**A REVIEW OF**

# UNPARTICLE PHYSICS


Author: MATTHEW KENZIE

matthew.kenzie09@imperial.ac.uk

Supervisor: KELLY STELLE

k.stelle@imperial.ac.uk


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*Department of Theoretical Physics*

*Blackett Laboratory*

*Imperial College London*

*Exhibition Road*

*London*

*SW13 7AZ*


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# UNPARTICLE PHYSICS

MATTHEW KENZIE

Abstract.

This document is a literature review of theoretical physics papers on *Unparticle Physics*. It is aimed at Masters level students of Theoretical Physics and Applied Maths. The first half of the document concentrates on a brief introduction to Conformal Field Theories as an aid for the latter half which focuses on coupling a conformal sector to the Standard Model. Howard Georgi proposed under such coupling that conformal operators emerging at low energies (in the infrared) will produce a phase space of a fractional number of massless particles, ergo the name unparticles. We go on to see what new physics emerges in the effective theory below the scale, $\Lambda$, where conformal invariance becomes apparent. This leads onto a discussion of unparticle propagators, unparticle interactions and some qualitive deliberation of how we can distinguish and observe unparticles and how likely it is that experimental evidence will be acquired for such features in the near future. An example of how missing energy might arise in standard model processes with a low energy conformal interaction is then demonstrated. In the final section several extensions in the field are explored. Introducing a colour charge to the unparticle operators and the effect this has on gauge boson-unparticle interactions is included. A mass gap is established via a conformal breaking Higgs mechanism and the implications of both a standard model Higgs and an approximately conformal Higgs (the Unhiggs) are examined. This mass gap compels us to consider the unparticle theory as a hidden valley scenario and the phenomenological implications of this are studied. Finally, a few comment are made on other approaches (AdS/CFT), adjustments and problems regarding unparticle physics.

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# 1. Introduction

## 1.1. **The Birth of Unparticle Physics.**

This dissertation provides a Master's level outline/summary to the theoretical aspects of *Unparticle Physics*. Initially proposed by Georgi in [1] we weakly couple a conformally invariant sector to the Standard Model at low energies. The conformal fields of this theory describe unparticles which, although unphysical in substance, may have low energy coupling to the standard model which cannot elude detection via missing energy in various quark decays and other elementary processes.

*Unparticle Physics* itself is a theoretical curiosity. It doesn't claim to fulfil any deep unresolved problems on the theoretical circuit[(i)] in comparison to many other physical theories which are born out of a desire to describe some unexplained phenomena. It is simply an investigation into *what we would see if there was a conformally invariant sector in the Standard Model which becomes apparent below a particular energy scale?* There is no real need for it in the sense that it fills a gap in our understanding. Having said that, since the release of Georgi's initial publications on *Unparticles*, [1] and [2], a plethora of papers have been published on the subject. The field is especially well populated by phenomenologists, examining and explaining how we might find unparticle stuff at the LHC, or super-LHC[(ii)], as missing energy/momentum in particle decays or via its own decay into standard model fields. Unparticle fields are unphysical - a scale invariant field cannot have a mass, it has no notion of a de Broglie wavelength. The phase space of unparticles describes a *fractional* number of massless particles. This *unparticle stuff* is weird sounding let alone weird looking, it isn't something we could observe directly, only infer from peculiar interactions.

## 1.2. **Intended audience.**

To introduce a scale invariant sector to the standard model the reader clearly needs some knowledge of how to manage and manipulate conformal fields. The principles of effective theory will also be highly useful if not completely necessary. This dissertation is aimed

(i) Although, the work discussed in Sec. 5.3 of this document could actually provide a solution to the little hierachy problem.
(ii) The LHC upgrade due around 2018.

at Master's level students (or above) in Theoretical Physics or Applied Mathematics. As such, a working knowledge of quantum field theory - commutation relation principles, symmetry, generators, conserved charges, Hilbert spaces, renormalisation schemes (especially Wilson and MS-bar) etc. - is a prerequisite and assumed by the author. Those readers who are more advanced may consider skipping the early sections which concentrate on much of the theoretical background in conformal field theory. Sec. 2 gives only the bare minimum of requirements in CFT, understanding of the principles is favoured over a deep mathematical description, and the reader should refer to Sec. 1.4 for some books and papers with which to enhance this explanation, if he/she feels unconfident.

It is at this point the author feels obliged to apologise for the disjointed nature which one may encounter in the proceeding pages. By the very nature of writing to interest students at the Masters level a fair bit of theory (mainly concerning conformally invariant fields) needs to be covered first. This is done with a relatively traditional methodology, following a similar path to much of the popular literature. As such one may perceive, and rightly so, a considerable jump between the intellectual levels of Sec. 2 and Sec. 4.

### 1.3. **Layout.**

The opening part of this dissertation (the one in which the reader is hopefully now fully engrossed) is an introduction to this fairly niche field, containing motivations for the author and expectations of the audience as well as a short word on some of the literature. Sec. 2 provides much of the theoretical background required which isn't already assumed - most of this is conformal field theory with some other aspects - and exists to provide less informed readers with the important underlying theory. The conformal group is discussed, its generators, operators and algebra, quantisation, correlation functions, the operator formalism and the significance of two dimensional conformal theories. Although much of this is not directly applicable to the later sections, it aims to provide some insight into the properties of CFTs. Those who are familiar with conformal invariant theories would be advised to skip this and continue reading from Sec. 4.

A few pages explaining the bare essentials and need for effective field theories is provided in Sec. 3. Once again, the more informed of readers will be able to skip this and move straight to Sec. 4.

In the latter section, the scheme is introduced. Unparticle physics is allowed to emerge by coupling Banks-Zaks fields to the standard model, which can flow to a conformal sector in the infrared. We discuss the ingredients required for a sensible unparticle theory, what new physics is apparent, the phase space of unparticles and also derive the form of unparticle propagators. We demonstrate that unparticle scattering problems can be related to those of a normal QFT using the conformal partial wave expansion and furnish this with a discussion of how unparticles can be observed - we consider the specific case of a top quark to an up quark decay with unparticle emission and the missing energy produced in this process.

In Sec. 5 we look at some extensions and different approaches to the unparticle domain. First, a colour charge is included, culminating in the realisation that unparticles can couple to an arbitrary number of gauge bosons and producing the result that the $q\bar{q} \rightarrow q\bar{q}$ cross section is supressed by a factor of $(d{-}2)$ when considering such interactions. A conformal breaking Higgs mecahnism is shown to provide the unparticle theory with a mass gap, giving the Higgs as well as the unparticle operator tadpole a vacuum expectation value. It is shown that making the Higgs itself an unparticle will still break the electro-weak symmetry and unitarize the $WW$ scattering as for the SM Higgs. The link between unparticle physics and hidden valley scenarios is briefly discussed at the end, whereupon some striking new phenomenology is proposed. Finally, a few problems and directions for future work are deliberated.

This piece does not cover any new ground. It is an attempt to explain the important aspects of an emerging field to a novice audience. Sec. 4 is a review of the literature that focuses on the theoretical aspects of unparticle physics, a collection of about 10-20 papers. Much of the work done on the phenomenology of unparticle stuff, of which there is a vast amount[(iii)], is not discussed here. To avoid 'cherry picking', literature which has

[(iii)]A search of 'unparticle' on `arXiv.org` returns 204 results (within the Title) and 234 (within the Abstract), on `sciencedirect.com` returns 60 and `webofknowledge.com` returns 142.

a theoretical, as opposed to a phenomenological, motivation has been reviewed, although some of the most interesting phenomenological implications of an unparticle theory are examined.

The final section concludes this paper, outlines the most important information and discusses future directions for the theory and potential for detection in the future. Acknowledgements and references are located at the end, followed by a few Appendices. Many of these will not interest the average reader - they contain several calculations in more detail and provide some of the elementary laws and caveats, the majority of which the audience will be happy to accept without proof.

### 1.4. **Literature.**

It is worth commenting on some of the important literature in the subjects of Conformal Field Theory and unparticle physics. As mentioned previously, the author is assuming a solid working knowledge of Quantum Field Theory, any readers in doubt should refer to these introductory texts; [3], [4], [5] and for a more advanced option, [6].

