

4. Waves

   4.1. The 1D wave equation

   4.2. 2D and 3D waves.

5. Laplace-Poisson equation

   5.1. Dirichlet and Neumann problems.

   5.2. Introduction to calculus of variations. The Dirichlet principle.

   5.3 Finite Element Method.

MATH60020 Function Spaces and Applications

Brief Description

The purpose of this course is to introduce the basic function spaces and to train the student in the basic methodologies needed to undertake the analysis of Partial Differential Equations and to prepare them for the course "Advanced topics in Partial Differential Equations'' where this framework will be applied. Most of the topics contained in the module do not require preliminary knowledge. However, knowledge of the material in the Y2 module on "Lebesgue Measure and Integration" (or a suitable equivalent) is recommended.

Learning Outcomes

On successful completion of this module, you will be able to:

- appreciate the main concepts of metric topology and integration theory (Fatou's lemma, monotone and dominated convergence theorems);

- manipulate concepts associated with Banach spaces (Cauchy sequence, completeness concept, bounded operators, continuous linear forms, dual space);

- apply the concept of uniform convergence of functions, and those related to spaces of differentiable functions;

- interpret the concept of convergence in Lebesgue spaces;

- manipulate convolutions and sequences of mollifiers to approximate continuous or Lebesgue integrable functions by infinitely differentiable functions with compact support;

- appreciate the notion of compactness and the difference between finite and infinite-dimensional normed vector spaces.

Module Content

The course will span the basic aspects of modern functional spaces: integration theory, Banach spaces, spaces of differentiable functions and of integrable functions, convolution and regularization, Hilbert spaces. The concepts of Distributions and Sobolev spaces will be taught in the follow-up course ''Advanced topics in Partial Differential Equations'' as they are tightly connected to the resolution of elliptic PDE's and the material taught in the present course is already significant.

The syllabus of the course is as follows:

1) Review of metric topology and Lebesgue's integration theory.

2) Normed vector spaces. Banach spaces. Continuous linear maps. Dual of a Banach space.

3) Examples of function spaces: continuously differentiable function spaces and Lebesgue spaces. Hölder and Minkowski's inequalities. Convolution and Mollification. Approximation of continuous or Lebesgue integrable functions by infinitely differentiable functions with compact support.

4) Compactness: Non- compactness of the unit ball in infinite-dimensional normed vector spaces. Criteria for compactness in space of continuous functions: the Ascoli theorem. Compact operators.

5) Hilbert spaces. The projection theorem. The Riesz representation theorem. The Lax-Milgram theorem. Hilbert bases and Parseval's identity. Application to Fourier series.

MATH60021 Advanced Topics in Partial Differential equations

Brief Description

This course develops the analysis of boundary value problems for elliptic and parabolic PDE's using the variational approach. It is a follow-up of 'Function spaces and applications' but is open to other students as well provided they have sufficient command of analysis. An introductory Partial Differential Equation course is not needed either, although certainly useful.

Learning Outcomes

On successful completion of this module you will be able to:

- appreciate the concepts of distribution (differentiation, convergence);

- manipulate the main properties of the Sobolev space H^m for integer m (inbeddings and compactness theorems, Poincaré inequality);

- derive the variational formulation of a specific elliptic boundary value problem and to provide the reasoning leading to the proof of the existence and uniqueness of the solution;

- develop the spectral theory of an elliptic boundary value problem;

- solve a parabolic boundary value problem using the spectral theory of the associated elliptic operator.

Module Content

The course consists of three parts. The first part (divided into two chapters) develops further tools needed for the study of boundary value problems, namely distributions and Sobolev spaces. The following two parts are devoted to elliptic and parabolic equations on bounded domains. They present the variational approach and spectral theory of elliptic operators as well as their use in the existence theory for parabolic problems.

The aim of the course is to expose the students to some important aspects of Partial Differential Equation theory, aspects that will be most useful to those who will further work with Partial Differential Equations be it on the theoretical side or on the numerical one.

The syllabus of the course is as follows:

1. Distributions. The space of test functions. Definition and examples of distributions. Differentiation. Convergence of distributions.

2. Sobolev spaces: The space H1. Density of smooth functions. Extension lemma. Trace theorem. The space H10. Poincare inequality. The Rellich-Kondrachov compactness theorem (without proof). Sobolev imbedding (in the simple case of an interval of R). The space Hm. Compactness and Sobolev imbedding for arbitrary dimension (statement without proof).

3. Linear elliptic boundary value problems: Dirichlet and Neumann boundary value problems via the Lax-Milgram theorem. Spectral theory. The maximum principle. Regularity (stated without proofs). Classical examples: elasticity system, Stokes system.

4. Linear parabolic initial-value problems: Bochner Spaces. Existence and uniqueness of weak solutions by the Galerkin method. Application to the incompressible Navier-Stokes equations.

MATH60022 Finite Elements: Numerical Analysis and Implementation

Brief Description

Finite element methods form a flexible class of techniques for numerical solution of PDEs that are both accurate and efficient. The finite element method is a core mathematical technique underpinning much of the development of simulation science. Applications are as diverse as the structural mechanics of buildings, the weather forecast, and pricing financial instruments. Finite element methods have a powerful mathematical abstraction based on the language of function spaces, inner products, norms and operators.

Learning Outcomes

On successful completion of this module, you will be able to:

- appreciate the core mathematical principles of the finite element method;

- employ the finite element method to formulate and analyse numerical solutions to linear elliptic PDEs;

- implement the finite element method on a computer;

- compare the application of various software engineering techniques to numerical mathematics.

Module Content

This module aims to develop a deep understanding of the finite element method by spanning both its analysis and implementation. In the analysis part of the module, students will employ the mathematical abstractions of the finite element method to analyse the existence, stability, and accuracy of numerical solutions to PDEs. At the same time, in the implementation part of the module students will combine these abstractions with modern software engineering tools to create and understand a computer implementation of the finite element method.

This module is composed of the following sections:

I - Basic concepts: weak formulation of boundary value problems, Ritz-Galerkin approximation, error estimates, piecewise polynomial spaces, local estimates;

II - Efficient construction of finite element spaces in one dimension: 1D quadrature, global assembly of mass matrix and Laplace matrix;

III - Construction of a finite element space: Ciarlet's finite element, various element types, finite element interpolants;

IV - Construction of local bases for finite elements: efficient local assembly;

V - Sobolev Spaces: generalised derivatives, Sobolev norms and spaces, Sobolev's inequality;

VI - Numerical quadrature on simplices: employing the pullback to integrate on a reference element;

VII - Variational formulation of elliptic boundary value problems: Riesz representation theorem, symmetric and nonsymmetric variational problems, Lax-Milgram theorem, finite element approximation estimates;

VIII - Computational meshes: meshes as graphs of topological entities, discrete function spaces on meshes, local and global numbering;

IX - Global assembly for Poisson equation: implementation of boundary conditions, general approach for nonlinear elliptic PDEs;

X - Variational problems: Poisson's equation, variational approximation of Poisson's equation, elliptic regularity estimates, general second-order elliptic operators and their variational approximation.

