THE PILOT WAVE THEORY

Dissertation by R.O.HAVERY submitted in partial fulfilment of the requirements for the degree of Master of Science of Imperial College.

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Contents

1. Introduction

Standard quantum theory is immensely successful: it predicts the gross results of quantum reactions to as high a degree of accuracy as can be measured. Yet philosophically and aesthetically it is objectionable in at least two respects. First, it asserts that only the probabilities, or frequencies, of particular outcomes of measurements are in principle predictable. Yet how can nature produce regular frequencies without any underlying reason? Second, it concerns measurements made by a classical observer on a quantum system using classical equipment. The distinction between those three things is ambiguous.

The Pilot Wave Theory is not open to the above objections. It was propounded by de Broglie at the Solvay Conference in 1927. De Broglie proposed the existence of electron trajectories with velocity given by the gradient of the phase of Schrödinger's wave-function or "pilot-wave" (a "hidden variable"). But he abandoned the theory. He could not see how a particle in free space guided by a superposition of plane waves could have rapidly varying velocity and energy consistently with the outcomes of quantum energy measurements, which would coincide with discrete eigenvalues.[1] Moreover, he considered that in some circumstances pilot wave theory predicted

[1] Bacciagaluppi and Valentini, Quantum Theory at the Crossroads, in preparation, p.254.

superluminal speeds for photons. Einstein adopted a similar proposal but abandoned it on the ground that the theory had to be non-local.[2] The pilot wave theory was resurrected by Bohm in 1952.[3] Bohm introduced a "quantum potential" Q,[4] and his theory, unlike de Broglie's, was essentially second order in time. In 1982 Bell supported the view that a deterministic quantum theory was possible.[5] The torch was again taken up by Valentini in 1991, who has developed a substantial theory, insisting that

P (probability) =□□□^{2}

is not an axiom, but a special case only. Valentini's theory is first-order in time, like that of de Broglie.

The pilot wave theory is contextual. That is, it recognizes that the outcome of measurements cannot be accounted for simply by hidden variables in the observed system alone. The variables of the measuring equipment (which expression can include human observers) and the interaction between the equipment and the system all play a part in bringing about the result. The relevant wave function is the total wave function of system and equipment. The evolution of the system follows the gradient of the phase of the wave function. If the wave function is

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[2] A. Valentini, On the Pilot-Wave Theory of Classical, Quantum and Sub-quantum Physics, doctoral thesis, 1992, p.2.

[3] D. Bohm, (1952) Phys. Rev. **85**, 2, p.166.

[4] "U" in the 1952 paper; "Q" in D. Bohm and B. J. Hiley, (1987) Phys. Rep. **144**, 6, p. 323.

[5] Bell, J. S. (1982) Found. Phys. **12**, 989.

entangled, the evolutions of the variables are interdependent. Bell showed that any hidden variable theory must imply non-locality. The pilot wave theory gives a unified explanation of contextuality and non-locality. But non-locality at the level of hidden variables ("the subquantum level") implies conflict with the theory of relativity.

Instantaneous communication at a distance appears to be impossible in practice, notwithstanding that non-locality has been demonstrated by experimental results[6] showing violation of Bell's inequalities. There appears to be a "conspiracy" in nature whereby noise arising from the uncertainty principle just masks quantum non-locality, so as to preclude instantaneous signalling across a distance. According to the pilot-wave theory, that preclusion is not fundamental. It is contingent on a state of equilibrium in which we find quantum systems in our locality in the universe. Special relativity and standard quantum theory are equilibrium theories only. Non-locality implies a preferred temporal cross-section of spacetime and hence an absolute simultaneity.

The state of quantum equilibrium is analogous to the state of thermal equilibrium envisaged in the so-called "heat death" of the universe. But it may be that the whole universe is not in a state of quantum

[6] Aspect et al. (1982)

equilibrium, and disequilibrium states may be accessible. Valentini makes one point which may be questioned. He says[7]

> ...it is only in nonequilibrium that the underlying details of the theory become visible (via measurements more accurate than those allowed by quantum theory). If instead the universe is always and everywhere in quantum equilibrium, the details of de Broglie-Bohm trajectories will be forever shielded from experimental test, *and the de Broglie-Bohm theory itself would be unacceptable as a scientific theory.* [Emphasis added].

The pilot-wave theory makes the same predictions at the quantum level as standard quantum theory. If the subquantum trajectories are forever shielded from experimental test, then the pilot-wave theory cannot be tested against standard quantum theory, and might be regarded as useless, given the existence of quantum theory. But the success and general use of standard quantum theory do not entitle it to priority of acceptability. The motions of the planets can still properly be described in terms of epicycles. But if Newtonian mechanics did nothing more than explain the motions of the planets it would still be an acceptable theory, if not superseded.

[7] Inflationary Cosmology as a probe of Primordial Quantum Mechanics (ArXiV: 0805.0163v1, [hep-th] 1st May 2008, p.11).

In 1987, Bell proposed that the Schrödinger wave-function ψ is a physical field in configuration space.[8] That view is followed by Valentini. The phase of ψ is represented by S as follows:

$$\psi = R\exp(iS/\hbar)$$

where R is a real constant and S is real. In Bell's and Valentini's formulation, the pilot-wave theory operates in terms of a physical guiding field (pilot-wave) in configuration space. The field is of the same nature as the phase function, or action, S of classical Hamilton-Jacobi theory. The gradient of that field determines the velocities of the variables of the system. For a particle of mass m, in one dimension x,

$$m\dot{x} = R\nabla S.$$

In that respect, the theory differs markedly from Newtonian mechanics, where the force, or gradient of the potential, determines the accelerations of the masses. In pilot-wave theory, classical forces are manifestations of EPR-entanglement in the phase function. The concepts of momentum, force and energy play no fundamental role.

[8] Bell wrote: "It is in the wavefunction that we must find an image of the physical world, and in particular of the arrangement of things in ordinary three-dimensional space. But the wavefunction as a whole lives in a much bigger space, of 3N-dimensions. It makes no sense to ask for the amplitude or phase or whatever of the wavefunction at a point in ordinary space. It has neither amplitude nor phase nor anything else until a multitude of points in ordinary three-space are specified". (Speakable and Unspeakable in Quantum Mechanics, p.204)

What is fundamental is the wave function: the rate of change of all physical variables is given by the gradient or functional derivative of S, with no need of further explanation. Further explanation may at some stage emerge, not necessarily using conventional mechanistic concepts. The wave function □ may be described as an "informative field", which "informs" the time evolution X(t) of the system.

The pilot-wave theory includes theories of subquantum electro-dynamics, subquantum cosmology, subquantum measurement, subquantum automata and parallel processing.

How can the theory be tested? That can be done definitively, i.e. by refuting standard quantum theory, only if non-equilibrium situations can be found. If there was quantum disequilibrium at the big bang, it can be shown that it would largely have disappeared by now. That explains the present prevalence of quantum equilibrium. But quantum noise may be a remnant of the big bang, and residual deviations from quantum noise may be found in the cosmic microwave background. It can be shown that inflationary expansion of the early universe can transfer microscopic non-equilibrium to macroscopic scales, resulting in anomalous power spectra for the cosmic microwave background. It is suggested that two-slit experiments can be carried out using cosmological microwave photons, WIMPs or [in the future?] cosmological neutrinos.

Another way of testing the pilot wave theory is suggested in section 16 below.

2. Pilot Wave Theory of a Single Particle.

For a single particle system with Hamiltonian $\frac{-\hbar^2}{2m}\nabla^2 + V$, we have the Schrödinger equation

$$i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\Psi + V\Psi .$$

If $j = \frac{\hbar}{m}\mathrm{Im}(\Psi * \nabla\Psi)$, then

$$\frac{\partial|\Psi|^2}{\partial t} + \nabla . j = 0 .$$

This is the continuity equation, and j(x,t) is the quantum probability current. The probability flow can be interpreted as the flow of an ensemble of particles, initially with distribution

$$\rho(x,t) = |\Psi(x,t)|^2$$

at t=0, where at time t a particle at x has velocity given by

$$\dot{x}(t) = \frac{\hbar}{m}\mathrm{Im}\frac{\nabla\Psi(x,t)}{\Psi(x,t)} .$$

For each particle, Ψ(x,t) is a “guiding field” or “pilot wave” which generates a velocity field v(x,t) throughout space, at each moment telling the particle how to move, wherever it may be.

The trajectory of the particle is the solution *x(t)* of the equation

$$\dot{x} = \frac{\nabla S(x,t)}{m}$$

evaluated at $x=x(t)$. Hence the velocity is orthogonal to the surfaces S=constant, and the particle moves orthogonally to the wavefront. Thus for a plane wave, the trajectory of the particle is a straight line (see fig 2.1)

The wave function ψ is given by

$$\psi = R\exp(iS/\hbar).$$

S represents the phase of the wave. $|\psi|^2$ (i.e. R^2) gives the probability density. Hence the probability P that the particle lies between the points x and $x+dx$ at time t is given by

$$P = R^2(x,t)dx$$

for one dimension.

The solution $\psi(x,t)$ of the wave equation in any given situation will in general depend on a set of parameters which characterize the environment that the propagating wave encounters, e.g. width of slit, radius of scattering centre. If those parameters change, ψ will change at every point in space-time, not just those points in the vicinity of the physical objects giving rise to the parameters[9].