Conformal Field Theory is now a cemented mathematical concept and there are several textbooks on the subject. It is usually taught from a very mathematical stand point or in the context of string theory - neither of which are particularly useful for application in unparticle physics. As for unparticle physics itself, there are no actual textbooks on the subject (because the theory is still in its infancy) but hundreds of papers have made use of the proposals in Georgi's first few papers on the topic, [1] and [2].

#### 1.4.1. *Conformal Field Theory.*

Chapters on conformal invariance can be found in almost any introductory string theory book or lecture course. The disadvantage of these is that many of the details which are not relevant in a string context are left out. As such, the author would suggest that the books and papers written on conformal field theory for and by mathematicians may be more useful. Many of the influential papers on the topic were written in the early 60's, such as [7–10]. These lay out the mathematical properties of the conformal group; their transformations, generators, Hilbert spaces etc., although work on wave equations in conformal space had been around decades before, see for example [11]. The author would recommend Mack and Salaam's paper [10] and also Ferrara et. al.'s short text [12] as excellent introductions to the appropriate tools for Conformal Invariant Fields. In the late 80s, Paul Ginsparg gave a series of lectures on Applied Conformal Field Theory [13], designed as an 'elementary introduction to conformal field theory'. The first three or so sections are helpful for us but the applications, discussed later in this series, are relevant to statistical mechanical systems and string theory, so not particularly useful for a student of unparticle physics. Interesting developments made in CFT that are made use of in unparticle publications include [14] and [15]. Francesco, Mathieu and Sénéchal's book *Conformal Field Theory*, [16], affectionately referred to by David Tong as 'The Yellow Pages' in [17], is an excellent book for any novice in the field. It is long yet incredibly detailed - it starts from the very basics and works through in a logical and precise manner. This is an ideal starting point for any readers who do not feel especially comfortable with conformal transformations and the conformal algebra in either a classical or quantum space-time.

1.4.2. *Effective Field Theory.*

Although most advanced field theory books mention the principles of Effective Theories, many do not divulge into it in particular detail. There are some excellent lecture notes on the topic of effective field theories by both Pich [18] and Kaplan [19]. Georgi has produced a review paper on the subject [20] and this covers all the bases of EFTs.

1.4.3. *Unparticle Physics.*

Unparticle Stuff was first proposed by Georgi in [1]. It was shortly followed by [2] and in the following three years some specific examples were released by Georgi and Kats in [21] and [22]. In the three years since Georgi's initial proposal a huge amount of material has been produced on unparticle phenomenology. We will look at some of the comments made by Grinstein et. al. in [23] and Deshpande et. al. in [24], and also discuss the matter of the unparticle spectrum - with examples provided by [25–27]. Our focus will then concentrate on some particularly interesting theoretical extensions - especially that of introducing a colour charge [28–35], Higgs/unparticle interplay [36–42] and the hidden valley link [27, 43].

1.5. **Notation.**

The notation adopted in this document attempts to follow that of the literature mentioned above. However, the reader may find that this changes slightly throughout although all equations will come with an explanation of the notation if it is different from that used in previous expressions. Note that sometimes we use the phrase *conformal transformation* and sometimes *scale transformation*, generally a scale transformation is a type of conformal transformation but when we are discussing this in an unparticle context i.e. from Sec. 4 onwards, we really mean that they are the same thing. The terms *unparticle* and *unparticle stuff* are also used interchangeably. In truth *unparticle stuff* is probably a better expression because it is unclear what *unparticles* actually are. In Feynman diagrams we will depict the propagation of an unparticle or unparticle stuff by a thick dashed line ( ▬ ▬ ▬ ▬ ▬ ). The document contains the occasional *aside*, these are encompassed within two horizontal lines and contain important information that doesn't really flow with the text. In general the following table may be a useful guideline.

Table 1. **Key to notation used**

| | | |
|---|---|---|
| $\phi$ | - | primary conformal field (not necessarily a scalar field) |
| $x^\mu$ | - | space-time coordinate |
| $g_{\mu\nu}(\eta_{\mu\nu})$ | - | dynamical background (Minkowski/Euclidean) metric |
| $z(\bar{z})$ | - | holomorphic (antiholomorphic) coordinates |
| $P_\mu$ | - | generator of translations |
| $L_{\mu\nu}(S_{\mu\nu})$ | - | generator (infinitesimal) of angular momentum |
| $K_\mu(\kappa_\mu)$ | - | generator (infinitesimal) of special conformal transformations (SCTs) |
| $D(\Delta)$ | - | generator (infinitesimal) of dilatations/scale transformations |
| $\mathcal{O}$ | - | a general operator (usually it will be conformal, with a subscript $U$ denoting an unparticle operator) |
| $\langle X \rangle$ | - | correlation function of $n$ fields $= \langle \phi_1(x_1) \ldots \phi_n(x_n) \rangle$ |
| $M$ | - | mass of unparticle-SM sector mediator |
| $\Lambda$ | - | scale at which conformal invariance emerges in hidden (unparticle) sector |
| $T$ | - | time ordering |
| $D$ | - | (when not being used as a generator) space-time dimensions |
| $d_\phi$ | - | scaling dimension of field $\phi$ (sometimes $\Delta$ is used instead) |
| $d_U$ | - | scaling dimension of unparticle operator $\mathcal{O}_U$ |
| $\theta(x)$ | - | heavyside step function of variable $x$ |
| $S$ | - | action |
| $\mathcal{L}$ | - | Lagrangian |
| $Y$ | - | hidden sector (i.e. unparticle) gauge boson |
| $H$ | - | higgs or unhiggs field |
| $\Rightarrow$ | - | therefore/hence |
|  | - | unparticle propagator in Feynman diagrams |
|  | - | standard model fermion propagator in Feynman diagrams |
|  | - | gauge boson propagator in Feynman diagrams (can be standard model or hidden sector) |
|  | - | standard model scalar propagator in Feynman diagrams |

# 2. Conformal Field Theory

The following section provides an introduction to conformal field theory. It follows the same structure as in much of the popular literature. The main sources for the author are [10], [13] and [16]. This section is aimed at students with no previous knowledge of conformal invariance. The aim is to introduce some of the basic principles and build a general understanding, not develop a specific knowledge. Much of the technical detail in this section will not directly apply to unparticle discussions.

We start by discussing the conformal group, its generators and its representations. We will then go on to develop what the fields look like and how we can expand them in a useful way. We will try to develop some of the important machinery that can be transferred into the unparticle field. Many of the techniques used will be familiar to those who have studied quantum field theory.

The mathematical details presented in this section hope to provide the reader with some insight into the workings of conformal field theories, especially the details of correlation functions of conformal operators which are vital in analysing unparticle processes.

CFT in 2D is given special interest. This is because local conformal invariance gives exact solutions in two dimensions. The benefits and results of this are discussed in Sec. 2.7. When Georgi first discusses unparticles he imagines them in the usual four dimensional space-time, with little consideration for exact solutions but concentration on the heuristics of such a theory. When we go on to discuss unparticle propagators and interactions in Secs. 4.4 and 4.5 we will extend this to $D$ space-time dimensions. However, to compute specific unparticle processes in a way that is consistent in both high and low-energy regimes, which has only been done very recently in an impressive collection of work by Georgi and Kats (presented in [22]), one requires restricting their attention to a 2D 'toy model' of unparticles coupled to a 'toy' Standard Model. The question of how transferable our calculations in 2D are to 4D is an important one that is usually ignored, we will ponder this more closely in Sec. 5.