MATH60023 Numerical Solution of Ordinary Differential Equations

Brief Description

The module is an introductory course in numerical methods for ordinary differential equations. The purpose of this module is to learn how to use the computer to find numerical solutions to ordinary differential equations as well as to provide you with theoretical knowledge and practical skills to lay the solid groundwork necessary to advance in scientific computing.

Learning Outcomes

On the successful completion of the module, you will be able to:

- use classical numerical methods for ordinary differential equations;

- analyse different properties of numerical methods (e.g. accuracy and stability);

- develop your own methods with prescribed properties;

- compare different methods with respect to accuracy, stability, computational and space complexity.

Module Content

This module will cover the following topics:

- Taylor series methods;

- Linear multi-step methods;

- Runge-Kutta methods;

- Adaptive step size control;

- Boundary value problems for ordinary differential equations.

MATH60024 Computational Linear Algebra

Brief Description

Linear systems of equations arise in countless applications and problems in mathematics, science and engineering. Often these systems are large and require a computer to solve. This course provides an overview of the algorithms used to solve linear systems and eigenvalue problems, in terms of their development, stability properties, and application.

Learning Outcomes

On successful completion of this module, you will be able to

- describe, select and use algorithms for QR decomposition of matrices;

- solve least-squares problems using QR decomposition;

- apply LU decomposition to solve linear systems;

- analyse and modify algorithms that take advantage of matrix structure;

- find numerical solutions to eigenvalue problems;

- critically analyse various iterative methods for solving linear systems.

Module Content

The module will cover the following topics:

1) Direct methods:

Triangular and banded matrices, Gauss elimination, LU-decomposition, conditioning and finite-precision arithmetic, pivoting, Cholesky factorisation, QR factorisation and their numerical implementation.

2) Eigenvalue problems:

power method and variants, Jacobi's method, Householder reduction to tridiagonal form, eigenvalues of tridiagonal matrices, the QR method.

3) Iterative methods:

Krylov subspace methods: Lanczos method and Arnoldi iteration, conjugate gradient method, GMRES, preconditioning.

MATH60025 Computational Partial Differential Equations

Brief Description

This module will introduce a variety of computational approaches for solving partial differential equations, focusing mostly on finite difference methods, but also touching on finite volume and spectral methods. Students will gain experience implementing the methods and writing/modifying short programs in Matlab or another programming language of their choice. Applications will be drawn from problems arising in areas such as Mathematical Biology and Fluid Dynamics.

Learning Outcomes

On successful completion of this module, you will be able to:

- appreciate the physical and mathematical differences between different types of PDES;

- design suitable finite difference methods to solve each type of PDE;

- outline a theoretical approach to testing the stability of a given algorithm;

- determine the order of convergence of a given algorithm;

- demonstrate familiarity with the implementation and rationale of multigrid methods.

Module Content

The module will cover the following topics:

1) Introduction to Finite Differences

2) Classificaton of PDEs

3) Explicit and Implicit methods for Parabolic PDEs

4) Iterative Methods for Elliptic PDEs. Jacobi, Gauss-Seidel, Overrelaxation

5) Multigrid Methods

6) Hyperbolic PDEs. Nonlinear Advection/Diffusion systems. Waves and PMLs

as well as various advanced practical topics from Fluid Dynamics, which will depend on the final project.

MATH60026 Methods for Data Science

Brief Description

This module provides an hands-on introduction to the methods of modern data science. Through interactive lectures, the student will be introduced to data visualisation and analysis as well as the fundamentals of machine learning.

Learning Outcomes

On successful completion of this module, you will be able to:

- Visualise and explore data using computational tools;

- Appreciate the fundamental concepts and challenges of learning from data;

- Analyse some commonly used learning methods;

- Compare learning methods and determine suitability for a given problem;

- Describe the principles and differences between supervised and unsupervised learning;

- Clearly and succinctly communicate the results of a data analysis or learning application.

Module Content

The module is composed of the following sections:

- Introduction to computational tools for data analysis and visualisation;

- Introduction to exploratory data analysis;

- Mathematical challenges in learning from data: optimisation;

- Methods in Machine Learning: supervised and unsupervised; neural networks and deep learning; graph-based data learning;

- Machine learning in practice: application of commonly used methods to data science problems; Methods include: regressions, k-nearest neighbours, random forests, support vector machines, neural networks, principal component analysis, k-means, spectral clustering, manifold learning, network statistics, community detection.

MATH60027 Scientific Computation

Brief Description

This module introduces students to the analysis and implementation of efficient algorithms used to solve mathematical and computational problems connected to a broad range of scientific topics. Mathematical tools and concepts from linear algebra, calculus, numerical analysis, and statistics will be utilised to develop and analyse computational solutions to mathematical and scientific problems.The objectives are that by the end of the module all students should have a good familiarity with the essential elements of the Python programming language and be able to undertake programming tasks in a range of areas.

Learning Outcomes

On successful completion of this module you will be able to:

- analyse the performance of simple sorting and searching algorithms and implement them in Python;

- computationally analyse complex networks and dynamical processes of complex systems;

- effectively utilise important tools for data analysis such as discrete Fourier transforms;

- evaluate and implement numerical methods for mathematical optimisation and the solution of differential equations;

- assess the correctness and efficiency of simple data structures and algorithms on graphs and implement them in Python.

Module Content

The module will cover the following topics:

1) Sorting and searching with scientific applications from fields such as bioinformatics;

2) Algorithms on graphs and basic data structures such as queues and hash tables;

3) Methods for data analysis using tools such as discrete Fourier transforms;

4) Analysis and use of common optimisation methods such as Simulated Annealing;

5) Numerical solution of differential equations arising in multiscale problems;

6) Computational analysis of complex systems.

MATH60028 Probability Theory

Brief Description

This module provides a rigorous approach to the fundamental properties of probability. It teaches fundamental notions and structures as well as tools relevant to modern probability theory and applications. The module is important for further study of probability theory and stochastic processes.

Learning Outcomes

On successful completion of this module, you should be able to:

- Demonstrate knowledge of fundamental notions and structures of Probability Theory;

- Use the tools developped in a variety of applications;

- Demonstrate problem solving skills in probability theory

- Communicate your knowledge of the area in a concise, accurate and coherent manner.

Module Content

An indicative list of topics is:

Probability spaces. Random variables: (Bernoulli, Rademacher, Gaussian variables with integration by parts formula). Probability Distributions.

Basic probability inequalities: Jensen, Tshebychev. Tail of Distribution Estimates.