For a plane wave, consider a particle of energy E and vector momentum p moving in three dimensions in a straight line. Following

[9] Holland, The Quantum Theory of Motion (1993) 79.

de Broglie, we associate with that particle a plane wave whose wave function, a solution of the Schrödinger equation with potential V=0, is given by

$$\Psi(x,t) = Ae^{\frac{i}{\hbar}(p.x-Et)} = e^{iS/\hbar}$$

where $E = \frac{p^2}{2m}$ and A is some constant. Then the phase

$$S(x,t) = p.x - Et\,.$$

The velocity field

$$\dot{x} = \frac{\nabla S}{m} = \frac{p}{m}\,.$$

Hence the trajectory of the particle is given by

$$x(t) = x_0 + \frac{p}{m}t\,.$$

This is uniform motion in a straight line, as for a classical free particle.

From the de Broglie equation

$$\dot{x}(t) = \frac{\hbar}{m}\operatorname{Im}\frac{\nabla\Psi(x,t)}{\Psi(x,t)}$$

we may write the velocity field as

$$v(x,t) = \frac{\hbar}{m}\operatorname{Im}\frac{\nabla\Psi(x,t)}{\Psi(x,t)} = \frac{\nabla S(x,t)}{m}\,.$$

The wave function

$$\Psi(x,t) = \left|\Psi(x,t)\right|e^{\frac{i}{\hbar}S(x,t)}$$

is a physical field on configuration space. Unlike Newtonian and Bohmian mechanics, the dynamics of the pilot wave theory is first order, based on velocities. The law of motion is

$$m\dot{x} = \nabla S \, .$$

Trajectories generally violate classical intuition, and the theory is fundamentally non-local.

3. Exactly sketchable

For a given position x at a given time t the velocity field is single-valued:

$$\dot{x} = \frac{\nabla S}{m} \, .$$

∇S is a single-valued function of position. Thus at each point in space at each instant there is a unique tangent vector associated with ∇S. It follows that no trajectory (plotted as position against time) can cross or even touch another. Thus if a beam of electrons is fired at a potential barrier or a potential well, where some are reflected and some are transmitted by way of tunnelling, the trajectories of the reflected electrons form a group distinct from those of the transmitted electrons. The incident electrons that will be transmitted are all ahead of those that will be reflected. See figs. 11.1 to 11.5.

4. No notion of energy

Pilot Wave Theory has first-order dynamics:

$$m\dot{x} = \nabla S$$

whereas Newtonian mechanics is second order:

$$m\ddot{x} = -\nabla V\ .$$

In pilot wave theory, the initial conditions are $\Psi(x,0)$ and $x(0)$. Those give $\Psi(x,t)$ and $x(t)$ for all t. In Newtonian mechanics, the initial conditions of a particle are $x(0)$ and $\dot{x}(0)$, which give $x(t)$ for all t.

Newtonian mechanics has a first integral of motion

$$\tfrac{1}{2} m\dot{x}^2 + V(x) = const.\ \mathrm{E}\ \text{(energy)}.$$

In Bohmian dynamics, there is the potential V+Q, but it is not conserved. For an individual system at the hidden variable level, there is no useful concept of energy in pilot wave dynamics.

5. Bohmian dynamics

In 1952 Bohm[10] revived de Broglie's pilot wave theory, introducing to it a "quantum potential" Q,[11] given by

$$Q = -\frac{h^2}{2m}\frac{\nabla^2 R}{R} \qquad \text{where R = □□□}$$

and $\quad$ □ = R$e^{\frac{iS}{\hbar}}$.

This leads to a second-order form of the pilot wave theory:

$$m\ddot{x} = \frac{d}{dt}\nabla S = (\frac{\partial}{\partial t} + \dot{x}.\nabla)\nabla S = \nabla\ \frac{\partial S}{\partial t} + \frac{\nabla S.\nabla}{m}\nabla S\ . \qquad (5.1)$$

With the classical Hamiltonian $H = \frac{p^2}{2m}$+V, the Schrödinger equation is

[10] Bohm, Hiley and Kaloyerou Phys. Rep. **144**, No. 6 (1987) 321. The paper appears to contain a misprint, stating that R=□□□².
[11] In the 1952 paper, the potential later designated as "Q" was designated as "U".

$$\mathrm{i}\square \frac{\partial}{\partial t}\square = (-\frac{h^2}{2m}\nabla^2 + V)\square .$$

Using □ = □□□$e^{\frac{iS}{\hbar}}$, we find that the real part of the Schrödinger equation yields

$$\frac{\partial S}{\partial t} + \frac{(\nabla S)^2}{2m} + V + Q = 0$$

hence, from equation (5.1),

$$m\ddot{x} = -\nabla(V + Q) .$$

6. Free Gaussian (Stationary)

We here consider the movement of a particle in a stationary spreading wave packet. Let the initial wave function (see fig. 6.1a) be represented by a Gaussian curve

$$\Psi_0(x) = \frac{1}{(2\pi\Delta_0^2)^{1/4}} e^{-x^2/4\Delta_0^2} .$$

Let the spread as a function of time (see fig. 6.1b) be given by

$$\Delta(t) = \Delta_0\sqrt{1 + t^2/\tau^2}, \text{ where timescale } \tau = \frac{2m\Delta_0^2}{\hbar} .$$

Then

$$\Psi(x,t) = \frac{1}{(2\pi[\Delta_0 + i\hbar t/2m\Delta_0]^2)^{1/4}} e^{-x^2/4\Delta_0^2(1 + i\hbar t/2m\Delta_0^2)} .$$

It follows that

$$|\Psi(x,t)|^2 = \frac{1}{(4\pi^2\Delta_0^2[1 + t^2/\tau^2])^{1/4}} \exp\frac{-x^2}{4\Delta_0^2(1 + t^2/\tau^2)}$$

and that the phase $S = \frac{mx^2\hbar^2 t}{2(4m^2\Delta_0^4 + \hbar^2 t^2)}$.

Hence $\qquad \dfrac{\nabla S}{m} = \dot{x} = \dfrac{xt}{t^2 + \tau^2}$;

therefore $\qquad x(t) = x_0\sqrt{1 + \dfrac{t^2}{\tau^2}}$.

Thus the particle moves to the right if $x_0 \rangle 0$, and to the left if $x_0 \langle 0$.

7. Free Gaussian (moving)

Consider a free Gaussian, treated as an ensemble of particles, whose centre is moving (fig. 7.1) with speed v as measured by the time of flight of a central particle across a known distance. That would be done by making a series of measurements and averaging the results.

We take

$$\Psi(x,t) = \{2\pi(\Delta_0[1 + it/\tau])^2\}^{-1/4} \exp\{\frac{i}{\hbar}(mv[x - \frac{1}{2}vt])\}\exp\frac{-(x - vt)^2}{4\Delta_0^2(1 + it/\tau)}$$

where, as before, $\tau = \dfrac{2m\Delta_0^2}{\hbar}$ and $\Delta(t) = \Delta_0\sqrt{1 + t^2/\tau^2}$.

Since $\qquad \dfrac{S}{\hbar} = \mathrm{Im}\{\dfrac{\Psi}{|\Psi|}\}$,

$$S = mv(x - \frac{1}{2}vt) + \frac{m(x - vt)^2 t}{2(t^2 + \tau^2)} .$$

In one dimension, $\qquad \dot{x} = \dfrac{1}{m}\dfrac{\partial S}{\partial x} = v + \dfrac{(x - vt)t}{t^2 + \tau^2}$.

Now $\qquad \dfrac{x - vt}{x_0} = \dfrac{x - x_c(t)}{x_0} = \dfrac{\Delta}{\Delta_0} = \sqrt{1 + t^2/\tau^2}$.

Therefore $\qquad x(t) = vt + x_0\sqrt{1 + t^2/\tau^2}$

and $\dot{x}(t) = v + \dfrac{x_0 t}{\tau^2 \sqrt{1 + t^2 / \tau^2}}$.

If $t \rangle\rangle \tau$, then $\dot{x} - v \approx \dfrac{x_0}{\tau} \approx \dfrac{\hbar x_0}{m\Delta_0^2}$

and if $x_0 \langle\approx \Delta_0$, then $\dfrac{\hbar x_0}{m\Delta_0^2} \langle\approx \dfrac{\hbar}{m\Delta_0}$.

Thus unless the particle is in the tail of the Gaussian, $\dot{x} - v$ is small, and the motion of the particle approximates to classical motion.

8. Energy eigenfunctions

Consider the energy eigenvalues E_n and eigenfunctions f_n of the Hamiltonian operator $\hat{H}$:

$$\hat{H} f_n(x) = E_n f_n(x) ,$$

$$\Psi_n(x,t) = f_n(x) e^{-\frac{i}{\hbar} E_n t}$$

If $f_n(x)$ is real, then S= $- E_n t$

and $m\dot{x} = \nabla S = 0$.

Thus, where

$$\Psi_n(x,t) = \sqrt{\frac{2}{L}} \sin(\frac{n\pi x}{L}).e^{-\frac{i}{\hbar} E_n t} .$$

the particle is at rest in a box.

Similarly, where

$$\Psi_{100} = \frac{1}{\sqrt{\pi a_0^3}} e^{-r/a_0} e^{-\frac{i}{\hbar}E_1 t}.$$

the particle is at rest in the ground state of hydrogen.

But the particle is not at rest in eigenstates where $\nabla S \neq 0$, e.g.

$$\Psi_{211} = -\frac{1}{8\sqrt{\pi a_0^3}} \frac{r}{a_0} e^{-r/2a_0}.\sin\theta.e^{i\phi}.e^{-\frac{i}{\hbar}E_2 t}.$$

Here, the phase $\mathrm{S} = \hbar\phi - E_2 t$. So

$$\dot{\phi} = \frac{\nabla_\phi S}{m} = \frac{1}{mr\sin\theta}\frac{\partial S}{\partial\phi} = \frac{\hbar}{mr\sin\theta}.$$

Thus, the orbital angular speed of the particle about the *z*-axis is proportional to the m quantum number, the exponent in the wavefunction being im□. [m is not to be confused with the mass *m* of the particle].