## 2.1. **The Conformal Group.**

Under a coordinate transformation, $x \to x'$, the metric transforms as,

$$g_{\mu\nu}(x) \to g'_{\mu\nu}(x') = \frac{\partial x^\alpha}{\partial x'^\mu}\frac{\partial x^\beta}{\partial x'^\nu} g_{\alpha\beta}(x). \tag{2.1}$$

A conformal field theory is defined as any theory which is invariant under the change of metric,

$$g_{\mu\nu}(x) \to g'_{\mu\nu}(x') = \Omega(x) g_{\mu\nu}(x). \tag{2.2}$$

That is to say, the conformal group is the subgroup of (2.1) which obeys the condition (2.2). This is a physical theory which 'looks the same at all length scales' [16]. i.e. one in which all angles are preserved. It is often much simpler to work in two dimensions in CFTs, and this is regularly the case in the literature. A nice way of doing this is to introduce the complex Euclidean worldsheet coordinates, $z = \sigma^1 + i\sigma^2$ and $\bar{z} = \sigma^1 - i\sigma^2$. Using these coordinates a conformal transformation looks like,

$$z \to z' = f(z),\ \bar{z} \to \bar{z}' = \bar{f}(\bar{z}) \ \text{ thus } \ ds^2 = dz d\bar{z} \to \left|\frac{df}{dz}\right|^2 dz d\bar{z}.^{\text{(iv)}} \tag{2.3}$$

It is immediately obvious that the Poincaré group is a subgroup of the conformal group, as it leaves the metric invariant, $g_{\mu\nu} \to g'_{\mu\nu}$. It is the subgroup with $\Omega = 1$ in (2.2). So clearly rotations and translations will leave the conformal group invariant (as they leave the Poincaré group invariant). So what other transformations do the same?

---

**ASIDE.** As a short aside lets briefly try to understand why the case in which $d = 2$ is so unique. Lets consider the effect the transformation, $x^\mu \to x^\mu + \epsilon^\mu(x)$, has on the metric to first order,

$$g_{\mu\nu} \to g'_{\mu\nu} = g_{\mu\nu} - \partial_\mu \epsilon_\nu - \partial_\nu \epsilon_\mu = \Omega g_{\mu\nu}, \tag{2.4}$$

and thus,

$$\partial_\mu \epsilon_\nu + \partial_\nu \epsilon_\mu = f(x) g_{\mu\nu} \ \text{ and by taking the trace } \ \frac{2}{d}\partial_\rho \epsilon^\rho = f(x), \tag{2.5}$$

where $d$ is the number of dimensions we are working in.

(iv) This can just as easily be done in Minkowski space but Euclidean space is chosen for simplification

As we are considering only infinitesimal transformations we will assume that the metric is simply a slight deformation of the usual metric, $\eta_{\mu\nu}$[(v)]. By taking the derivative of the left hand equation we can write,

$$\partial_\rho f(x)\eta_{\mu\nu} = \partial_\rho\partial_\mu\epsilon_\nu + \partial_\rho\partial_\nu\epsilon_\mu \;\Rightarrow\; 2\partial_\mu\partial_\nu\epsilon_\rho = \eta_{\nu\rho}\partial_\mu f + \eta_{\mu\rho}\partial_\nu f - \eta_{\mu\nu}\partial_\rho f. \tag{2.6}$$

If we then contract this with $\eta^{\mu\nu}$ we find,

$$2\partial^2\epsilon_\mu = (2-d)\partial_\mu f(x) \;\Rightarrow\; (2-d)\partial_\rho\partial_\mu f(x) = 2\partial_\rho\partial^2\epsilon_\mu. \tag{2.7}$$

Comparing this to the double derivative of the first equation in (2.5) we see that,

$$2\partial_\rho\partial^2\epsilon_\mu = \partial^2 f(x)\eta_{\mu\nu} \;\Rightarrow\; (2-d)\partial^2 f(x) = d\,\partial^2 f(x) \;\Rightarrow\; \boxed{(d-1)\partial^2 f(x) = 0}. \tag{2.8}$$

- if $d = 1$ - There are no constraints on f which makes sense as in one dimension there are no angles.
- if $d \geq 3$ - (2.8) implies that $\partial_\mu\partial_\nu f = 0$ and as a result $\partial_\mu\partial_\nu\epsilon_\rho$ will equal a constant. So $\epsilon$ is going to be at most quadratic in $x$. This allows us to construct the conformal transformations below (see Sec. 2.1.1).
- if $d = 2$ - Using (2.5) we can write $\partial_\mu\epsilon_\nu + \partial_\nu\epsilon_\mu = \frac{d}{2}(\partial_\rho\partial^\rho\eta_{\mu\nu})$ which in two dimensions will produce the Cauchy-Riemann equations[(vi)]. Any function satisfying these equations is known as holomorphic and will satisfy the constraints in (2.3). So holomorphic functions, $f(z) = z + \epsilon(z)$, produce infinitesimal conformal transformations in 2D. Crucially, in two dimensions any coordinate transformation will be locally conformally invariant, this local symmetry enables exact solutions of CFTs in 2D. More on these in Sec. 2.7.

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2.1.1. *Conformal transformations and their generators.*

Let's consider an infinitesimal coordinate transformation of the form $x \to x' = x + \epsilon$. In

(v) We are actually using the Cartesian metric here for simplicity but the Minkowski metric can be used just as easily.

(vi) $\partial_0\epsilon_0 = +\partial_1\epsilon_1$ and $\partial_0\epsilon_1 = -\partial_1\epsilon_0$.

order to conserve the condition (2.2) the third derivative (and above) of $\epsilon$ must vanish. Therefore, $\epsilon$ cannot be more than quadratic in $x$. Our coordinate transformation is,

$$x^{\mu} \to x'^{\mu} = x^{\mu} + \epsilon^{\mu}(x) \tag{2.9}$$

and so,

| | | | |
|---|---|---|---|
| To $0^{\text{th}}$ order: | $\epsilon^{\mu} = a^{\mu}$ | Translations | |
| To $1^{\text{st}}$ order: | $\epsilon^{\mu} = \omega^{\mu}{}_{\nu} x^{\nu}$ | Rotations | |
| | $\epsilon^{\mu} = \lambda x^{\nu}$ | Scale transformations (dilatations) | (2.10) |
| To $2^{\text{nd}}$ order: | $\epsilon^{\mu} = b^{\mu} x^2 - 2x^{\mu} b \cdot x$ | The so called special conformal transformations (SCTs) | |

So then the full conformal group with finite transformations consists of four operations, which are explicitly,

| | | | | |
|---|---|---|---|---|
| $x'^{\mu} = x^{\mu} + a^{\mu}$ | Translations | $\Omega = 1$ | $P_{\mu} = i\partial_{\mu}$ | |
| $x'^{\mu} = \omega^{\mu}{}_{\nu} x^{\nu}$ | Rotations | $\Omega = 1$ | $L_{\mu\nu} = i(x_{\mu}\partial_{\nu} - x_{\nu}\partial_{\mu})$ | (2.11) |
| $x'^{\mu} = \lambda x^{\nu}$ | Dilatations | $\Omega = \lambda^{-2}$ | $D = i x^{\mu}\partial_{\mu}$ | |
| $x'^{\mu} = \frac{x^{\mu} - b^{\mu} x^2}{1 - 2b\cdot x + b^2 x^2}$ | SCT[(vii)] | $\Omega = (1 - 2bx + b^2x^2)^2$ | $K_{\mu} = i(2x_{\mu}x^{\nu}\partial_{\nu} - x^2\partial_{\mu})$ | |

where, in the right most column, we have introduced the generators for each operation - $P_{\mu}$, $L_{\mu\nu}$, $D$ and $K_{\mu}$ for translations, rotations, dilatations and SCTs respectively. Using the form of the generators in (2.11) we produce the following commutation relations for the conformal algebra,

[(vii)] An SCT is a translation, preceded and followed by an inverstion

$$
\begin{aligned}
&[D, P_\mu] = -iP_\mu, \\
&[D, K_\mu] = iK_\mu, \\
&[D, L_{\mu\nu}] = 0, \\
&[K_\mu, P_\nu] = -2i(\eta_{\mu\nu} D + L_{\mu\nu}), \\
&[K_\mu, K_\nu] = 0, \\
&[K_\rho, L_{\mu\nu}] = i(\eta_{\rho\mu} K_\nu - \eta_{\rho\nu} K_\mu), \\
&[P_\rho, L_{\mu\nu}] = i(\eta_{\rho\mu} P_\nu - \eta_{\rho\nu} P_\mu), \\
&[L_{\mu\nu}, L_{\rho\sigma}] = i(\eta_{\nu\rho} L_{\mu\sigma} + \eta_{\mu\sigma} L_{\nu\rho} - \eta_{\mu\rho} L_{\nu\sigma} - \eta_{\nu\sigma} L_{\mu\rho}).
\end{aligned} \tag{2.12}
$$