Convergence in probability, in p-th moment, almost everywhere. 0-1 Law.

Mutual Independence of Events/Random Variable and Vieta Formula. Product Probability Spaces. Conditional Expectations and Independence. Borel-Cantelli Lemmas.

Weak and Strong Laws of Large Numbers for Random Sequences and Series of Mutually Independent or Weakly Correlated Random Variables.

Applications : [Probabilistic proof of Weierstrass Theorem, Monte Carlo Method for Large Dimensional Integration, Macmillan's Theorem, Infinitely Often Events: Decay and Recurrence of Human Civilisations, Normal Numbers...]

Weak Convergence & Characteristic Functions. Central Limit Theorem.

Infinite Product of Bernoulli measures versus Gaussian measure.

MATH60029 Functional Analysis

Brief Description

This module brings together ideas of continuity and linear algebra. It concerns vector spaces with a distance, and involves linear maps. The vector spaces are often spaces of functions. It is an important requirement for further study of many areas of Mathematical Analysis including PDEs, Stochastic Analysis, Dynamical Systems and Quantum Mechanics.

Learning Outcomes

On successful completion of this module, you should be able to:

- Demonstrate knowledge of fundamental notions and structures of Functional Analysis by proving a range of results;

- Use the tools developped in a variety of applications;

- Demonstrate problem solving skills in functional analysis

- Communicate your knowledge of the area in a concise, accurate and coherent manner.

Module Content

An indicative list of topics is:

Metric Linear Spaces and basic examples of topological spaces with non metrisable topology.

Minkowski and Hoelder Inequality.

Existence of Hamel basis (axiom of choice 1st time).

Normed vector spaces

& example of not normed Frechet space (Schwartz test functions).

Banach spaces.

Classical Banach Spaces: l_p, c, c_0, $L_p(\mu)$, $C(\Omega)$, $C^{(m)}(\Omega)$.

Closed Subspaces, Completness, Separability and Compactness in Classical Spaces.

Schauder Basis.

Continuous linear maps.

Banach contraction mapping principle and applications to integral equations (Frdholm+Volterra). Finite dimensional spaces.

The Hilbert space (orthonormal basis).

The Riesz-Fisher Theorem.

The Hahn-Banach Theorem. (Banach Limit.)

Dual spaces: Dual spaces of classical spaces. Reflexive Non-reflexive spaces.

Baire Cathegory Theorem (axiom of choice again).

Principle of Uniform Boundedness. (Application to Fourier Series).

Open Mapping and Closed Graph Theorems.

Compact operators.

Hermitian operators and the Spectral Theorem.

The module provides a general orientation in contemporary research problems in Mathematical Analysis includ- ing PDEs , Stochastic Analysis, Dynamical Systems and Quantum Mechanics.

MATH60030 Fourier Analysis and the Theory of Distributions

Brief Description

Fourier analysis is an important tool used in various branches of mathematics and beyond. The module provides a deeper understanding of it than what is briefly mentioned in general analysis courses. It also connects it to the theory of distributions. As a result of studying the module, students will understand the basics of the Fourier analysis and theory of distributions which will be sufficient for most branches of mathematics.

Learning Outcomes

On successful completion of this module, you will be able to:

-understand the issues of convergence for Fourier series,

-apply the Fourier and Laplace transforms,

-understand the motivation behind the notion of distribution,

-be in command of the basics of Fourier analysis and distribution theory sufficient for working in many areas of mathematics

Module Content

The module will assume familiarity with measure theory and functional analysis, especially L^p spaces and linear functionals.

Indicative content: Orthogonal systems in infinite-dimensional Euclidean spaces, Bessel inequality, Parseval equality, general Fourier series, trigonometric basis in L_2[-Pi,Pi], convergence of trigonometric Fourier series, Fejer's theorem and applications, Fourier transform and its properties, application to solution of differential equations, Plancherel theorem, Laplace transform, linear functionals, distributions, basic properties of distributions and applications, Fourier transform for distributions.

Those students who decide to do a PhD in a closer related area of analysis will be able to use the acquired basic knowledge and skills to relatively easily extend their knowledge to more sophisticated areas of the theory.

MATH60031 Markov Processes

Brief Description

Markov processes are widely used to model random evolutions with the Markov property `given the present, the future is independent of the past'. The theory connects with many other subjects in mathematics and has vast applications.

Learning Outcomes

On successful completion of this module, you should be able to:

- demonstrate your understanding of the concepts and results associated with the elementary theory of Markov processes, including the proofs of a variety of results
- apply these concepts and results to tackle a range of problems, including previously unseen ones
- apply your understanding to develop proofs of unfamiliar results
- communicate your knowledge of the area in a concise, accurate and coherent manner.

Module Content

Markov processes are widely used to model random evolutions with the Markov property `given the present, the future is independent of the past'. The theory connects with many other subjects in mathematics and has vast applications. This course is an introduction to Markov processes. We aim to build intuitions and good foundations for further studies in stochastic analysis and in stochastic modelling.

The module is largely self-contained, but it is strongly advised that students should have taken the second-year module Lebesgue Measure and Integration (or equivalent). A good knowledge of real analysis and metric spaces will be assumed.

The module is related to a number of modules in stochastic analysis, probability theory, dynamical systems and mathematical finance.

An indicative list of contents is:

1. Discrete time and finite state Markov chains : Chapman-Kolmogorov equations, irreducible, Perron-Froebenius theorem for stochastic matrices, recurrent and transient.

2. Discrete time Markov processes on general state space. Conditional expectations, Chapman-Kolmogorov equation, Feller property, strong Feller property, Kolmogorov's theorem, stopping times, strong Markov, stationary process, weak convergence and Prohorov's theorem, Existence of invariant measures : Krylov-Bogolubov method, Lyapunov method. Ergodicity by contraction method and Doeblin's criterion. Structures of invariant measures, ergodic theorems.

MATH60032 Geometry of Curves and Surfaces

Brief Description

This module is an introduction to classical theory of differential geometry, where we disucss geometric features of curves and surfaces in (mostly) three dimensional Euclidean spaces.