9. Simple harmonic oscillator

In standard quantum theory, for a position measurement we write

probability density = $|\Psi(x,t)|^2$;

For momentum measurement, we write

probability density = $|\widetilde{\Psi}(p,t)|^2$.

For example, in the ground state of a simple harmonic oscillator we have

$|\Psi_0(x)|^2 \propto e^{-\frac{m\omega}{\hbar}x^2}$; and

$$\left|\widetilde{\Psi}_0(p)\right|^2 \propto e^{-\frac{p^2}{m\hbar\omega}}.$$

But in pilot wave theory we have an ensemble of positions whose density is

$$\left|\Psi(x,t)\right|^2.$$

Each x has a definite momentum

$$p = \nabla S(x,t).$$

For the ground state of the simple harmonic oscillator

$$\Psi_0(x,t) \propto e^{-m\omega x^2/2\hbar} e^{iS/\hbar}, \text{ where } S = -E_0 t.$$

Thus $\quad p = \nabla S = 0$ for all x.

This apparent inconsistency was explained by Bohm[12] in 1952. He wrote:

> …the measurement of an observable is not really a measurement of any physical property belonging to the observed system alone. Instead, the value of an "observable" measures only an incompletely predictable and controllable potentiality belonging just as much to the measuring apparatus as to the observed system itself. At best, such a measurement provides unambiguous information only at the classical level of accuracy, where the disturbance of the □-field by the measuring apparatus can be neglected.

The measured momentum is not equal to the actual prior momentum. The momentum may be measured, for example, by the time-of-flight method (see fig. 6.1). The potential confining the system to the state of a simple harmonic oscillator is switched off at t=0. The Gaussian

[12] Bohm, 1952b, p.183.

$|\Psi(x,t)|^2$ spreads freely, □ develops a phase with $\nabla S \neq 0$, and the trajectories are given by

$$x(t) = x_0\sqrt{1 + t^2/\tau^2} \ .$$

At large t, the measured momentum p is $m\frac{x}{t}$ where x is the measured position at t. The outcome p depends on x_0, the position of the particle when $t = 0$. The distribution $|\Psi_0(x)|^2$ for x_0 implies the distribution $|\tilde{\Psi}_0(p)|^2$ for p.

We have

$$\Psi_0(x) = (\frac{m\omega}{\pi\hbar})^{1/4}\exp(\frac{-m\omega}{2\hbar}x^2) \ ;$$

the initial width Δ_0 of the Gaussian is given by

$$\Delta_0 = \sqrt{\frac{\hbar}{2m\omega}}$$

and the width spreads on the timescale

$$\tau = \frac{2m\Delta_0^2}{\hbar} = \frac{1}{\omega} \ .$$

The trajectories are given by

$$x(t) = x_0\sqrt{1 + t^2/\tau^2} = x_0\sqrt{1 + \omega^2\tau^2} \ .$$

The measured momentum is given by

$$p_{meas} \equiv \lim_{t\to\infty}\frac{mx(t)}{t} = mx_0\omega \ .$$

Then $$\tilde{\rho}(p_{meas})dp_{meas} = |\Psi_0(x_0)|^2 dx_0 \ .$$

$$\therefore \tilde{\rho}(p_{meas}) = \frac{1}{m\omega}\left|\Psi_0(\frac{p_{meas}}{m\omega})\right|^2$$

$$= \frac{1}{\sqrt{\pi m\hbar\omega}} \exp(\frac{-p_{meas}^2}{m\hbar\omega}) = \left|\widetilde{\Psi}_0(p_{meas})\right|^2 .$$

The r.m.s. width $\Delta p_{meas} = \sqrt{\frac{m\hbar\omega}{2}}$.

The product of the statistical dispersions $\Delta_0 \Delta p_{meas} = \frac{\hbar}{2}$. It is the distribution of the positions $\rho(x,t) = \left|\Psi(x,t)\right|^2$ which leads to this "uncertainty".

10. Reflection and transmission by barrier

A beam of free particles travels along the x-axis in the direction of increasing x towards a potential barrier of height V and width b-a. The interaction takes place in the interval x=(a,b) (see fig. 10.1). The general solution of the Schrödinger equation for a free particle is

$$\psi(x) = Ae^{ikx} + Be^{-ikx} \qquad \text{for } x\langle a$$

and $$\psi(x) = Ce^{ikx} \qquad \text{for } x\rangle b .$$

The reflected wave has the negative exponent. There being no reflected wave where $x\rangle b$, in that region there is no wave with negative exponent.

We are considering a steady state, and the probability current density j is given by

$$j = \frac{\hbar}{m}|\psi|^2 \operatorname{Im}(\frac{\nabla\psi}{\psi})$$

$$= \frac{\hbar k}{m}(|A|^2 - |B|^2) \qquad \text{where } x\langle a$$

$$= \frac{\hbar k}{m}|C|^2 \qquad \text{where } x\rangle b.$$

The reflection coefficient R, the ratio of the intensity of the reflected probability current density to the intensity of the incident probability current density, is given by

$$R = \frac{|B|^2}{|A|^2}$$

and the transmission coefficient T is given by

$$T = \frac{|C|^2}{|A|^2}.$$

If we consider a group of incident particles with Gaussian distribution, the first particles to arrive at the barrier at x=a are transmitted (whether or not first lingering in the barrier) and all those behind them are reflected (again, whether or not lingering in the barrier). That must be so, since the trajectories do not cross. The probability T that a particle will be transmitted is equal to the probability that it will be in the region $x\rangle b$, that is

$$\int_b^\infty |\psi|^2 \, dx.$$

Similarly, the probability that it will be reflected is

$$\int_{-\infty}^{a} |\psi|^2 \, dx = \int_{-\infty}^{b} |\psi|^2 \, dx .$$

Particles do not remain indefinitely in the barrier, hence

$$\int_{a}^{b} |\psi|^2 \, dx = 0 .$$

The Gaussian being normalized, the sum T+R=1. See fig. 10.2.

11. Potential barriers and wells

Using the Bohm quantum potential, Dewdney and Hiley carried out calculations of the effect of square potential barriers and wells on bunches of incident particles represented by Gaussian packets.[13] They found that in the case of the barrier, not only were there direct reflection and transmission of particles, but also a finite probability of a particle being trapped inside the barrier for a finite time. In the discussion that follows, we shall ignore the reference to potential and concentrate on the trajectories.

In the case of barriers, Dewdney and Hiley examined three cases: (1) where the incident energy was half the height of the barrier; (2) where the incident energy was equal to the height of the barrier; and (3) where the incident energy was twice the height of the barrier.

In case (1), figure 11.1 presents an ensemble of trajectories chosen to show all classes of motion. In order to achieve tunnelling the initial

[13] Dewdney and Hiley, (1982) Found. Phys. **12**, 27.

position of the incident particle must be very close to the front of the packet. The trajectory which starts from position 0.67 (see figure), which is still close to the front, exhibits a complicated behaviour. It does not pass through the barrier, but is temporarily trapped, oscillating back and forth in the barrier. Depending on their precise starting positions in the vicinity of 0.67, trapped particles escape from the barrier after one or more oscillations, causing a spread of emerging trajectories.

The trajectories starting between 0.57 and 0.66 remain inside the barrier without oscillating and are eventually reflected by it. The relevant particles are almost stationary while within the barrier. The trajectories starting between 0.5[14] and 0.57 just penetrate into the barrier before being reflected. They are closely bunched there and give rise to a skin effect.

The particles that follow trajectories that start between the centre and the rear of the packet are reflected without ever reaching the barrier. The trajectories become bunched near the barrier and produce Wiener-type fringes, a matter-wave analogue of Wiener's optical fringes, produced by standing light waves.[15]

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[14] Figure 11.1 indicates a lower-bound starting figure in excess of 0.50.
[15] O. Wiener, (1890) Ann. d. Physik **40**, 203.

The particles in the rear of the packet are decelerated and reflected before reaching the barrier. The trajectory from the centre of the packet follows its classical path, except in the region of interference.

The trajectories in case (2) are shown in fig. 11.2. It can be seen from figure 11.2 that the particles in the rear are reflected slightly earlier than are reflected particles ahead of them, unlike case (1) depicted in fig. 11.1. Fig. 11.2 also shows considerably more tunnelling than fig. 11.1. Particles just in front of the centre, i.e. those starting at about 0.52, enter the barrier and form a wave packet which remains within the barrier slowly decreasing in amplitude as the individual particles are eventually either transmitted (those at the front of this subgroup) or reflected (those at the rear of this subgroup) out of the barrier.

Case (3). The trajectories here are illustrated in fig. 11.3. The particles in the front half of the packet are transmitted classically. There are two separate reflected wave packets, those whose trajectories do not reach the barrier and those which enter the barrier and are reflected out of it.

Two cases of square wells were considered: case (4), where the incident energy was half the depth of the well; and case (5), where the incident energy was twice the depth of the well. Case (4) is illustrated in fig. 11.4. The first point to note is that the particles that penetrate the well move faster inside it than outside, in contrast to the situation where there was a barrier. All of the particles whose initial positions were in the front half of the wave packet were transmitted. The particles whose initial positions lay on the tail side of the centre of the packet spent more time in the well, thereby spreading out the emerging trajectories. The density of trajectories between the initial positions 0.47 and 0.46 was increased in fig.11.4 in order to illustrate a bifurcation between transmitted and reflected particles. It can also be seen that some reflected particles entered the well.