It takes minimal inspection to see that parity requires the properties,

$$
\Pi D \Pi^{-1} = D, \;\; \Pi K_\mu \Pi^{-1} = \pm K_\mu, \;\; \Pi P_\mu \Pi^{-1} = \pm P_\mu
$$
$$
+ \text{ for } \mu = 0 \;\text{ and }\; - \;\text{ for } \mu = 1, 2, 3. \tag{2.13}
$$

If we redefine our generators, in the following way, then this symmetry can be explicitly seen to resemble that of SO($d$+1,1) in $d$ dimensions. First we define,

$$
J_{\mu\nu} = L_{\mu\nu}, \; J_{-1,\mu} = 1/2(P_\mu - K_\mu), \; J_{-1,0} = D, \; J_{0,\mu} = 1/2(P_\mu + K_\mu) \tag{2.14}
$$
$$
\text{for } J_{ab} = -J_{ab} \text{ and } a, b \in \{-1, -, 1, ..., d\}.
$$

Then the generators, $J_{ab}$, obey the familiar SO($d$+1,1) commutation relations,

$$
[J_{ab}, J_{cd}] = i(\eta_{ad} J_{bc} + \eta_{bc} J_{ad} - \eta_{ac} J_{bd} - \eta_{bd} J_{ac}) \tag{2.15}
$$
$$
\text{with metric } \eta = \text{diag}(-1, 1, ....., 1) \quad .
$$

---

ASIDE: Note that Mack and Salaam [10] make the symmetry explicitly O(4,2) by defining, $J_{\mu\nu} = L_{\mu\nu},\ J_{65} = D,\ J_{5\mu} = 1/2(P_\mu - K_\mu)$ and $J_{6\mu} = 1/2(P_\mu + K_\mu)$ which gives the same commutation relation as (2.15) with the metric instead, $\eta = \text{diag}(+---,-+)$. We will adopt this notation from now on.

---

The conformal group has two abelian subgroups, one generated by $P_\mu$ and one generated by $K_\mu$ and so has two Poincaré subalgebras, $(L_{\mu\nu}, P_\mu)$ and $(L_{\mu\nu}, K_\mu)$. We also have the following relation (derived from (2.12) and proved in Appendix A),

$$e^{i\alpha D} P^2 e^{-i\alpha D} = e^{2\alpha} P^2, \tag{2.16}$$

which implies that exact dilatation symmetry will give rise to a continuos mass spectrum or all masses are zero. Hence we see that exact dilatation symmetry is a physical impossibility in a massive theory and therefore we will need to specify how conformal symmetry is broken to describe the dynamics of a conformal invariant system. To look at this in another way, imagine there is a discrete massive state such that $p^2 = m^2$. Using (2.12) we can write,

$$[P^2, D] = P^\mu [P_\mu, D] + [P^\mu, D] P_\mu = 2iP^2 \tag{2.17}$$

and so,

$$\begin{aligned} \langle p|[P^2, D]|p\rangle &= 2i\langle p|P^2|p\rangle = 2im^2 \\ \langle p|[P^2, D]|p\rangle &= \langle p|m^2 D - Dm^2|p\rangle = 0 \end{aligned} \tag{2.18}$$

which implies that $m^2 = 0$ and therefore all discrete massive states will break the conformal symmetry. We are allowed a continuum of massive states but no mass gap. This has particular consequences in unparticle physics because our hidden scale invariant sector (in which unparticles emerge) will have a continuous mass spectrum. Some kind of conformal breaking Higgs mechanism is required to obtain a mass gap in the theory - this is discussed further in Sec. 5.2.

So far we have limited our conformal group to a classical description. The quantum description does not follow naturally from this,

> *"A Quantum Field Theory needs a regularisation description that provides the theory with a scale. This scale breaks conformal invariance except at particular parameter values which give a renormalisation group fixed point (the IR fixed point)"* [16].

2.1.2. *Action of generators on a field.*

Slighty subtly in CFTs, when we mention a field we don't mean it in quite the same sense as we would in a normal QFT. What we really mean is any thing we can write down in terms of the field. So we're not just refering to $\phi$ but also things like $e^{\phi}$ and $\partial_\mu \phi$. As with any field theory, we are seeking a representation, $T(g)$, such that,

$$\phi'(x') = exp(iw_g T_g)\phi(x) \simeq (1 - iw_g T_g)\phi(x) \tag{2.19}$$

for some infinitesimal $g \in O(2,4)$. Firstly, consider the Lorentz group ($\subseteq$ Poincaré group $\subseteq$ Conformal group) which leaves $x = 0$ invariant, this is the stability subgroup which is isomorphic to a Poincaré algebra + dilations. So our general conformal group looks like,

$$\underbrace{(SO(3,1) \otimes \{D\})}_{\substack{\text{spin part of} \\ \text{Lorentz + dilatations}}} \underbrace{\otimes T_4}_{\text{SCTs}}. \tag{2.20}$$

In other words we have,

$$(T(g)\phi)_\alpha(x) = \Sigma_{\alpha\beta}(g,x)\phi_\beta(g^{-1}x), \tag{2.21}$$

where $\Sigma(g,0)$ is the subgroup which leaves $x = 0$ invariant.

If we pick a matrix representation for each generator which leaves $x = 0$ invariant we can then induce the action of the generators (2.11) on fields. Firstly consider infinitesimal generators of the group (2.20). We will call these $S_{\mu\nu}$, $\Delta$ and $\kappa_\mu$ for rotations, dilatations and SCTs respectively. Naturally, they will obey exactly the same commutation relations expressed in (2.12). We can then use (2.22) to translate from the action of the infinitesimal generators on fields at $x = 0$ to the action of conformal generators on fields at any space-time point. If we pick a space-time translation independent basis, $P_\mu\phi(x) = i\partial_\mu\phi(x)$ (the familiar representation for the momentum operator), then the following will hold,

$$X\phi(x) = e^{-ix^\rho P_\rho} X'\phi(0) \tag{2.22}$$

where,

$$\begin{aligned}
X' &= e^{ix^\mu P_\mu} X e^{-ix^\mu P_\mu} \\
&= \sum_n \frac{(ix^\mu P_\mu)^n}{n!} \cdot X \cdot \sum_n \frac{(-ix^\mu P_\mu)^n}{n!} \\
&= \left[1 + ix^\mu P_\mu + \frac{(ix^\mu P_\mu)^2}{2} + \frac{(ix^\mu P_\mu)^3}{3!} + \ldots\right] X \\
&\quad \times \left[1 - ix^\mu P_\mu + \frac{(ix^\mu P_\mu)^2}{2} - \frac{(ix^\mu P_\mu)^3}{3!} + \ldots\right] \\
&= X + i(x^\mu P_\mu X - X x^\mu P_\mu) + \frac{i^2}{2}\Big((x^\mu P_\mu)^2 X + X(x^\mu P_\mu)^2 - x^\mu P_\mu X x^\mu P_\mu\Big) \\
&\quad + \frac{i^3}{3!}\Big((x^\mu P_\mu)^3 X - X(x^\mu P_\mu)^3 + 3(x^\mu P_\mu)X(x^\mu P_\mu)^2 - 3(x^\mu P_\mu)^2 X(x^\mu P_\mu)\Big) + \ldots \\
&= X + ix^\mu[P_\mu, X] + \tfrac{i^2}{2!}x^\mu x^\nu\left[P_\mu, [P_\nu, X]\right] + \tfrac{i^3}{3!}x^\mu x^\nu x^\rho\Big[P_\mu, \left[P_\nu, [P_\rho, X]\right]\Big] + \ldots \\
&= \sum_n \frac{i^n}{n!} x^{\mu_1} \ldots x^{\mu_n}\left[P_{\mu_1}, [\ldots[P_{\mu_n}, X]]\right].
\end{aligned} \tag{2.23}$$