Learning Outcomes

On successful completion of this module, you will be able to:

- identify regular curves and implement different re-parametrisations of curves in two and three dimensional spaces,

- learn about and calculate the geometric quantities of curvature and torsion of a regular curve,

- identify regular surfaces in 3 dimensional spaces using the notions of charts,

- analyse the regularity of maps from one surface into another surface, and also of functions on surfaces,

- use partitions to calculate the basic topological invariant of Euler characteristic,

- learn about the topological classification of compact surfaces, and identify them,

- calculate the first and second fundamental forms of a surface,

- learn about the existence and uniqueness of geodesics on general surfaces,

- link the Gaussian curvature to the local shape of a surface, and present different kinds of examples,

- analyse the global topological features of a surface by integrating local geometric features (Gauss-Bonnet and winding numbers)

Module Content

This module is an introduction to classical theory of differential geometry, where we disucss geometric features of curves and surfaces in (mostly) three dimensional Euclidean spaces. A curve, which is the trajectory of a particle moving in a smooth fashion, may twist in two manners described by the values called curvature and torsion. The twists of a surface in three dimensional space is naturally more involved. There are different notions of curvature: the Gaussian curvature and the mean curvature. The Guassian curvature describes the intrinsic geometry of the surface, and the mean curvature describes how it bends in space. We look at several examples of surfaces, and calculate their curvatures. We study the local shapes of surfaces based on their curvatures. For example, the Gaussian curvature of a sphere is strictly positive, which explains why any planar illustration of the countries distorts shapes. Remarkably, these local gometric notions can be combined to derive global information about the topology of the surface (for example the Gauss-Bonnet formula). This module starts with the basic real analysis taught in years 1 and 2, and leads into the more modern and general theory of manifolds.

An indicative list of sections and topics is:

• Curves in two and three-dimensional spaces: re-parametrizations, curvature and torsion, Frenet-Serret formulae, curves are determined by curvature and torsion, winding number and the total curvature,

• Surfaces: Charts, Tangent vectors, and tangent planes, Smooth maps from one surface into another surface, smooth functions on a surface, Normal vectors,

• Curvature of a surface: the first and second fundamental forms, Christoffel symbols, normal curvature, Gaussian curvature, and mean curvature, Gauss’s Theorema Egregium,

• Area of a surface,

• Geodesics on a surface: length-minimising curves, existence, non-existence and examples, geodesic curvature,

• Gauss-Bonnet Theorem and applications

• Topological classification of surfaces

• Vector fields and the Poincare-Hopf Theorem

The module will assume familiarity with material in the second-year module Analysis II

MATH60033 Algebraic Curves

Brief Description

This module is meant as a first encounter with algebraic geometry, through the study of affine and projective plane curves over the field of complex numbers. We will also discuss some complex-analytic aspects of the theory (Riemann surfaces). Important results include the definition of local intersection multiplicities and Bézout's theorem, inflection points and the classification of plane cubics, linear systems of curves, and the degree-genus formula.

Learning Outcomes

On successful completion of this module, you will be able to:

- solve geometric problems about affine and projective plane curves with algebraic techniques;

- determine the projectivizations of affine plane curves and the points at infinity;

- determine the tangent lines of plane curves at smooth and singular points;

- compute projective transformations and find convenient coordinate systems;

- compute intersection multiplicities using resultants and the axiomatic characterization;

- formulate, prove and apply Bézout's theorem;

- find inflection points of projective plane curves and use them to classify cubic curves;

- solve enumerative problems by means of the theory of linear systems;

- work with holomorphic charts to determine local and global properties of Riemann surfaces and morphisms;

- compute ramification degrees of morphisms of Riemann surfaces;

- formulate, prove and apply the degree-genus formula for smooth projective plane curves.

Module Content

An indicative list of sections and topics is:

- Affine plane curves;

- The geometry of projective spaces;

- Projective plane curves;

- Smooth and singular points, tangent lines;

- Projective transformations and the classification of conics;

- Intersection multiplicities (resultants and axiomatic characterization)

- Bézout's theorem on intersections of projective plane curves;

- the Legendre family of cubics, inflection points and the classification of non-degenerate smooth cubics;

- linear systems of projective plane curves, projective duality and enumerative geometry;

- Riemann surfaces;

- local description of morphisms of Riemann surfaces (ramification);

- classification of topological surfaces and genus (informal introduction);

- Riemann-Hurwitz and the degree-genus formula.

Some related topics will appear in the problem sheets and the coursework (e.g., dual curves, group structure on smooth cubics).

MATH60034 Algebraic Topology

Brief Description

This module gives a first introduction to algebraic topology. After some preliminary results on quotient spaces and CW-complexes, we discuss fundamental groups and the Galois correspondence for covering spaces. We then move on to homology theory and study simplicial and singular homology, as well as some applications like the Jordan curve theorem and invariance of domain. Throughout the module, we pay special attention to algebraic and categorical aspects.

Learning Outcomes

On successful completion of this module, you will be able to:

- define the basic invariants in algebraic topology and prove their main properties;

- use algebraic techniques to distinguish different homotopy types and classify topological objects;

- compute fundamental groups, simplicial homology groups and singular homology groups;

- apply the Galois correspondence to classify covering spaces of topological spaces;

- apply fundamental groups and homology groups to prove fundamental topological properties (Brouwer's fixed point theorem, Jordan's curve theorem, invariance of domain);

- formulate topological and algebraic constructions in a categorical language (universal properties)

Module Content

An indicative list of sections and topics is:

Preliminaries:

- Homotopy and homotopy type

- Cell complexes

- Operations on spaces

The Fundamental Group:

- Paths and Homotopy

- Presentations of groups, amalgamated products and Van Kampen's Theorem

- Covering Spaces

- The Galois correspondence

Homology

- Δ-complexes and simplicial homology

- Singular homology

- Homotopy invariance

- Relative homology, exact sequences and excision

- The equivalence of simplicial and singular homology

- Mayer-Vietoris Sequences

- Applications

The main reference for this course is "Algebraic topology" by Hatcher.

The module will assume familiarity with material in the second-year modules: Groups and Rings, Analysis II

MATH60140 Geometric Complex Analysis

Brief Description

In this module we look at the subject of complex analysis from a more geometric point of view. We shall look at geometric notions associated with domains in the plane and their boundaries, and how they are transformed under holomorphic mappings. In turn, the behaviour of conformal maps is highly dependent on the shape of their domain of definition.

Learning Outcomes

On successful completion of this module, you will be able to:

– identify features of, and develop arguments about, certain holomorphic maps;

– state, apply, and explain aspects of the Riemann Mapping Theorem for arbitrary simply connected plane domains;

– explain the automorphisms of the disk and the upper half plane;

– Explain hyperbolic geometry, and basic notions of length, geodesics, isometries;

– apply area theorem, and derive distortion estimates for arbitrary conformal mappings;

– acquire deeper understanding of holomorphic mappings through generalisation to quasi-conformal mappings;

– appreciate significance of universal bounds in geometric function theory;

– explain the statement of the Beltrami-equation, and generalisation of the Riemann mapping theorem;

Indicative Module Content

Complex analysis is the study of the functions of complex numbers. It is employed in a wide range of topics, including dynamical systems, algebraic geometry, number theory, and quantum field theory, to name a few. On the other hand, as the separate real and imaginary parts of any analytic function satisfy the Laplace equation, complex analysis is widely employed in the study of two-dimensional problems in physics such as hydrodynamics, thermodynamics, Ferromagnetism, and percolations.