Case (5) is illustrated in fig. 11.5. Of those trajectories that are reflected, there is a group that are reflected at some distance in front of the well, and there is a second group that includes trajectories that enter the well before being reflected.

Figures 11.1 to 11.5 constitute a graphic illustration of the fact that no two trajectories can cross.

Dewdney carried out calculations in relation to neutrons showing comparable results.[16]

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[16] Dewdney, Particle Trajectories and Interference in a Time-dependent Model of Neutron Single Crystal Interferometry, (1985) **109A** Phys. Lett., no.8, p.377.

12. Creating the Past

A beam of particles is split by a half-silvered mirror so that half the particles are reflected by fully-silvered mirror number 1 and go into counter number 1 and the other half are reflected by fully-silvered mirror number 2 and go into counter number 2 (see fig. 12.1). At any time a second half-silvered mirror, shown by a dashed line, may be inserted. Before it is inserted, it is known that particles entering counter 1 have travelled by via mirror 1, and that particles entering counter 2 have travelled via mirror 2. The count rates in the two counters will be equal. After insertion, there will be interference between the two half-beams. The respective count rates will depend on the phase difference represented by the two routes; and it will not be possible to tell by which route any particle has travelled. In the Copenhagen interpretation of quantum theory, after insertion of the second half-silvered mirror each particle has travelled by both routes. But in principle insertion can take place after a particle has passed the first half-silvered mirror. Thus on the Copenhagen interpretation, the history of a particle can be changed after the event. Or it might be said that the particle travelled by one route only and by both routes. Wheeler suggested that the past has no existence except as it is recorded in the present. The passage of an electron is not a phenomenon until it is an observed phenomenon.[17] These

[17] J. A. Wheeler (1978), quoted by Holland, p. 189.

conclusions, at best mystifying, do not arise on pilot wave theory. On pilot wave theory, each particle travels along one route only. The pilot wave is split into two parts at the first half-silvered mirror. The pilot wave is not affected unless and until the second half-silvered mirror is inserted. Its history, and that of the particle, before that point are unchanged.

13. Quantum Measurement

Standard quantum theory provides that when a measurement is made on a quantum system, there is a collapse of the wavefunction, giving rise to a specific reading. Pilot wave theory, on the other hand, provides that when a measurement is made on a quantum system, the system behaves as though guided by a reduced wavefunction.

Specifically, the standard theory works as follows. Consider the following example. At $t = 0$, the wavefunction of a particle is a superposition

$$\psi_0(x) = c_1\phi_1(x) + c_2\phi_2(x)$$

of two eigenfunctions ϕ_1, ϕ_2 of an operator $\hat{Q}$:

$$\hat{Q}\phi_1(x) = q_1\phi_1(x), \qquad \hat{Q}\phi_2(x) = q_2\phi_2(x)$$

(with $|c_1|^2 + |c_2|^2 = 1$, eigenvalues q_1, q_2).

To measure the value of $\hat{Q}$, introduce a "pointer" particle with position *y*, initially with wavefunction $g_0(y)$ localized around $y = 0$. Couple *x* and *y* via the von Neumann interaction

$$\hat{H}_I = a\hat{Q}\hat{P}_y$$

where $\hat{P}_y = -i\hbar\frac{\partial}{\partial y}$ is the momentum operator conjugate to *y*. Initially,

$$\psi_0(x,y) = [c_1\phi_1(x) + c_2\phi_2(x)]g_0(y).$$

With a large coupling *a* over a short time we can ignore the Hamiltonians of *x* and *y*. The Schrödinger equation

$$i\hbar\frac{\partial\psi(x,y,t)}{\partial t} = -a\hat{Q}i\hbar\frac{\partial\psi(x,y,t)}{\partial y}$$

has the solution

$$\psi(x,y,t) = c_1\phi_1(x)g_0(y - aq_1t) + c_2\phi_2(x)g_0(y - aq_2t).$$

For sufficiently large *at*, the alternative pointer packets $|g_0(y - aq_it)|^2$ centred on $y = aq_it$ for $i = 1,2$ do not overlap. There is thus a collapse to one pointer reading y_i with probability $|c_i|^2$. y lies either in $|g_0(y - aq_1t|^2$ or in $|g_0(y - aq_2t)|^2$ corresponding to the respective outcome q_1 or q_2.

The pilot wave theory account is this. The system has a configuration (*x(t), y(t)*) with a velocity given by $\frac{j}{|\psi|^2}$, where j is the probability current. An initial ensemble

$$P_0(x,y) = |\psi_0(x,y)|^2$$

evolves into a final ensemble

$$P(x,y,t) = |\psi(x,y,t)|^2 .$$

Hence

$$P(x,y,t) = |c_1|^2 |\phi_1(x)|^2 \, |g_0(y - aq_1 t)|^2 + |c_2|^2 |\phi_2(x)|^2 |g_0(y - aq_2 t)|^2$$

and there is no overlap between the alternative pointer packets. The distribution separates in configuration space into two non-overlapping regions, with the actual $x(t)$, $y(t)$ in one region only. The probabilities for the two possible regions are

$$\int dx \int dy |c_i|^2 |\phi_i|(x)^2 |g_0(y - aq_i t)|^2 = |c_i|^2$$

where the integral over *y* is limited to the support of $|g_0(y - aq_i t)|^2$.

Thus the quantum probabilities $|c_i|^2$ are derived from the initial $P_0(x,y) = |\psi_0(x,y)|^2$ together with the dynamics of the actual configurations. Once $(x(t), y(t))$ enters one non-overlapping region, the velocity $(\dot{x}(t), \dot{y}(t))$ is not affected by the other branch of the wavefunction with no support there. There is no real collapse of the wavefunction ψ . The velocity $(\dot{x}(t), \dot{y}(t))$ due to the product

$$c_i \phi_i(x) g_0(y - aq_i t)$$

for the relevant value of *i* is the same as though *x* had the wavefunction $\phi_i(x)$ and *y* had the wavefunction $g_0(y - aq_i t)$. So once separation in configuration space has occurred, the system *x* behaves as if it were guided by the “reduced wavefunction” $\phi_i(x)$ for the particular value of *i*.

Standard quantum theory asserts that the observable $\hat{Q}$ has the value q_i; pilot wave theory asserts that the trajectory $x(t)$ is guided by $\phi_i(x)$. But it is notable that in pilot wave theory, the obtaining of the outcome q_i on making a measurement does not imply that any actual attribute of the system had that value before or after the measurement. A quantum "measurement" is the product of the system and the apparatus.[18]

14. Subquantum measurement

Non-equilibrium particles could be used to perform measurements on ordinary equilibrium systems.[19] We assume an ensemble of "apparatus" particles with known wavefunction $g_0(y)$ and known non-equilibrium distribution $\pi_0(y) \neq |g_0(y)|^2$. The position y may be regarded as the position of a "pointer". We wish to use the apparatus particles to measure the position of "system" particles with known wavefunction $\psi_0(x)$ and known equilibrium distribution $\rho_0(x) = |\psi_0(x)|^2$.

We consider a model which has an exact solution. At $t = 0$, we take a system particle and an apparatus particle and switch on an interaction between them described by the Hamiltonian $\hat{H} = a\hat{x}\hat{p}_y$,

[18] See, e.g. Bell (2004), p.166.
[19] Antony Valentini, Subquantum Information and Computation, arXiv:quant-ph/0203049v2 12th April 2002.

where a is a coupling constant and p_y is the momentum canonically conjugate to y. This is the standard interaction Hamiltonian for an ideal quantum measurement of x using the pointer y. For simplicity, we neglect the Hamiltonians of x and y themselves. We can assume that the interaction lasts for so short a time that the changes of the apparatus and the system that would have taken place in the absence of interaction can be neglected, and that we can use that part of the Hamiltonian that represents only the interaction.[20]

For $t\rangle 0$ the joint wavefunction $\Psi(x,y,t)$ satisfies the Schrödinger equation

$$i\hbar\frac{\partial\Psi}{\partial t} = \hat{H}\Psi = -ia\hat{x}\frac{\partial\Psi}{\partial y}$$

i.e.
$$\frac{\partial\Psi(x,y,t)}{\partial t} = -ax\frac{\partial\Psi(x,y,t)}{\partial y}, \qquad (14.1)$$

hence the continuity equation

$$\frac{\partial\left|\Psi(x,y,t)\right|^2}{\partial t} + ax\frac{\partial\left|\Psi(x,y,t)\right|^2}{\partial y} = 0\,. \qquad (14.2)$$

The general solution of (14.1) is $\Psi(x,y,t) = \Psi(x,[y-axt],0)$.

The continuity equation for the equilibrium probability density $\left|\Psi(x,y,t)\right|^2$ is

$$\frac{\partial\left|\Psi(x,y,t)\right|^2}{\partial t} + \frac{\partial(\left|\Psi(x,y,t)\right|^2\dot{x})}{\partial x} + \frac{\partial\left|\Psi(x,y,t)\right|^2\dot{y}}{\partial y} = 0\,. \qquad (14.3)$$

[20] Bohm, A Suggested Interpretation of the Quantum Theory in Terms of “Hidden” Variables II, (1952) Phys. Rev. **85**, 180,181.

From (14.2) and (14.3) we deduce that $\dot{x} == 0$ and $\dot{y} = ax$. Hence the trajectories are $x(t) = x_0$, $y(t) = y_0 + ax_0 t$.