This sum is actually finite[(vi)] but moreover in our specific case it will only include a few terms. Looking back at the commutations relations (2.12) one can see that nested commutators of $P_\mu$ with any of the generators will only produce a handful of non-zero terms. Using the above result (2.23) we can deduce the action of $L_{\mu\nu}$, $D$ and $K_\mu$ on $\phi(x)$ since we know the action of $S_{\mu\nu}$, $\Delta$ and $\kappa_\mu$ on $\phi(0)$ (note that we have already picked the basis in which the action of $P_\mu$ on $\phi$ is such that $P_\mu\phi(x) = i\partial_\mu\phi(x)$). As an example we will explicitly compute the case for the SCT generator, i.e. for $X = K_\mu$,

(vi)see Appendix B for a proof of this.

$$
\begin{aligned}
K'_\mu &= e^{ix^\nu P_\nu} K_\mu e^{-ix^\nu P_\nu} \\
&= \sum_n \frac{i^n}{n!} x^{\nu_1} \dots x^{\nu_n} \left[P_{\nu_1}, [\dots [P_{\nu_n}, K_\mu] \dots]\right] \\
&= K_\mu + ix^\nu [P_\nu, K_\mu] + \tfrac{i^2}{2!} x^\nu x^\rho \big[P_\nu, [P_\rho, K_\mu]\big] + [\text{all other terms vanish}] \\
&= K_\mu - 2x^\nu (\eta_{\mu\nu} D + L_{\mu\nu}) - ix^\nu x^\rho \big(\eta_{\mu\rho}[P_\nu, D] + [P_\nu, L_{\mu\rho}]\big) \\
&= K_\mu - 2x^\nu (\eta_{\mu\nu} D + L_{\mu\nu}) + 2x_\mu x^\nu P_\nu - x^2 P_\mu. \qquad (2.24)
\end{aligned}
$$

So in summary, the action of each generator on the field is,

$$
\begin{aligned}
&P_\mu \phi(x) = i\partial_\mu \phi(x), \\
&L_{\mu\nu} \phi(x) = [i(x_\mu \partial_\nu - x_\nu \partial_\mu) + S_{\mu\nu}]\phi(x), \\
&D\phi(x) = (ix^\nu \partial_\nu + \Delta)\phi(x), \\
&K_\mu \phi(x) = [\kappa_\mu + 2x_\mu \Delta + 2x^\nu S_{\mu\nu} + 2ix_\mu x_\nu \partial^\nu - ix^2 \partial_\mu]\phi(x) \qquad (2.25)
\end{aligned}
$$

An interesting point to make here is that,

**Definition 1.** *All field theoretically admissible representations of the conformal algebra are induced by a representation of the algebra of the little group. [10]*

The little group being the one which leaves $x = 0$ invariant. Hence, we are left with only the following types of representations,

(1) finite-dimensional representations of the little group. Where either a) $\kappa_\mu = 0$ or b) $\kappa_\mu \neq 0$ but is nilpotent
(2) infinite-dimensional representations of the little group

In the proceeding arguments we will assume a representation of the kind 1a) or 1b) above and ignore infinite-dimensional reps. The case 1a) helps us to construct local conformal currents (see Section (2.2)) by allowing us to enforce the condition,

**Condition 1.** $\Delta = i l \mathbf{1}$ *- from Schur's lemma, as $\phi(x)$ must be an irreducible representation of the Lorentz group. $\Delta$ must be proportional to the identity. Also $\kappa_\mu = 0$.*

We can use all of this information to restrict how conformal fields will change under a conformal transformation, $x \to x'$. In $D$ space-time dimensions, where we define the scaling dimension of the field as $d$, the conformal transformation of the field will look like(vii),

$$\begin{aligned} \phi(x) \to \phi'(x') &= \left|\frac{\partial x'}{\partial x}\right|^{-d/D} \phi(x) \\ \text{where } \left|\frac{\partial x'}{\partial x}\right| &= \Lambda(x)^{-D/2} \\ \Rightarrow \phi'(x') &= \Lambda(x)^{d/2}\phi(x). \end{aligned} \tag{2.26}$$

This is an important point, and is regularly made use of when analysing conformal fields. We will use this property of conformal fields as part of our discussion of unparticles.

### 2.2. **Conformal Currents.**

**Definition 2.** Noether's Theorem. *Every symmetry transformation which leaves a system invariant has a corresponding conserved charge and conserved current.* [44]

Noether's theorem simply states that for conformal transformations specified by $\delta\omega_A$ the following statements (2.27) - (2.29) will hold,

$$S = \int d^4x \mathcal{L}(x) \;\text{ and }\; \delta S = \sum_A \delta\omega_A \int d^4x \partial_\mu J^{A\mu}. \tag{2.27}$$

If the symetry is exact then $\delta S = 0$ and the currents $J_\mu$ will have no divergence. Defining the energy-momentum tensor as,

$$T_{\nu\rho}(x) = -\eta_{\nu\rho}\mathcal{L} + \frac{\partial \mathcal{L}}{\partial \partial^\nu \phi}\partial_\rho \phi, \tag{2.28}$$

the Noether currents will take the form,

(vii)This is for a spinless field, $\phi(x)$

$$\sum_A J^A_\mu \delta\omega_A = \frac{\partial\mathcal{L}}{\partial(\partial^\mu\phi)}\delta\phi - T_{\mu\nu}\delta x^\nu. \tag{2.29}$$

Using (2.11) and (2.25) we can write down the local currents; $\mathcal{D}_\nu$ for dilations, $\mathcal{K}_{\nu\mu}$ for SCTs and $\mathcal{M}_{\mu\nu\rho}$ for the usual angular momentum,

$$\begin{aligned}
\mathcal{D}_\mu(x) &= x^\nu T_{\mu\nu} - l\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)}\phi \\
\mathcal{K}_{\mu\nu}(x) &= 2x_\nu x^\rho T_{\mu\rho} - x^2 T_{\mu\nu} - \frac{\partial\mathcal{L}}{\partial(\partial^\mu\phi)}(2lx_\nu + 2ix^\rho S_{\rho\nu})\phi \\
\mathcal{M}_{\mu\nu\rho}(x) &= x_\nu T_{\mu\rho} - x_\rho T_{\mu\nu} - i\frac{\partial\mathcal{L}}{\partial(\partial^\mu\phi)}S_{\nu\rho}\phi
\end{aligned} \tag{2.30}$$

Since the zeroth component of each of these is hermitean, the corresponding generators, are also hermitean. One can check, by making use of ETCRs, that the following satisfy the required properties of (2.25), regardless of whether the Lagrangian is conformally invariant or not.

$$\begin{aligned}
D\phi(x) &= -\int d^3x[\mathcal{D}_0(t,\vec{x}),\phi(t,\vec{x}\,')] \\
K_\mu\phi(x) &= -\int d^3x[\mathcal{K}_{0\mu}(t,\vec{x}),\phi(t,\vec{x}\,')]
\end{aligned} \tag{2.31}$$

In this appraisal we have assumed minimal coupling (there are no derivatives of fermion fields and only first derivatives of boson fields). By examining the divergence of the currents we can uncover information about the symmetry of the theory, specifically whether it is exact or broken. The divergences of the dilatation current and conformal current are shown below. (Note: energy-momentum conservation, $\partial^\nu T_{\nu\mu} = 0$, and angular momentum conservation, $\partial^\nu \mathcal{M}_{\nu\mu\rho} = 0$, have been used).

$$\partial^\mu \mathcal{D}_\mu(x) = T_\nu{}^\nu - \partial^\mu\Big(l\frac{\partial \mathcal{L}}{\partial(\partial_\mu\phi)}\phi\Big)$$
$$= -4\mathcal{L} - \Big(l\frac{\partial \mathcal{L}}{\partial\phi}\phi + (l-1)\frac{\partial \mathcal{L}}{\partial(\partial_\rho\phi)}\partial_\rho\phi\Big) \tag{2.32}$$
$$\partial^\mu \mathcal{K}_{\mu\nu}(x) = 2x_\nu\partial^\mu\mathcal{D}_\mu - 2l\frac{\partial \mathcal{L}}{\partial(\partial^\nu\phi)} + 2i\frac{\partial \mathcal{L}}{\partial(\partial_\mu\phi)}S_{\mu\nu}\phi$$
$$= 2x_\mu\partial^\nu\mathcal{D}_\nu + V_\mu. \tag{2.33}$$

This implies that the divergences of the dilatation and SCT currents are non-zero meaning that these symmetries are only approximate. In principle this isn't really the case since, using a method outlined in [12], one can redefine the energy-momentum tensor as $\theta_{\mu\nu}$ such that,

$$\mathcal{M}_{\mu\nu\rho} = x_\rho\theta_{\nu\mu} - x_\mu\theta_{\nu\rho}$$
$$\mathcal{D}_\mu = x^\nu\theta_{\mu\nu}$$
$$\mathcal{K}_{\mu\nu} = x_\nu\mathcal{D}_\mu + x^\rho\mathcal{M}_{\mu\nu\rho}, \tag{2.34}$$

and these currents will all be conserved in a conformally invariant theory and will still give rise to the charges $P_\mu$, $L_{\mu\nu}$, $K_\mu$ and $D$. However, recall that exact dilatation symmetry is not possible in a massive theory. If we want to include a mass gap in the theory we need a conformal symmetry breaking mechanism.