In this module we look at the subject of complex analysis from a more geometric point of view. We shall look at geometric notions associated with domains in the plane and their boundaries, and how they are transformed under holomorphic mappings. In turn, the behaviour of conformal maps is highly dependent on the shape of their domain of definition.

An indicative list of topics is:

- Schwarz lemma and automorphisms of the disk,
- Riemann sphere and rational maps,

- Conformal geometry on the disk, Poincare metric, Isometries, Hyperbolic contractions,
- Conformal Mappings, Conformal mappings of special domains, Normal families, Montel's theorem, General form of Cauchy integral formula, Riemann mapping theorem,
- Growth and Distortion estimates, Area theorem,
- Quasi-conformal maps and Beltrami equation, Linear distortion, Dilatation quotient, Absolute continuity on lines, Quasi-conformal mappings, Beltrami equation, application of MRMT,

There will also be extra self-study of extension material (in the form of a book chapter, additional notes or a research paper) applying or extending material from the above topics.

Prerequisites: It will be helpful if you have taken (or are taking) one or more of the following courses:

Functional Analysis, Measure and Integration (or Lebesgue Measure and Integration), Fourier Analysis and Distributions.

MATH60035 Algebra 3

Brief Description

This course continues the study of commutative rings and introduces the notion of R-module, which is an analogue over rings of the notion of a vector space over a field. Using these ideals we prove fundamental results about various classes of rings, particularly polynomial rings in several variables.

Learning Outcomes

On successful completion of this module, you will be able to:

- Understand the detailed theory of finite fields, their classification, and factorization of polynomials over finite fields

- Understand the theory of R-modules and their presentations

- Understand the classification of modules over Euclidean Domains, and how to use Smith Normal Form to determine the isomorphism class of such a module given a presentation

- Use this classification, in the case of K[T]-modules, to prove fundamental results in linear algebra

- Apply several different criteria for irreducibility of polynomials over various base rings

Module Content

An indicative list of sections and topics is:

- Chinese Remainder Theorem
- Field Extensions and Finite Fields
- R-modules
- Free modules and presentations
- Modules over Euclidean Domains
- Noetherian rings
- Gauss's Lemma and Factorization in polynomial rings
- If R is a UFD, so is R[X]
- Irreducible Polynomials and factorization of polynomials

MATH60036 Group Theory

Brief Description

This module builds on the Group Theory from the 1st year module Linear Algebra & Groups and the 2nd year module Groups and Rings. We start with a discussion of isomorphism theorems, and proceed to further example of groups and operations on them, including automorphism groups and semidirect products. Special attention is given to group actions and permutation groups: primitivity, multiple transitivity etc. Further we discuss solvable and nilpotent groups and their characterizations.

Learning Outcomes

On successful completion of this module, you will be able to:

– identify features of, and develop arguments about, certain groups and classes of group;

– explain the principles of group actions, and work with elementary examples;

– construct and work with direct and semidirect products of groups;

– state the definition of and extraspecial group and construct small examples of them;

– construct certain classical series of doubly and triply transitive groups;

– state and prove the structure theorem for nilpotent groups;

– determine the normal structure and calculate the automorphism groups of symmetric groups;

– work independently and with peers to articulate their understanding of abstract concepts in algebra.

Module Content

An indicative list of sections and topics is:

Definition and basic properties of groups. Isomorphism Theorems. Sylow subgroups. Group actions, primitivity and multiple transitivity. Composition series. Nilpotent groups. Solvable groups. Symmetric groups. Automorphism group of a group and semidirect products. Linear groups: centres and commutator subgroups, with small examples.

MATH60037 Galois Theory

Brief Description

The formula for the solution to a quadratic equation is well--known. There are similar formulae for cubic and quartic equations but no formula is possible for quintics. The module explains why this happens.

Learning Outcomes

On successful completion of this module you should be able to:

-- state, prove, and apply the fundamental theorem of Galois theory (aka the "Galois correspondence").

-- work with examples such as cubic polynomials, cyclotomic polynomials, and polynomials over finite fields.

-- compute Galois groups of splitting fields of cubic and bi-quadratic polynomials in arbitrary characteristic.

-- state and apply the formulas for solving cubic and quartic equations, and prove that there are no such formulas for equations of degree 5 or larger.

-- compute Galois groups over the rationals by the method of Frobenius elements.

Module Content

Familiarity with the following topics from Algebra 3

will be assumed: irreducible polynomials and factorization of polynomial;

Gauss's Lemma and factorization in polynomial rings.

An indicative list of topics is:

Field extensions, degrees and the tower law

Splitting fields, normal extensions, separable extensions

Automorphisms, fixed fields and the fundamental theorem

Examples: cubic and biquadratic extensions, finite fields

Extensions of the rationals and Frobenius elements

Cyclotomic extensions

Kummer theory and the insolubility of quintic equations

MATH60038 Graph Theory

Brief Description

A graph is a structure consisting of vertices and edges. Graphs are used in many areas of Mathematics, and in other fields, to model sets with binary relations. In this module we study the elementary theory of graphs; we discuss matters such as connectivity, and criteria for the existence of Hamilton cycles. We treat Ramsey's Theorem in the context of graphs, with some of its consequences. We then discuss probabilistic methods in Graph Theory, and properties of random graphs.

Learning Outcomes

On successful completion of this module, you will be able to:

- Demonstrate facility with the terminology of graphs and simple graph constructions to analyse examples and prove results

- Explain the proofs of the theorems of König and Menger, and certain other related results. Apply these results to appropriate problems.

- State, prove and apply Turán's Theorem. Describe and apply certain results in the theory of Hamilton cycles, including Dirac's Theorem.

- Explain and reason about Ramsey's Theorem and related results in the context of graph colourings.

- Describe various models of random graphs and apply probabilistic arguments to situations in graph theory.

Module Content

An indicative list of sections and topics is:

Standard definitions and basic results about graphs. Common graph constructions: complete graphs, complete bipartite graphs, cycle graphs.

Matchings and König's Theorem. Connectivity and Menger's Theorem.

Extremal graph theory. The theorems of Mantel and Turán. Hamilton cycles, and conditions for their existence.

Ramsey Theory for graphs, with applications.

The Probabilistic Method and random graphs. Evolution of random graphs.

MATH60039 Group Representation Theory

Brief Description

This module defines and begins the study of representations of groups, focusing on finite-dimensional complex representations of finite groups. These structures encode ways that groups can act as symmetries, which appear throughout mathematics (notably in algebra, number theory, and geometry, but also in analysis) as well as in physics and chemistry, among other places.