The initial product wavefunction $\Psi(x,y,t) = \psi_0(x)g_0(y)$ evolves into the entangled wavefunction $\Psi(x,y,t) = \psi_0(x)g_0(y - axt)$. In the limit as $at \to 0$ we have $\Psi(x,y,t) \approx \psi_0(x)g_0(y)$, and the system wavefunction $\psi_0(x)$ is undisturbed. Yet no matter how small at may be, provided that it is non-zero, at the subquantum level the pointer position $y(t) = y_0 + ax_0 t$ contains information about the value of x_0, and of $x(t) = x_0$. And this subquantum information about x will be visible to us if the initial pointer distribution $\pi_0(y)$ is sufficiently narrow. For consider an ensemble of similar experiments, where x and y have the initial joint distribution $P_0(x,y) = |\psi_0(x)|^2 \pi_0(y)$. The continuity equation

$$\frac{\partial P(x,y,t)}{\partial t} + ax\frac{\partial P(x,y,t)}{\partial y} = 0$$

implies that at later times $P(x,y,t) = |\psi_0(x)|^2 \pi_0(y - axt)$. If $\pi_0(y)$ is localised (say $\pi_0(y) \approx 0$ for $|y| \rangle \frac{w}{2}$) then from a standard measurement of y we may deduce that x lies in the interval $(\frac{y}{at} - \frac{w}{2at}, \frac{y}{at} + \frac{w}{2at})$, where the margin of error $\frac{w}{2at} \to 0$ as the width $w \to 0$. Thus if the apparatus distribution $\pi_0(y)$ is arbitrarily narrow, we can measure the position

x_0 of the system without disturbing its wavefunction $\psi_0(x)$ to arbitrary accuracy, in violation of the uncertainty principle.

15. Hot water

Standard quantum theory interprets quantum measurements on the basis of classical theory. Einstein explained that that is not a proper way to proceed.

Valentini[21] quotes a conversation between Einstein and Heisenberg in 1926. In that conversation, Heisenberg stated that a good theory must be based on directly observable magnitudes. Einstein replied:[22]

> ...it may be heuristically useful to keep in mind what one has actually observed. But on principle, it is quite wrong to try founding a theory on observable magnitudes alone. In reality the very opposite happens. It is the theory which decides what we can observe. You must appreciate that observation is a very complicated process. The phenomenon under observation produces certain events in our measuring apparatus. As a result, further processes take place in the apparatus, which eventually and by complicated paths produce sense impressions and help us to fix the effects in our consciousness. Along this whole path – from the phenomenon to its fixation in our consciousness – we must be able to tell how nature functions, must know the natural laws at least in practical terms, before we can claim to have observed anything at all. Only theory, that is, knowledge of natural laws, enables us to deduce the underlying phenomena from our sense impressions. When we claim that we can observe something new, we ought really to be saying that, although we are about to formulate new natural laws that do not agree with the old ones, we nevertheless assume that the existing ones – covering the whole path from

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[21] Valentini (2008) De Broglie-Bohm Pilot-Wave Theory: Many Worlds in Denial?, citing Heisenberg, W. (1971) Physics and Beyond, New York: Harper & Row, pp. 62-69.

[22] Heisenberg (1971) p. 63. The passage is Heisenberg's recollection more than 40 years after the event of what Einstein had said to him. Heisenberg wrote "…..Needless to say, conversations cannot be reconstructed literally after several decades" [Heisenberg (1971) p. v].

> the phenomenon to our consciousness – function in such a way that we can rely upon them and hence speak of 'observations'.

But once the new theory has been formulated, we must use the new theory to design and interpret measurements. Failure to do that is likely to cause difficulties and inconsistencies. Observations of quantum systems should not be interpreted in terms of classical physics. As Einstein said[23]

> I have a strong suspicion that, precisely because of the problems we have just been discussing, your theory will one day get you into hot water. .....When it comes to observation, you behave as if everything can be left as it was, that is, as if you could use the old descriptive language.

The Schrödinger cat paradox is an example of what Einstein had in mind by his use of the expression "hot water".

16. The two-slit experiment

Phillipidis, Dewdney and Hiley[24] calculated the explicit form of the Bohm quantum potential in the case of the two-slit interference experiment for electrons. They calculated the trajectories of the electrons from the slits to the screen for various initial positions of the electrons in the slits. It was assumed that the slits were Gaussian [i.e. that the slits had 'soft' edges that generated pilot waves having identical Gaussian profiles across the slit].[25] The trajectories are shown in fig. 16.1. The angular distribution of the trajectories as they

[23] Ib., p. 66.
[24] (1979) *Nuovo Cimento* **52B**, 15.
[25] Holland, (1993) The Quantum Theory of Motion, p.177.

leave the slits appears to be uniform, and Holland[26] describes the distribution of initial positions at each slit as uniform. In a subsequent paper Phillipidis, Bohm and Kaye[27] showed the trajectories for a Gaussian distribution of the particles in the slits. The result is illustrated in fig.16.2. In figs. 16.1 and 16.2, the scale of the x-axis (the axis of symmetry) is smaller than the scale of the y-axis (containing the slits) by a factor of about 10□.

It is apparent from figs. 16.1 and 16.2 that the calculated trajectories of the electrons are bunched, showing how the fringes are produced on the screen. The trajectories between the bunches show the effect of minima in the Bohm quantum potential. The pattern of fringes does not contradict the statistical predictions of standard quantum mechanics[28]. But the most obvious prediction is that the electrons do not cross the central axis of the apparatus. Thus according to the theory it is obvious through which slit each electron has passed.

We suggest that an experiment be carried out accurately counting the electron distribution on the screen for each of a series of different positions of the screen along the axis of the apparatus. For each run, there should be sufficient electrons to produce clear fringes, to eliminate the effect of random fluctuations in the paths of the electrons. It would then be possible to compare the positions and

[26] Ib., p.181.
[27] (1982) *Nuovo Cimento* **71B**, 75.
[28] Holland p.185

densities of the fringes with the predictions of the theory. It would thus be possible to disprove the theory. If it were not disproved, then the results would constitute persuasive inductive evidence in support of the theory. That is so notwithstanding that both the Copenhagen interpretation and pilot wave theory make the same predictions as to the intensity $|\psi|^2$ at each point. For the Copenhagen theory denies the existence of trajectories or that one can deduce the existence or location of trajectories from the fringes. Given that both theories have non-zero a priori probabilities of being correct, a theory that predicts the trajectories actually deduced from experimental observations thereby acquires support. Whereas pilot wave theory could be disproved if the experimental observations did not support the theory for a credible distribution of electrons at the slits, the Copenhagen interpretation could not thereby be disproved.

Further, the Bohm quantum potential repels particles from the central (x-) axis of the apparatus.[29] This is represented by the narrow central gaps in figures 16.1 and16.2. Thus it must be the case that along the central axis

□□□² =0.

Given that the predictions of the pilot wave theory concerning individual behaviour do not contradict the statistical predictions of quantum mechanics, it follows that according to the latter theory, too,

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[29] Holland p.185

the particles cannot cross the x-axis. □□□^{2} = 0 in both cases. Thus the slit through which a given electron, recorded at the screen, has passed is known on both theories.

Moreover, if the patterns of fringes predicted by the two theories are the same at all possible positions of the target screen, the trajectories predicted (implicitly or otherwise) by the two theories must be the same, assuming continuous smooth trajectories. It is unreasonable to assert that the electrons have no, or indeterminate, trajectories, or that it is not the case that any electron goes through one slit or the other.

Whilst it is true that the disturbance involved in identifying the precise trajectory of a particle would wash out the interference pattern[30], one can in principle confirm through which slit the particle passed by direct observation at the slit without any appreciable disturbance of the orbit.[31] For example, one could use the fact that the moving electron produces an electromagnetic field. A detector sensitive enough to detect not only the passage of a single electron but its distance from the detector accurately enough to distinguish the slits may not exist, but the possibility should be investigated.

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[30] Holland, p.186; §8.8.2. p.374.
[31] Holland p.184.

17. Conclusion

The following argument[32] sums up the position in standard quantum theory:

Proposition A is that each electron *either* goes through hole1 *or* it goes through hole 2. This idea is checked by experiment. Block off hole 2 and make a measurement of those electrons that come through hole 1. Similarly, make a measurement of those electrons that come through hole 2 when hole 1 is blocked off. It is found that the number that arrives at a particular point is *not* equal to the number that arrives through hole1 plus the number that arrives through hole 2. Thus proposition A is false.

The above argument fails to take account of the change in the wavefunction that takes place when one hole is blocked off. Proposition A is correct according to pilot wave theory. The logic of the (thought-) experiment referred to is flawed. One should measure the number of electrons arriving at a particular point that go through hole 1 when neither hole is blocked and the number arriving at that point that go through hole 2 when neither hole is blocked. The sum is indeed equal to the total number arriving at that point (though for any particular point of arrival one of the two numbers is zero).

[32] Feynman, Leighton and Sands, The Feynman Lectures on Physics, Vol. 1 (1963) pp. 37-6, 37-7, Addison-Wesley, Reading, Massachusetts.

Dewdney and Hiley[33] conclude with a passage which expresses our view:

> "One of the less helpful aspects of the Copenhagen school was their insistence on the uniqueness of the usual interpretation which leads to the supposition that what happens to the individual [system?] between measurements is somehow inherently indescribable and is not subject to detailed analysis. The ambiguous nature of the individual process has given rise to many speculations as to its origins, some of which ultimately resort to the introduction of human consciousness in some way…."