### 2.3. **The Energy-Momentum Tensor,** $T_{\mu\nu}$.

In conformal invariant theories, the stress-energy (or energy-momentum) tensor, has the very distinct property of its trace vanishing, $T^\mu{}_\mu = 0$. This is true of conformal theories in any dimension. Notice that by virtue of the fact that $T_{\mu\nu}$ is traceless the action is invariant under conformal transformations. For $x^\mu \to x^\mu + \epsilon^\mu$,

$$\delta S = \int d^dx T^{\mu\nu}\partial_\mu\epsilon_\nu = \frac{1}{2}\int d^dx T^{\mu\nu}(\partial_\mu\epsilon_\nu + \partial_\nu\epsilon_\mu) = \frac{1}{d}\int d^dx T^\mu{}_\mu\partial_\nu\epsilon^\nu = 0. \tag{2.35}$$

### 2.4. **Correlation Functions.**

A quantum description means that we will need to derive correlation functions, ward identities and the like for conformally invariant fields [(viii)]. In a conformal field theory the S-matrix description is rendered useless as we cannot define in and out states as being infinitely far away. Clearly, in a theory which is independent of scale the notion of being very, very far away cannot exist as it is equivalent to being very, very close. As such we must work with correlation functions (these will help us with unparticle scattering problems later) whereupon the operator product expansion will be very useful. The correlation functions are restricted by conformal invariance and so take a very particular form. Below we simply write down the 2 and 3-point functions for conformal fields with the scaling dimensions for each field, $\phi_a$ given by $d_a$. The 2-point function is only non-zero if the two fields have the same scaling dimension.

$$\text{2-pt function: } \langle \phi_1(x_1)\phi_2(x_2)\rangle = \frac{C_{12}}{|x_1 - x_2|^{2d}}$$

$$\text{3-pt function: } \langle \phi_1(x_1)\phi_2(x_2\phi_3(x_3))\rangle = \frac{C_{123}}{x_{12}^{d_1+d_2-d_3} x_{23}^{d_2+d_3-d_1} x_{13}^{d_1+d_3-d_2}}$$

$$\text{where } \; x_{ij} = |x_i - x_j| \tag{2.36}$$

It is very important to realise that correlation functions are restricted when the fields within them are conformal. This is what leads Georgi to first propose unparticles - the fact that correlation functions between two unparticle operators take a very particular, and *recognisable*, form. Using these unparticle correlators he argues that the phase space for unparticle stuff looks like that of a *fractional* number of massles fields. This is the whole point of calling them unparticles - there is a non-integer number of them! This is just a taster of what is to come in Sec. 4, for now we will return to our discussion of CFTs.

### 2.5. **Ward Identities.**

The ward identities can help us encapsulate Noethers theorem for quantum field theories

[(viii)] Recall use of the term field in CFTs describes any object that can be written down. In QFT we would restrict our fields to just $\phi$ or $\psi$ but in CFT we could easily take $\partial\phi$ or $e^{i\phi}$

in the form of operator equations. In general, for any current $J_a^\mu$ the corresponding Ward identity states that,

$$\frac{\partial}{\partial x^\mu}\langle J_a^\mu(x)\phi(x_1)\dots\phi(x_n)\rangle = -i\sum_{i=1}^{n}\delta(x-x_i)\langle\phi(x_1)\dots T_a\phi(x_i)\dots\phi(x_n)\rangle \tag{2.37}$$

where $T_a$ is the generator of some infinitesimal transformation $\phi'(x) = \phi(x) - i\omega_a T_a\phi(x)$.

We can now deduce three Ward Identities associated with conformal invariance using the generators derived in Sec. 2.1.2 and the currents defined in (2.34). Here we will use notation that $X$ is a product of $n$ operators, i.e. $\langle X\rangle = \langle\phi_1(x_1)\phi_2(x_2)\dots\phi_n(x_n)\rangle$. The Ward identity for translations takes the usual form, written down in (2.38). To derive the WI for Lorentz transformations we use the conserved current given in (2.34) and the generator given in (2.25). This then reduces to (2.39) using (2.38). We use the same method to find the WI associated to scale transformations which is given in (2.40).

$$1.)\ \partial_\mu\langle T^\mu \nu X\rangle = -\sum_i \delta(x-x_i)\frac{\partial}{\partial {x_i}^\nu}\langle X\rangle \tag{2.38}$$

and using the generator of rotations from (2.25)

$$\partial_\mu\langle T^{\mu\nu}x^\rho - T^{\mu\rho}x^\nu X\rangle = \sum_i \delta(x-x_i)\Big[({x_i}^\nu{\partial_i}^\rho - {x_i}^\rho{\partial_i}^\nu - i{S_i}^{\nu\rho})\langle X\rangle\Big]$$

then using (2.38),

$$2.)\ \langle(T^{\rho\nu} - T^{\nu\rho})X\rangle = -\sum_i \delta(x-x_i){S_i}^{\nu\rho}\langle X\rangle \tag{2.39}$$

Using the dilatation current $j_D^\mu = T^\mu{}_\nu x^\nu$, and scaling dimension, $d$, we expect

$$\partial_\mu\langle T^\mu{}_\nu x^\nu X\rangle = -\sum_i \delta(x-x_i)\big(x_i^\nu\partial_i^\nu + d_i\big)\langle X\rangle$$

Using (2.38) once more,

$$3.)\ \langle T^\mu{}_\mu X\rangle = -\sum_i \delta(x-x_i)d_i\langle X\rangle \tag{2.40}$$

for translations, rotations and scale transformations respectively.

### 2.6. **Operator Product Expansion.**

Inclusive scattering processes in unparticle physics take advantage of the correlation functions for conformal fields. As a result there are many cases where we have products of conformal operators inside correlation functions. In general, correlation functions will contain singularities when the positions of the fields within them overlap. One can argue that this embodies the nature of quantum mechanics as a whole - the inability for us to take precise position measurements of fields. The product of two operators which lie at the same space-time point is singular and therefore highly useless. The OPE is a Laurent expansion of the two operators which allows us to express the product of two operators, at separate points which are approaching each other, as a sum of operators at one of the points. It is at this stage that we will restrict our discussion to conformal fields in just two dimensions in order to simplify the explanation. After a brief deliberation of CFTs in 2D we will return to the OPE and examine some examples to highlight its use in conformal theories and more importantly unparticle physics.

### 2.7. **Conformal Invariance in 2D.**

The following section gives a few examples of the way CFTs are constructed in two dimensions. Much of the mathematics here is not applicable directly to unparticle physics where we will be more heuristic with our arguments. Instead, this section hopes to provide the reader with a better general understanding of *how* one goes about constructing CFTs in 2D.