Learning Outcomes

On successful completion of this module you will be able to:

- Recall and use basic definitions in group representations, their character theory, and modules over algebras, particularly finite-dimensional semisimple algebras;

- Explain and work with the features of complex representations of finite groups that allow one to simplify the theory (e.g., semisimplicity, character tables, etc.);

- Apply these results to classify representations of finite groups and semisimple algebras and compute the character tables of finite groups;

- perform basic constructions of representations of groups and to apply them to obtain all finite-dimensional representations of certain basic groups up to isomorphism;

- Explain the relationship between finite-dimensional irreducible representations of algebras and of finite-dimensional semisimple algebras, and the basic properties of their characters;

- Relate endomorphisms of representations to central elements in groups and semisimple algebras;

- Work independently and with peers to formulate and solve problems in algebra and geometry using tools of representation theory;

Module Content

This module defines and begins the study of representations of groups, focusing on finite-dimensional complex representations of finite groups. These structures encode ways that groups can act as symmetries, which appear throughout mathematics (notably in algebra, number theory, and geometry, but also in analysis) as well as in physics and chemistry, among other places. We explain how to understand and classify these representations through characters, or traces. In the final unit we generalise the theory to finite-dimensional modules over rings, particularly semisimple algebras, whose theory retains many of the features of that of representations of groups.

Indicative list of contents:

- Basic theory: definitions, Maschke's theorem, Schur's Lemma, classification and construction of representations of finite abelian groups, dihedral groups, and small symmetric and alternating groups;

- Tensor products of representations and homomorphism spaces, the regular representation;

- Character theory: behaviour under direct sums and tensor products, orthogonality relations, computation of character tables of certain groups;

- Finite-dimensional modules over algebras: definition of finite-dimensional semisimple algebras and relationship of finite-dimensional modules of general algebras

to those of finite-dimensional semisimple algebras; constructions of projections via the center;

- Other important examples of modules over algebras.

MATH60040 Formalising Mathematics

Brief Description

Computer theorem provers are mature enough now to tackle most undergraduate level mathematics, and also some much harder level mathematics. As these systems evolve, they will inevitably become useful as tools for researchers, and some believe that one day they will start proving interesting theorems by themselves. This project-assessed course is an introduction to the Lean theorem prover and during it we will learn how to formalise proofs of undergraduate level theorems from across pure mathematics.

Learning Outcomes

On successful completion of this module you will be able to

- understand the basics of how type theory can be used as a foundation for pure mathematics;

- understand how to "modularise" mathematical arguments, breaking them up into simple chunks, thus leading to clarity of understanding;

- understand how to "abstract" mathematical arguments, finding the correct generality in which statements should be made, thus again leading to clarity of understanding;

- state and prove many results from undergraduate pure mathematics courses in the Lean theorem prover;

- develop mathematical theories of your own in the Lean theorem prover;

- write formal proofs of theorems which other mathematicians can understand and follow.

Module Content

Note that the aim is to both learn the mathematics and to learn how to teach it to a computer. No experience in programming will be assumed. Lean is a functional programming langauge, so we will be picking up functional programming along the way. If you want to get a feeling for the kind of coding which will be involved, try playing the natural number game.

The following is an indicative list of areas where the mathematics could be drawn from:

*) Logic, functions, sets.

*) Lattice theory, complete lattices, Galois insertions and Galois connections. Examples in mathematics.

*) Groups and subgroups.

*) Closure operators in group theory and topology.

*) Filters as generalised subsets. Filters form a complete lattice.

*) Applications of filters to topology. New proofs of basic results in topology. Tychonoff's theorem.

*) Application of filters to analysis. New proofs of basic results in analysis.

*) What is cohomology? Group cohomology in low degree

MATH60132 Mathematical Logic

Brief Description

The module is concerned with some of the foundational issues of mathematics: formal logic and set theory. In propositional and predicate logic, we analyse the way in which we reason formally about

mathematical structures. In set theory, we will look at the ZFC axioms and use these to develop the notion of cardinality.

Learning Outcomes

On successful completion of this module, you should be able to:

- Understand how the notion of truth is defined precisely in propositional and predicate logic and apply the definitions and accompanying results in a variety of contexts.

- Demonstrate understanding of formal systems for propositional and predicate logic by constructing examples of formal proofs and deductions, and by applying and deriving general results about these.

- Appreciate the expressive power of a first-order language (and its limitations) and compare structures via their first-order theories.

- Relate the semantic and syntactic aspects of formal logic, have an understanding of powerful general results such as the completeness and compactness theorems, and be able to apply these in a variety of ways.

- Use the ZFC axioms to justify constructions in set theory, ranging from elementary applications, to constructions involving transfinite recursion, ordinals, cardinals and applications of these.

- Use general results to compute and compare cardinalities of infinite sets.

- Communicate your knowledge of the area in a concise, accurate and coherent manner.

Module Content

The module is concerned with some of the foundational issues of mathematics. In propositional and predicate logic, we analyse the way in which we reason formally about mathematical structures. In set theory, we will look at the ZFC axioms and use these to develop the notion of cardinality. These topics have applications to other areas of mathematics: formal logic has applications via model theory and ZFC provides an essential toolkit for handling infinite objects.

An indicative list of sections and topics is:

Propositional logic: Formulas and logical validity; a formal system; soundness and completeness.

Predicate logic: First-order languages and structures; satisfaction and truth of formulas; the formal system; Goedel’s completeness theorem; the compactness theorem; the Loewenheim- Skolem theorem.

Set theory: The axioms of ZF set theory; ordinals; cardinality; the Axiom of Choice.

There are no formal prerequisites for the module although a level of mathematical understanding such as would be provided by a second year algebra or analysis module, together with an appetite for abstraction and proofs, will be assumed. We will use basic notions from algebra (groups, rings and vector spaces) in examples.

MATH60041 Number Theory

Brief Description

The module is concerned with properties of natural numbers, and in particular of prime numbers, which can be proved by "elementary" methods (such as basic group theory and ring theory).

Learning Outcomes

On successful completion of this module, you will be able to:

– form arguments about and solve congruences, particularly modulo primes, and apply this to the RSA algorithm;

- compute with quadratic residues;

- solve some particular Diophantine equations, including Pell's

equation;

- compute continued fractions;

- construct transcendental numbers.

Module Content

An indicative list of topics is:

Euclid's algorithm, unique factorization, linear congruences, Chinese Remainder Theorem.

The structure of $(Z/nZ)^\times$, including the Fermat-Euler theorem,

Lagrange's theorem, the existence (and non-existence) of primitive

roots.

Primality testing, factorization, and the RSA algorithm (including the basics of the

Miller-Rabin test).

Quadratic reciprocity, Legendre symbols, Jacobi symbols.

Sums of 2 and 4 squares, using unique factorization in the Gaussian integers.

Pell's equation, existence of solutions via Dirichlet's theorem.

Continued fractions, periodicity for quadratic irrationals, algorithm for solving Pell's equation via continued fractions.

Irrationality, Liouville's theorem, construction of a transcendental number.

Elementary results on primes in arithmetic progressions.