The pilot wave theory answers the above objections. Dewdney and Hiley continue:

> "Take for example the Schrödinger cat paradox. In terms of the quantum potential approach it is the position of the triggering particle within the incident packet that effectively determines its future, and whatever the final outcome of the process, it will leave the cat in some definite state. In other words the cat will be dead or alive regardless of our knowledge of its final state".

18. Simultaneity

Aspect's experiment[34] showed that quantum phenomena could be non-local, in the sense that one quantum measurement could entail another quantum measurement made with a spacelike separation from the former. It is natural, though not necessary, to posit that the two phenomena so measured occur simultaneously in the laboratory frame, and absolutely simultaneously. That implies a preferred slicing, or foliation, of spacetime, contrary to the special theory of relativity. This represents a field for possible fruitful research.

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[33] (1982) pp.46, 47.
[34] Aspect (1982)

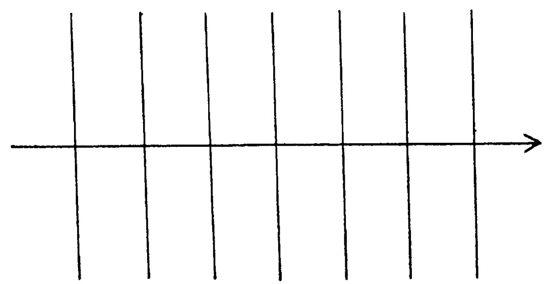

Fig. 2.1. Plane wave. Uniform motion in a straight line, as for a classical particle.

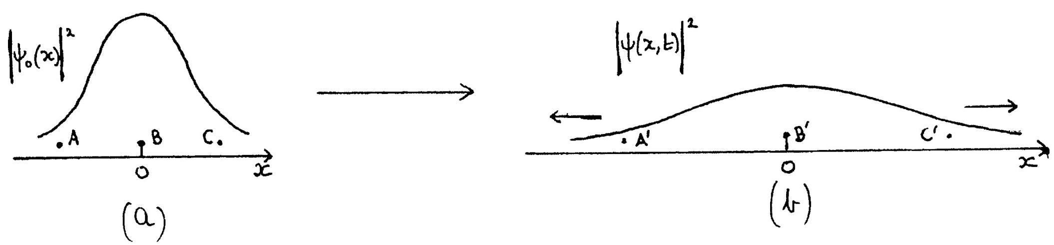

Fig. 6.1. Particle moves to the right if the initial value of x is greater than zero; to the left if it is less than zero.

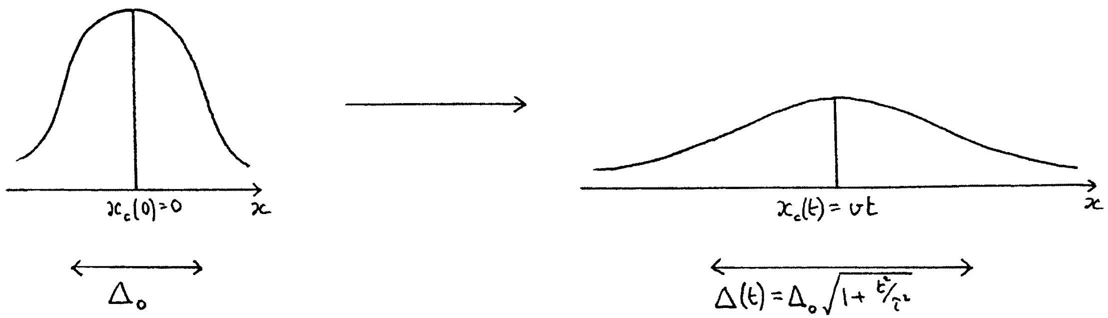

Fig. 7.1. Free Gaussian (moving).

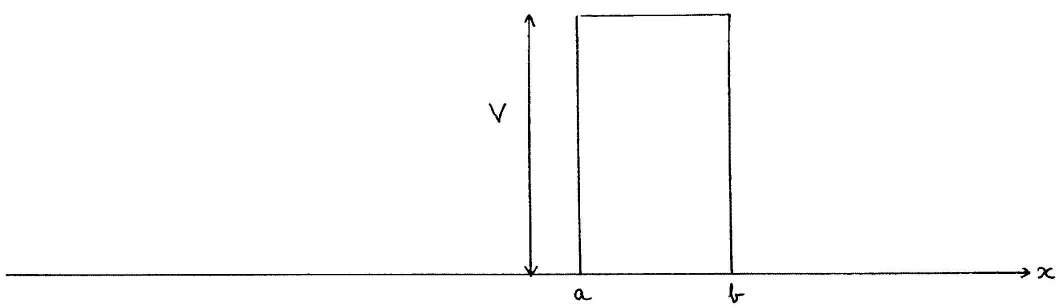

Fig. 10.1. Potential barrier of height V.

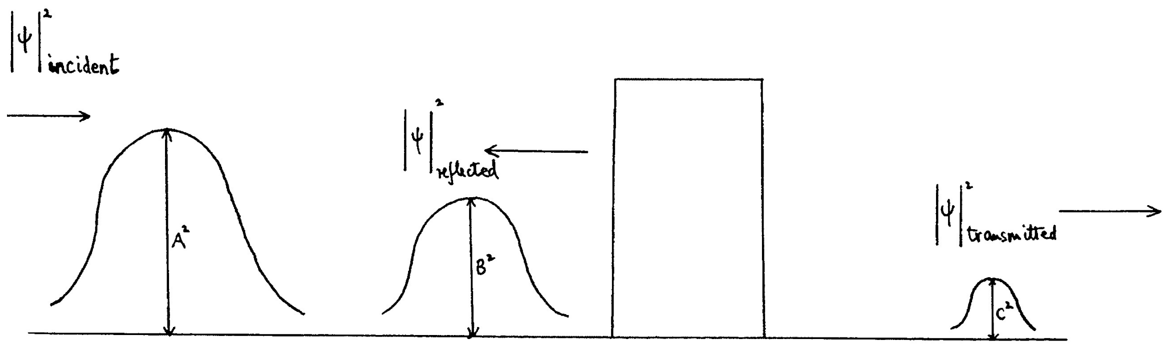

Fig. 10.2. Reflection and transmission by potential barrier.

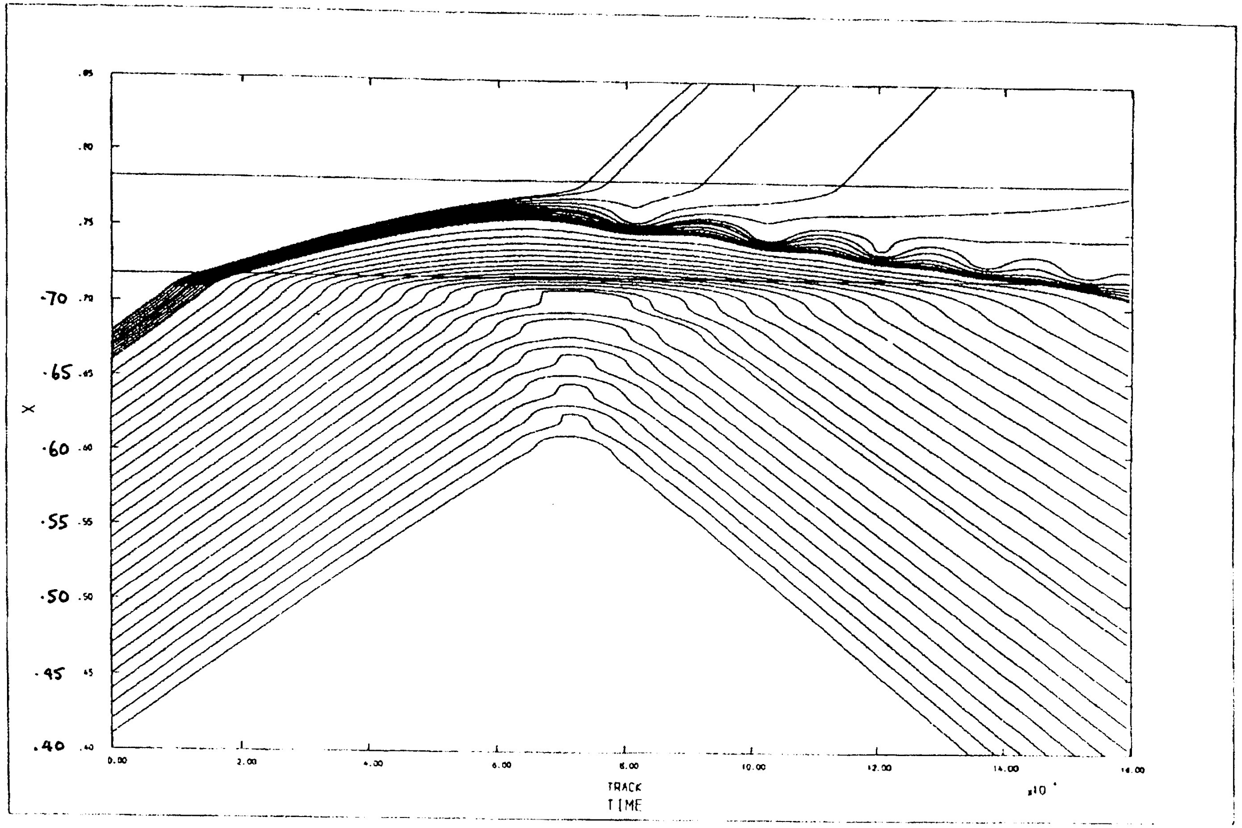

Fig. 11.1. Individual trajectories in case (1): Incident energy half height of barrier.