We will work with Euclidean worldsheet coordinates, $z_0$ and $z_1$ under mappings of the form $g^{\mu\nu} \longrightarrow \frac{\partial w^\mu}{\partial z^\alpha}\frac{\partial w^\nu}{\partial z^\beta}g^{\alpha\beta}$, as is the convention for conformal theories in two dimensions. We define holomorphic (left-moving) and anti-holomorphic (right-moving) coordinates as,

$$z = z^0 + iz^1, \quad \bar{z} = z^0 - iz^1, \quad \text{with} \quad \partial_z = \partial = \tfrac{1}{2}(\partial_0 - i\partial_1), \quad \partial_{\bar{z}} = \bar{\partial} = \tfrac{1}{2}(\partial_0 + i\partial_1).^{(\text{ix})} \tag{2.41}$$

(ix)Using these coords, $g^{\mu\nu} = \begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix}$, $g_{\mu\nu} = \begin{pmatrix} 0 & \frac{1}{2} \\ \frac{1}{2} & 0 \end{pmatrix}$, $\epsilon^{\mu\nu} = \begin{pmatrix} 0 & -2i \\ 2i & 0 \end{pmatrix}$, $\epsilon_{\mu\nu} = \begin{pmatrix} 0 & \frac{1}{2}i \\ -\frac{1}{2}i & 0 \end{pmatrix}$.

**Definition 3.** *A primary field is one which cannot be represented as the derivative of another field. Its descendants are called secondary fields which can always be represented as derivatives of the primary field.*

In two dimensions a primary field, of scaling dimension $d$ and spin $s$ has *holomorphic (antiholomorphic) conformal dimension* $h(\bar{h})$ given by,

$$h = \frac{1}{2}(d+s) \qquad \bar{h} = \frac{1}{2}(d-s). \tag{2.42}$$

2.7.1. *The Witt Algebra.*

For an infinitesimal holomorphic transformation, $z' = z + \epsilon(z)$, $\epsilon(z) = \sum_{-\infty}^{\infty} c_n z^{n+1}$, defining the generators as $l_n = -z^{n+1}\partial$ and $\bar{l}_n = -\bar{z}^{n+1}\bar{\partial}$ one finds that,

$$[l_n, l_m] = (n-m)l_{n+m}, \qquad [\bar{l}_n, \bar{l}_m] = (n-m)\bar{l}_{n+m} \quad \text{and} \quad [l_n, \bar{l}_m] = 0 \tag{2.43}$$

The finite subalgebra from, $l_{-1}, l_0, l_1$ forms the global conformal group,

$$\begin{aligned}
&\text{Translations:} && l_{-1} = -\partial \\
&\text{Scale \& Rotations:} && l_0 = -z\partial \\
&\text{SCTs:} && l_1 = -z^2\partial \\
&\text{Dilatations:} && l_0 + \bar{l}_0 \\
&\text{Rotations:} && i(l_0 - \bar{l}_0)
\end{aligned} \tag{2.44}$$

2.7.2. *The Stress-Energy tensor in 2D.*

By changing from real coordinates to complex coordinates, as in (2.41), and bearing in mind that that $T_{\mu\nu}$ is symmetric, we can write the energy-momentum tensor as,

$$\begin{aligned}
T_{zz} &= \frac{1}{4}(T_{00} - 2iT_{10} - T_{11}) \\
T_{\bar{z}\bar{z}} &= \frac{1}{4}(T_{00} + 2iT_{10} - T_{11}) \\
T_{z\bar{z}} = T_{\bar{z}z} &= \frac{1}{4}(T_{00} + T_{11}) = \frac{1}{4}T^{\mu}{}_{\mu} = 0
\end{aligned} \tag{2.45}$$

and therefore,

$$T_{zz} = \frac{1}{2}(T_{00} - iT_{10}) \qquad T_{\bar{z}\bar{z}} = \frac{1}{2}(T_{00} + iT_{10}). \tag{2.46}$$

It takes minimal effort to see that $\partial_{\bar{z}} T_{zz} = 0$ and similarly that $\partial_z T_{\bar{z}\bar{z}} = 0$ so the result is that the only non-zero parts of the energy-momentum tensor are a left handed and a right handed field, $T(z) = T_{zz}(z, \bar{z})$ and $T(\bar{z}) = T_{\bar{z}\bar{z}}(z, \bar{z})$.

2.7.3. *Ward Identities.*

We have so far already gathered the Ward Identities for a conformally invariant theory in a general number of dimensions. We can make this explicit for a two dimensional theory using the holomorphic coordinates. Combining the three produces the so called *Conformal Ward Identity*,

$$\delta_{\epsilon\bar{\epsilon}}\langle X\rangle = -\frac{1}{2\pi i}\oint_c dz\epsilon(z)\langle T(z)X\rangle + \frac{1}{2\pi i}\oint_c d\bar{z}\bar{\epsilon}(\bar{z})\langle \bar{T}(\bar{z})X\rangle. \tag{2.47}$$

where the holomorphic part is given by,

$$\langle T(z)X\rangle = \sum_{i=1}^{n}\Big[\frac{1}{z-w_i}\partial_{w_i}\langle X\rangle + \frac{h_i}{(z-w_i)^2}\langle x\rangle\Big] + [\text{terms regular at } z = w_i], \tag{2.48}$$

with a similar expression for the antiholomorphic component. The derivation of (2.47) (refer to Sec. 5.2 of [16]) is not very important but its implications on the operator product expansion are what we are going to concentrate on.

2.7.4. *The Operator Product Expansion.*

We can see from the expression (2.48) that there is singular behaviour in the correlation function of the renormalised energy-momentum tensor with primary fields as $z \to w_i$. OPEs are always assumed to be inside correlators and so often the $\langle \dots \rangle$ notation is dropped. We can write the general OPE of two fields $A(z)$ and $B(w)$ as,

$$A(z)B(w) = \sum_{n=-\infty}^{N}\frac{\{AB\}_n(w)}{(z-w)^n}, \tag{2.49}$$

where $\{AB\}_n$ might for example be $\{T\phi\}_n = \partial_w\phi(w)$. Even more generally for two local CFT operators in a set, $\mathcal{O}_i$,

$$\mathcal{O}_i(z,\bar{z})\mathcal{O}_j(w,\bar{w}) = \sum_k C_{ij}^k(z-w,\bar{z}-\bar{w})\mathcal{O}_k(w,\bar{w}). \tag{2.50}$$

The operator product expansion means that we can express products of operators in a non-singular way. The OPE is used in many QFTs for similar reasons, however, in CFTs it is especially useful because the operator product expansion is exact, not just an approximation [17]. There is a generalisation of the OPE which we make use of when considering some unparticle interactions. This is called the *Conformal Partial Wave Expansion*. It can be used to rewrite the time ordered product of arbitrarily separated conformal operators as a sum of $n$ point functions and primary operators of the CFT. There will be more on this in Sec. 4.5 where will make explicit use of the conformal wave expansion.

### 2.8. **What do we take from this?**

The hope is that this section has given the reader some intuitive sense of conformal theories, particularly the restrictions they inflict on the quantum construction of scale invariant fields. The fact that conformal operators fix the form of the correlation functions is very useful, as is the operator product expansion and its generalisation to the conformal partial wave expansion. Conformal fields, or conformal operators, are those which are invariant under a change of scale where all dimensional quantities are multiplied by a scaling factor or indeed by fractional powers of the scaling parameter. After a short tour of the principles underlying effective field theories we will begin our discussion of unparticle physics.

# 3. Effective Field Theory

Here follows some brief comments on the principles of Effective theories, why we need them and why they are particularly useful. We have made use of lecture notes by Pich [18] and Kaplan [19] as well as a review by Georgi [20].

We would like to be able to describe all physics with one theory. In practice this is actually very difficult and in reality it is much easier to consider a system at the energy scale which is most appropriate to the particular problem we want to solve. *Effective Field Theory* is the tool with which we can put this into practise. We study the low energy theory below a certain scale, $\Lambda$. We keep excitations with $m \ll \Lambda$ and use a method called *integrating out* to get rid of states with $M \gg \Lambda$. The result of this is a load of non-renormalisable interaction terms supressed by inverse powers of the scale, $\Lambda$. However, the low-energy theory has a finite number of couplings which allows us to renormalise the theory term by term. In other words we only need a finite number of parameters to calculate physical processes at an energy of $\Lambda$ or below. Effective Field Theory is an incredibly useful tool and unparticle physics takes advantage of it by introducing a conformally invariant sector to the standard model at *low energy scales*.