Other topics of the lecturer's choice as time permits, e.g. quadratic forms; Möbius inversion and

Dirichlet Convolution; the Selberg sieve; particular examples of Diophantine equations.

MATH60042 Algebraic Number Theory

Brief Description

An introduction to algebraic number theory using quadratic fields as the main source of examples. We will study rings of integers in finite extensions of the rational and discuss unique factorization and its failure, the decomposition of primes, the finiteness of the ideal class group, and Dirichlet’s unit theorem.

Learning Outcomes

On successful completion of the module you will be able to:

- explain how unique factorization domains, principal ideal domains and Euclidean domains are related.

- give examples of rings of integers in quadratic fields that are Euclidean domains and also counter-examples.

- define a Dedekind domain and explain why rings of integers in number fields are Dedekind domains.

- write a basis for the rings of integers in any given quadratic number field.

- explain what it means for a prime to be split, inert or ramified in an extension and, given a quadratic ring of integers and a prime, to say what happens to that prime.

- explain why the class group in a number field is finite and compute examples of class groups of quadratic number fields.

- state Dirichlet's unit theorem and to describe explicitly the group of units in a real or imaginary quadratic field.

Module Content

An indicative list of topics is as follows.

We will review / introduce some background from ring theory, discuss unique factorization domains, principal ideal domains and Euclidean domains. We will study Gaussian and Eisenstein integers in detail and see several other examples of quadratic rings of integers. We will then discuss the structure theorem for finitely generated abelian groups, the notion of integral closure, Dedekind domains and study the ideal class group. We will prove that the ideal class group in a number field is finite and compute many examples. We will study the decomposition of primes in number fields and in quadratic fields in particular. We will end by discussing Dirichlet's unit theorem.

The module will assume familiarity with a number of topics in algebra (such as modules)..

MATH60043 Statistical Theory

Brief Description

This module seeks to provide a more unified perspective of the core statistical problems introduced in earlier modules by developing a general mathematical theory for parametric statistical models. We will deal with the criteria and theoretical results necessary to develop and evaluate statistical procedures for point estimation, hypothesis testing and confidence intervals. We will consider several approaches, including maximum likelihood estimation and Bayesian approaches, and study when they are provably optimal.

Learning Outcomes

On sucessful completion of this module, you will be able to-

-Apply key results related to optimal statistical procedures

- Evaluate and compare estimators using their sampling properties

- Explain what it means for statistics to be sufficient and complete

- Explain the Rao-Blackwell theorem and how it can be used to improve an estimator

- Use elementary ideas from decision theory to evaluate statistical procedures

- Explain the Bayesian approch to estimation

Module Content

An indicative list of sections and topics is:

- Theories of point estimation and hypothesis testing
- Exponential families
- Sufficiency and the Rao-Blackwell theorem
- The Cramer-Rao lower bound
- Maximum likelihood estimation and its asymptotic theory
- Bayesian estimation
- Decision theory
- Completeness and minimum variance unbiased estimators
- The Neyman-Pearson lemma and likelihood ratio tests

MATH60044 Applied Statistical Inference

Brief Description

In this module, you will use statistical models to explore data from a wide range of problem domains: biology, medicine, engineering, finance. The emphasis throughout will be on developing practical skills for working with data. The module centres around two different extensions of the linear model. First, the idea of Generalized Linear Models, used to handle data where errors cannot be assumed normally distributed with constant variance. Examples include Poisson or binomial counts, proportions and waiting times. Second, we relax the assumption of independence, introducing Linear Mixed Models to accommodate correlated observations. We will introduce widely applicable methods and principles useful for the applied data scientist, such as diagnostic plots, bootstrapping and regularization.

Learning Outcomes

On successful completion of this module, you will be able to:

- Use R to fit linear models.

- Select appropriate generalized linear models for modelling data with with non-normal error distributions, e.g. poisson, binomial, gamma.

- Use models to give predictions, together with an associated confidence interval.

- Identify models that fit poorly, and suggest improved models.

- Model data with correlated observations using linear mixed models.

- Explain the properties of different estimators of random effects variances, such as maximum likelihood and REML.

- Interpret output from R using plain language.

Module Content

An indicative list of sections and topics is:

- Linear models: Least squares, normal equations, Gauss-Markov theorem, goodness of fit, diagnostics. Studentized residuals. Confidence intervals and prediction intervals.

- Generalized linear models: Specification, estimation (iterated reweighted least squares), inference, diagnostics.

- Mixed models: Specification, estimation (ANOVA, maximum likelihood, REML) and inference (parametric bootstrap) for variance components.

MATH60045 Applied Probability

Brief Description

This module introduces stochastic processes and their applications. The theory of different kinds of processes will be described and illustrated with applications in several areas. The groundwork will be laid for further deep work, especially in such areas as genetics, engineering and finance.

Learning Outcomes

On successful completion of this module, you will be able to:

- Use the Poisson process to model random arrivals.

- Extend the Poisson process to accommodate common depatures from the Poisson assumptions.

- Work with Markov chains in continous time.

- Determine the long-term behaviour of a continuous-time Markov chain.

- Determine whether states are recurrent or transient.

- Solve differential and difference equations to determine quantities of interest for stochastic processes.

- Explain basic properties of Brownian motion.

Module Content

An indicative list of sections and topics is:

- Review of probability and discrete time Markov Chains

- Random walks

- Poisson processes and their properties: superposition, thinning of Poisson processes; Non-homogeneous, compound, and doubly stochastic Poisson processes.

- Autocorrelation functions.

- Probability generating functions.

- General continuous-time Markov chains: generator, forward and backward equations, holding times, stationarity, long-term behaviour, jump chain, explosion; birth and death processes, reversibility, recurrence/transience.

- Differential and difference equations and pgfs. Embedded processes. Time to extinction.

- Queues.

- Brownian motion and its properties.

MATH60046 Time Series Analysis

Brief Description

A time series is a series of data points indexed and evolving in time. They are prevalent in many areas of modern life, including science, engineering, business, economics, and finance. This module is a self-contained introduction to the analysis of time series. Weight is given to both the time domain and frequency domain viewpoints, and important structural features (e.g. stationarity, reversibility) are treated rigorously. Attention is given to estimation and prediction (forecasting), and useful computational algorithms and approaches are introduced.

Learning Outcomes

On successful completion of this module, you will be able to:

- Appreciate that time series should be considered observations from an underlying stochastic process.

- Define what it means for a time series to be stationary.

- Identify autocorrelation within time series data.

- Work with standard models of time series.

- Appreciate that time series can exhibit trend and seasonality and know how to adjust for these.

- Determine the spectral representation of stationary time series and use the spectral density function to provide an alternative viewpoint of second-order structure.

- Derive and implement estimators of mean, correlation and spectral properties.