825/12/1-3

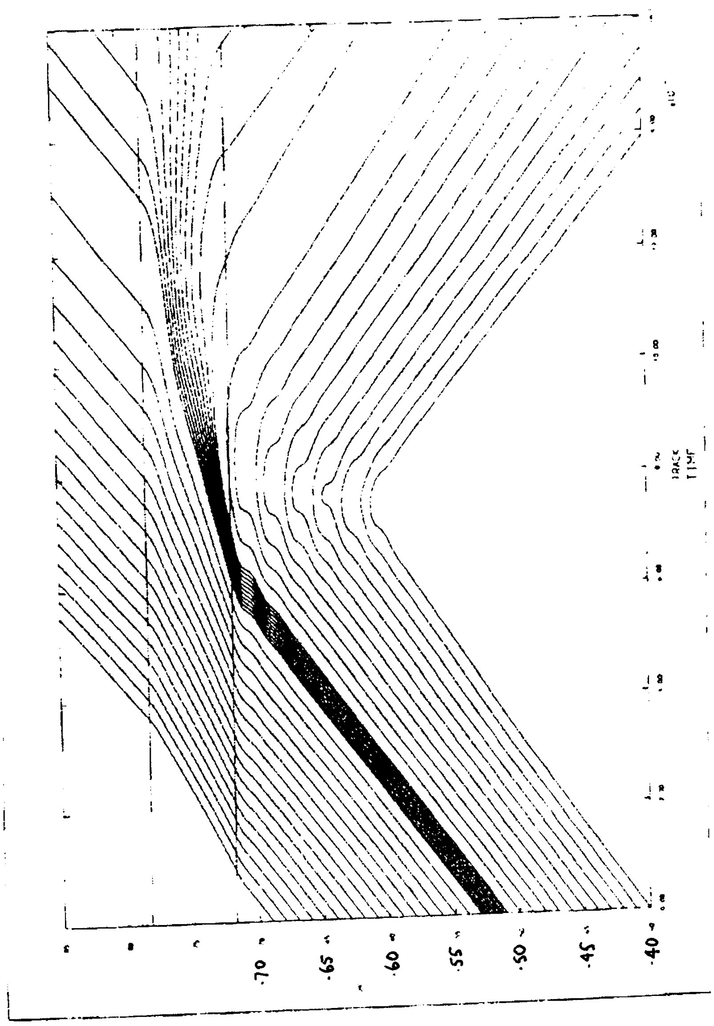

Fig. 11.2. Individual trajectories in case (2): Incident energy equal to height of barrier.

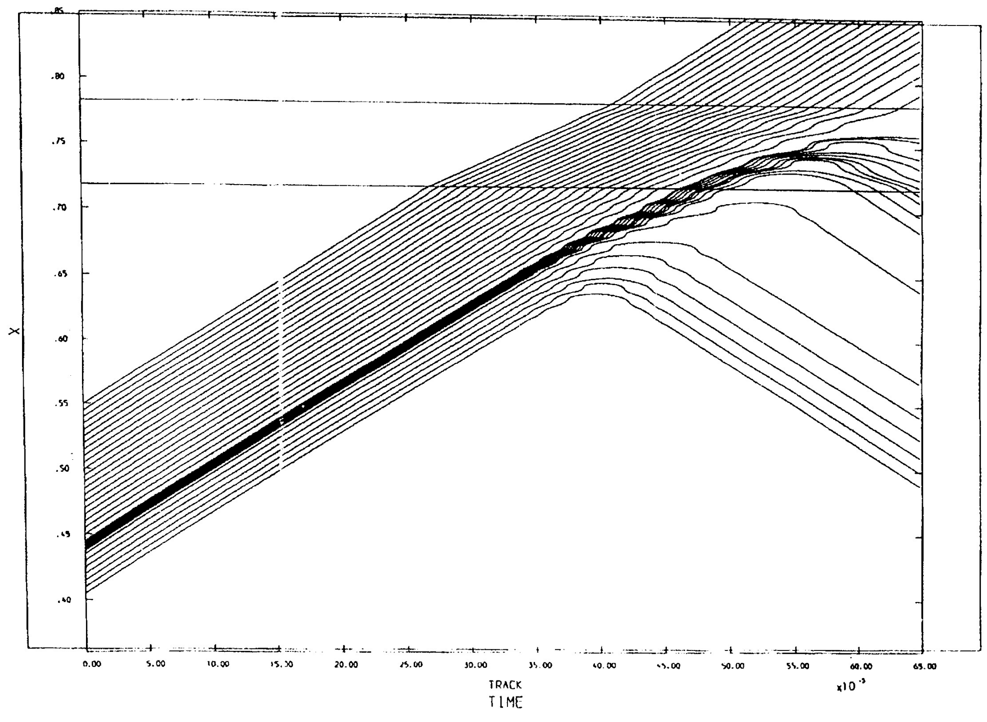

Fig. 11.3: Individual trajectories in case (3): Incident energy twice height of barrier

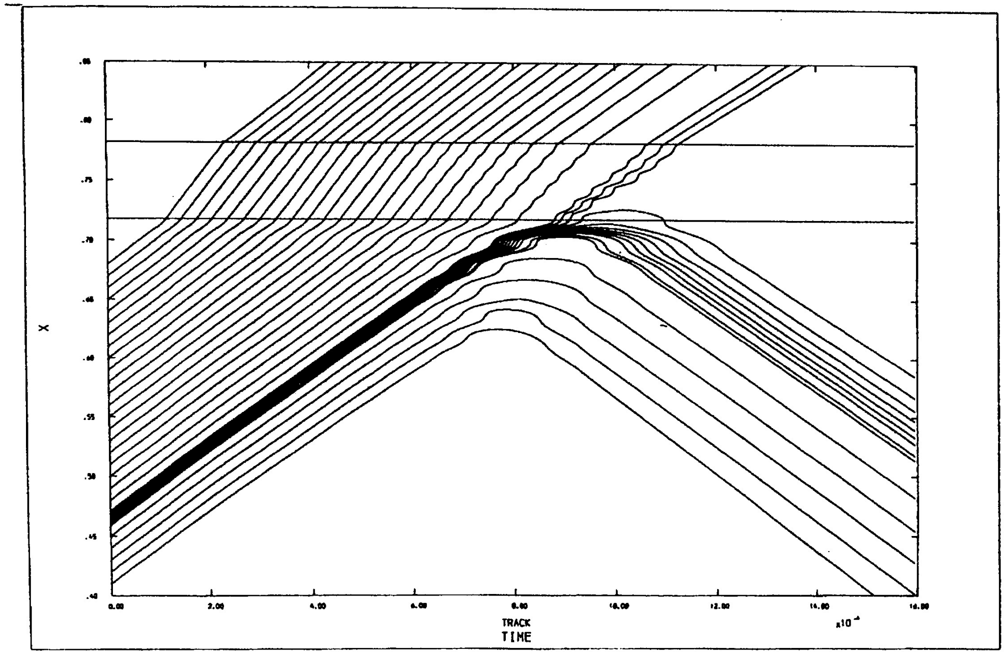

Fig. 11.4. Individual trajectories in case (4): Incident energy half depth of well.