- Extend time series models, the notion of stationarity, and frequency domain representations to multivariate time series.

- Derive forecasts from standard time series models and quantify their uncertainty."

Module Content

An indicative list of sections and topics is:

- Discrete time stochastic processes and examples.

- Autocovariance, autocorrelation, stationarity.

- Trend removal and seasonal adjustment.

- AR, MA and ARMA processes, characteristic polynomials, general linear process, invertibility, directionality and reversibility.

- Spectral representation, aliasing, linear filtering.

- Estimation of mean and autocovariance sequence, the periodogram, tapering for bias reduction.

- Parametric model fitting.

- Forecasting.

MATH60047 Stochastic Simulation

Brief Description

Computational techniques have become an important element of modern statistics. Computation particularly underpins simulation methods, which are widely applied when studying models of complex systems, e.g. in biology and in finance. This module provides an up-to-date view of such simulation methods, covering areas from basic random variate generation to advanced MCMC (Markov Chain Monte Carlo) methods. The implementation of stochastic simulation algorithms will be carried out in R, a language that is widely used for statistical computing and well suited to scientific programming more generally.

Learning Outcomes

On successful completion of this module, you will be able to:

- Use algorithms for efficient generation of pseudo-random numbers.

- Evaluate intractable definite integrals using Use Monte Carlo methods.

- Implement MCMC algorithms to draw samples from intractable distributions.

- Assess the performance of MCMC algorithms using suitable diagnostic procedures.

- Explain how sequential Monte Carlo methods can be used to understand time-structured problems.

Module Content

An indicative list of sections and topics is:

- Pseudo-random number generators.

- Generalized methods for random variate generation.

- Monte Carlo integration.

- Variance reduction techniques.

- Markov chain Monte Carlo methods (including Metropolis-Hastings and Gibbs samplers).

- Monitoring and optimisation of MCMC methods.

- Introduction to sequential Monte Carlo methods.

MATH60048 Survival Models

Brief Description

Survival models are fundamental to actuarial work, as well as being a key concept in medical statistics. This module will introduce the ideas, placing particular emphasis on actuarial applications.

Learning Outcomes

On successful completion of this module, you will be able to:

- Appreciate survival analysis models as a framework that deals with lifetimes and censored observations;

- Use several methods to define event time distributions and the relation between these definitions;

- Describe, select and use methods for fitting parametric, semi-parametric and non-parametric survival analysis models, including regression models and multi state models.

- Explain the counting process approach to survival analysis and its benefits;

- Use and critically analyse methods occurring in actuarial applications, such as methods for the construction and use of life tables.

Module Content

An indicative list of sections and topics is:

- Concepts of survival models,

- Right and left censored data and randomly censored data.

- Estimation procedures for lifetime distributions:

- Empirical survival functions,

- Kaplan-Meier estimates,

- Cox model.

- Statistical models of transfers between multiple states,

- Maximum likelihood estimators.

- Counting process models.

- Actuarial Applications:

- Life table data and expectation of life,

- Binomial model of mortality,

- The Poisson model,

- Estimation of transition intensities that depend on age,

- Graduation and testing of crude and smoothed estimates for consistency.

MATH60049 Introduction to Statistical Learning

Brief Description

This module provides an introduction to methods of statistical learning. That is, using statistical and artificial intelligence (AI) methods to learn from data, often when the data set is large, complex or of high dimension. We will consider both supervised and unsupervised learning. For the former we use a training set of data to learn patterns within data and then use our knowledge of those patterns to devise methods for predicting the outcomes of those patterns for new data. For the latter, there is not (usually) an outcome measure, but we seek to learn about the patterns within the data set itself. The methods in this module are of immense interest in academic and business circles and underpins much of the modern data tech industry to achieve tasks such as suggesting movies you might like to watch given information on those that you have watched, and from people with similar viewing patterns, developing machine methods for identifying cancer and seeking new ways to understand the economy. It is strongly recommended that students will have already passed the module Statistical Modelling I, or similar.

Learning Outcomes

On successful completion of this module, you will be able to:

Appreciate the structure and likely distribution(s) of various forms of data and which kinds of methods might be suitable for their analysis.

Understand the concept of multivariate data and situations where supervised and unsupervised learning might be employed.

Know and deploy a variety of important statistical and AI methods, using the R language to solve data related problems.

Understand the limitations of methods and how their application can be checked and or corrected.

Module Content

An indicative list of sections and topics is:Linear Models for Regression (variable selection, Lasso, Ridge Regression); Linear methods for classification (Logistic Regression); Basis expansions (piecewise polynomials and splines; smoothing splines, wavelets); Kernel regression (local linear and local polynomial); Additive models and Trees (boosting); Projection pursuit regression and Neural Networks; Support Vector Machines; Cluster Analysis and Multidimensional Scaling; Random Forests; Data Ethics.

MATH60139 Spatial Statistics

Brief Description

Data collected in space are common in many applications including climate science, epidemiology, and economics. This module covers theoretical and methodological statistical foundations for spatial data. The module is structured to cover in detail the three fundamental forms in which spatial data are collected: gridded data, network data, and point pattern data. For each data type, stochastic models will be defined and explored including random fields and point processes. Properties including isotropy, stationarity and homogeneity will be formalised and explored in the context of each model and data type. In addition, techniques for spatial interpolation will be studied.

Learning Outcomes

On successful completion of this module students will be able to:

- Recognise the key differences between different types of spatial data (gridded, network and point pattern)

- Apply different classes of spatial covariance models to gridded and network data

- Formulate different intensity functions for point pattern data

- Define the concepts of homogeneity, stationarity and isotropy in the context of spatial models

- Select, derive and apply appropriate methods for spatial interpolation

Indicative Module Content

1. Introduction to spatial data (gridded, network and point pattern data)
2. Random Fields and Covariance Functions
3. Spatial Interpolation and Kriging
4. Network Data and Markov Random Fields
5. Spatial Point Processes

MATH60050 Research Project in Mathematics

Brief Description

The main aim of this module is to give a deep understanding of a particular area/topic by means of a supervised project in some area of mathematics. The project may be theoretical and/or computational and the area/topic for each student is chosen in consultation with the Department. The project is designed to be an introduction to research work in mathematics.

Learning Outcomes

On successful completion of this module you will:

- have pursued a topic of mathematical research, beyond the scope of lectured modules, and engaged with ideas at the frontier of present mathematical knowledge.

- have produced a written report, finished to a professional standard, containing an exposition of the topic of your project together with an account of any new results or investigations.

- have demonstrated your ability to communicate mathematics effectively in both written and spoken formats.

Module Content

The project will be on a topic agreed between the student and the supervisor and (where appropriate) the Director of Studies in Mathematics. Work on th project takes place in terms 2 and 3. It will offer scope for the student to master material well beyond that offered in lectured modules, and to reach the frontiers of research.