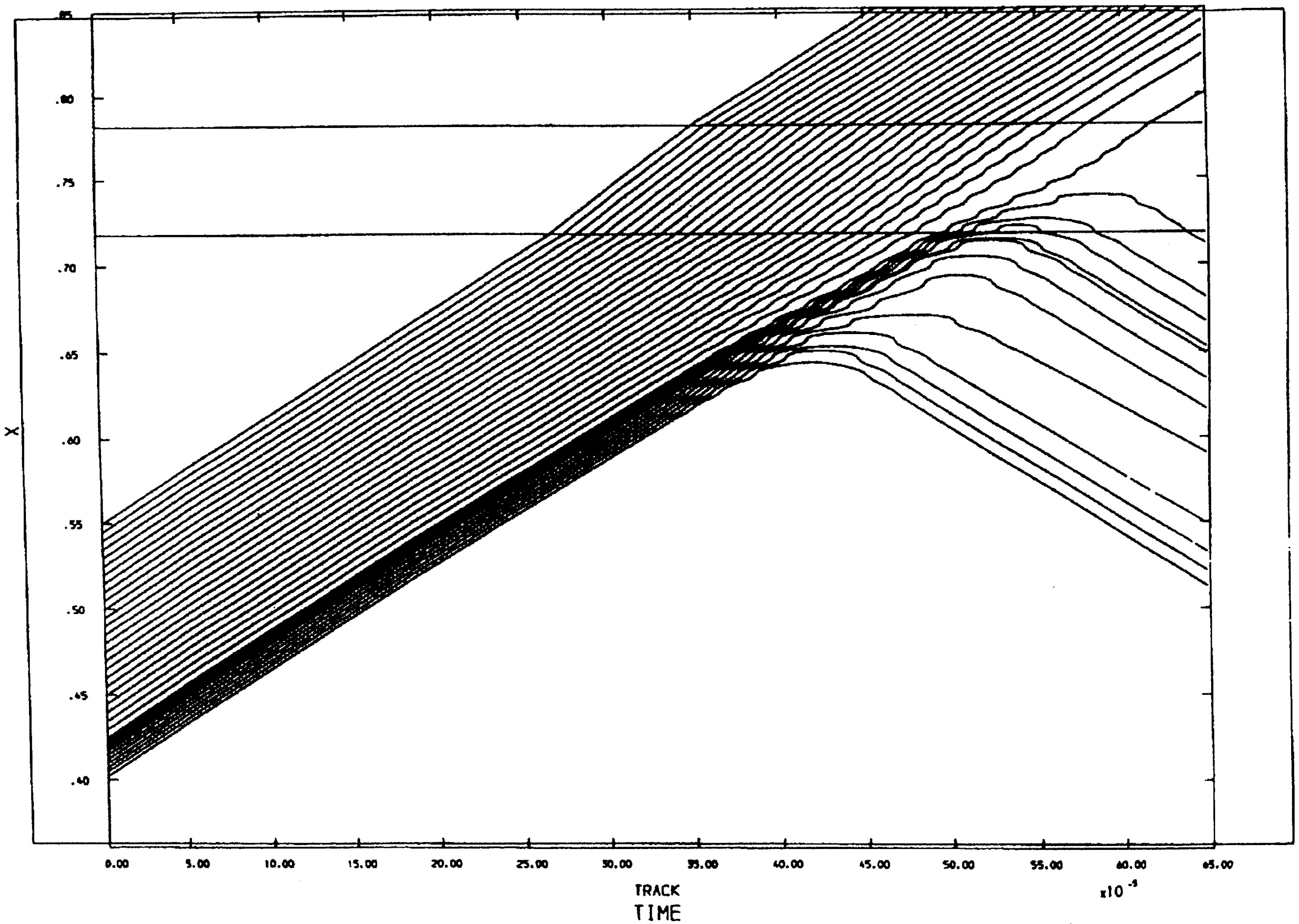

Fig. 11.5: Individual trajectories in case (5): Incident energy twice depth of well.

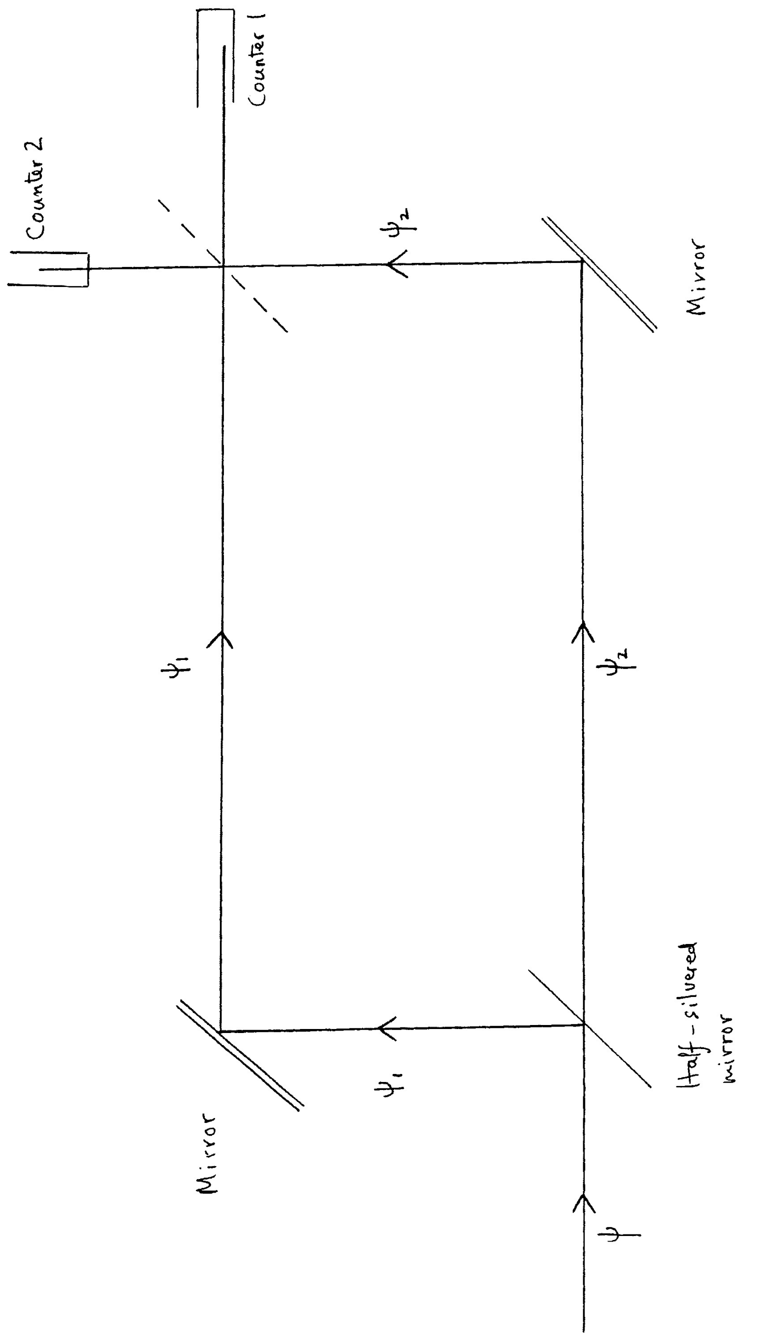
Counter 1
Counter 2
ψ_2
Mirror
ψ_1
ψ_2
Half-silvered mirror
ψ_1
Mirror
ψ

Fig. 12.1.

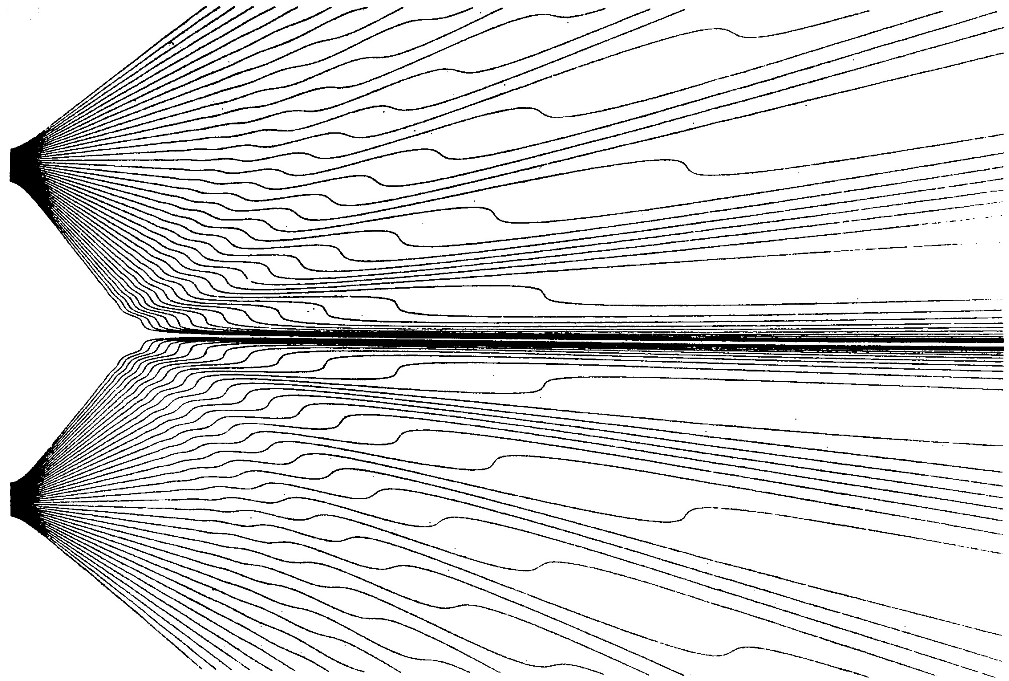

Fig. 16.1. Double slit: calculation of particle trajectories with apparently uniform distribution of particles in the slits.

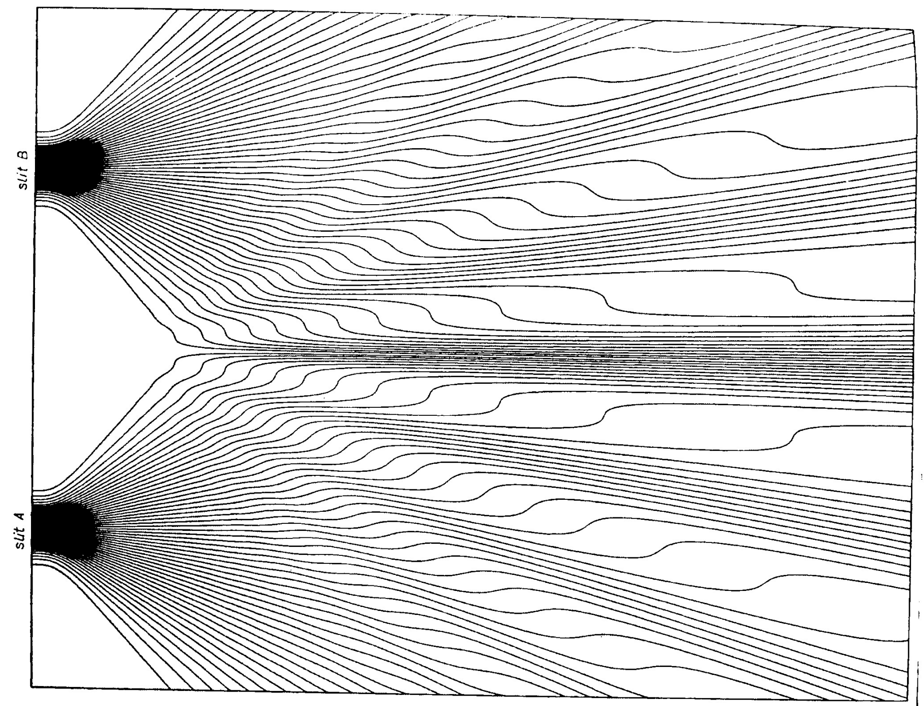

Fig. 16.2. Double slit calculation: Gaussian distribution of particles in the slits.

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