We distinguish an effective field theory by some Lagrangian of the form,

$$\mathcal{L} = \sum_i c_i \mathcal{O}_i, \tag{3.1}$$

where $\mathcal{O}_i$ are the operators of the theory (with dimension $[\mathcal{O}_i]$) made up of the light fields, and the coupling constants $c_i$ actually obscure information on the heavier states. The operators are arranged by their dimension which then fixes the dimension of the $c_i$'s,

$$c_i \propto \frac{1}{\Lambda^{[\mathcal{O}_i]-4}}. \tag{3.2}$$

Below the scale $\Lambda$, the importance of the operators is defined as,

(1) $[\mathcal{O}_i] < 4$ - Relevant
(2) $[\mathcal{O}_i] = 4$ - Marginal
(3) $[\mathcal{O}_i] > 4$ - Irrelevant

The couplings of operateors with dimension greater than four become irrelevant because their effects are negligible in the low energy theory. In contrast, operators with small dimensions have a much larger effect on the theory as the energy scale gets lower. As a result, in effective theories, the main interest is in operators with the smallest dimensions and in our unparticle discussions below we will often only account for operators of the smallest possible dimension and ignore all others for simplicity.

The basic ingredients for an EFT are summarised by Pich in [18]. They are as follows,

(1) The dynamics of the system at low energies are completely independent of the dynamics of the system at high energies.

(2) We pick the energy scale most relevant to the system we are considering. If there are masses much smaller than this scale we put them to zero. If there are masses much larger than this scale we put them to infinity[(x)],

$$0 \leftarrow m \ll \Lambda \ll M \rightarrow \infty. \tag{3.3}$$

(3) We replace any massive particle exchanges by a tower of non-renormalisable couplings amongst the lighter fields. The constraints on this *tower of interactions* (see (3.4) below) replace the need to renormalise every term.

(4) Clearly our theory is only useful in certain energy regimes so there is a particular accuracy to which our low-energy theory is defined. Each interaction (with dimension $[\mathcal{O}_i]$) will contribute,

$$\left(\frac{\Lambda}{M}\right)^{[\mathcal{O}_i]-4}, \tag{3.4}$$

and so we only need interactions up to,

$$\epsilon \approx \left(\frac{\Lambda}{M}\right)^{[\mathcal{O}_i]-4} \quad \Rightarrow \quad [\mathcal{O}_i] \approx 4 + \frac{\ln(1/\epsilon)}{\ln(M/\Lambda)}. \tag{3.5}$$

(5) The EFT must behave the same as the underlying theory in the infrared (but this doesn't need to be the case, in fact most likely won't be, in the ultraviolet).

(6) The only residue of the theory above the scale $\Lambda$ is hidden in the constants $c_i$.

[(x)]If necessary we can incorporate corrections using perturbation theory.

This is all very well for tree level processes but actually when we introduce higher order corrections we need to deal with an infinite number of counterterms. This requires the introduction of a mass-independent renormalisation scheme (such as the $MS$ or $MS$-bar schemes and the method of dimensional regularisation discussed in the AQFT course).

Clearly as the energy scale, $\Lambda$, increases then each interaction (3.4) becomes more influential. As you approach the scale of $M$, previously non-renomalisable couplings will become renormalisable, you get to some *new* physics and you must then start the business of constructing your EFT from the beginning. Quite an obvious question is how long does this go on for? Can we just keep on going and going to infinitely high energy scales? One assumes that theoretically the answer is yes, it seems rather unlikely *a priori* that at some point all of the nonrenormalisable couplings simply vanish. Having said that, if string theory is to be believed we will eventually reach the Planck scale at which point we can ditch Effective Field Theory and a whole new domain takes over (we don't care much for string theory in this document for further reading one might consider [45], [46] or even [47]). In reality we don't know, and as Georgi puts it in [20], "Who knows? Who cares?" And in some sense he has a point, does it really matter that much now? Whether it does or not, we certainly don't care for the remainder of this discussion, we need effective field theory for unparticle physics.

Philosophical discussions aside, EFT is highly advantageous. It allows us to investigate the physical effects we are interested in, without the annoyance of getting caught up with non-renormalisable terms in arbitrarily high energy regimes that we don't even really care about.

# 4. Unparticle Physics

> "*Unparticle physics describes a situation in which standard model physics is weakly coupled, at high energies, to a sector that flows to a scale-invariant theory in the infrared.*" [21]

## 4.1. **The Unparticle scheme - What are unparticles?**

Unparticles are unphysical, massless conformal invariant fields that become apparent as a scale invariant sector emerges from the standard model at low energies. As they are scale invariant they must be massless, but really we should be more strict with our description. They are not massless in the sense that they have $m = 0$ like a photon, but in the sense that there is no notion of mass what-so-ever (recall Sec. 2.1.1 where we stated that the mass spectrum must be continuous for an exact, non-broken conformal symmetry). The physics actually describes a fractional number of massless particles hence the name unparticle. If their existence is confirmed it would not be by direct detection but as missing energy in decays or unexpected phenomenology.

Many theoretical extensions to the standard model, often referred to as BSM (Beyond the Standard Model) Physics, entail coupling a hidden sector to the normal visible sector of the Standard model. In his first paper, [1], Georgi contemplates one such coupling of the standard model to a hidden interacting conformal field theory. The two sectors communicate via exchange of a very heavy field, of mass $M$. For a general standard model extension (in $D$ space-time dimensions) with hidden and visible operators, $\mathcal{O}_h$ and $\mathcal{O}_v$, with dimensions $d_h$ and $d_v$, the interaction terms in the Lagrangian will be, after integrating out the high energy terms (see Sec. 3),

$$\mathcal{L} \propto \frac{c_0}{M^{d_v+d_h-D}}\mathcal{O}_v\mathcal{O}_h + \frac{c_1}{M^{2d_v-D}}\mathcal{O}_v\mathcal{O}_v + \frac{c_2}{M^{2d_v-(D-2)}}\mathcal{O}_v\partial^2\mathcal{O}_v + \dots \tag{4.1}$$

where the coefficients, $c_i$, are dimensionless couplings.

Georgi proposes one such low energy, weak coupling of standard model operators to conformally invariant operators by considering an interaction like the first term in (4.1). The scheme is more precisely as follows. We take our standard model fields and weakly couple them to Banks-Zaks fields, [48]. Banks-Zaks fields have the very useful property that they have a non-trivial IR fixed point which can flow to an interacting conformal fixed point(xi). In other words, we can make scale invariance emerge in our theory at low energies by introducing these types of fields. In our high energy system the two fields interact via exchange of a massive messenger, with scale $M$. As we lower the energy, and drop below the scale $M$, we get non-renormalisable couplings of the form expressed in (4.1), dictated by powers of $M$. These will take the generic form,

$$\frac{1}{M^c}\mathcal{O}_{SM}\mathcal{O}_{BZ}. \tag{4.2}$$

If we continue to decrease our energy scale, we eventually get to a point where the scale invariant sector emerges in our BZ fields. Lets call this scale, $\Lambda$. As we drop below this scale, dimensional transmutation occurs. That is, our previously dimensionless coupling constants *transmute* into dimensionful coupling parameters. So we must enforce an effective theory where the BZ operators now match onto conformally invariant unparticle operators, $\mathcal{O}_U$, with interaction terms like,

$$\frac{C\Lambda^{d_{BZ}-d_U}}{M^c}\mathcal{O}_{SM}\mathcal{O}_U, \tag{4.3}$$

which match the interactions expressed in (4.2), *a lá* (3.4) and (3.5).

This is all a very nice way of introducing a scale invariant sector to the standard model, but does this scheme seem reasonable? Well there are two clear advantages of introducing interactions of the form in (4.3),

(1) In the low energy theory, below the scale $\Lambda$, the BZ fields decouple from the ordinary matter fields of the standard model. The advantage of this is that our non-renormalisable couplings, (4.2), don't effect the IR scale invariance of the unparticle stuff, which would make things a lot more complicated.
(2) Provided we make the mass of our messenger, $M$, large enough, then the coupling between ordinary matter and unparticle matter will be so small that we simply

(xi)As an *aide-memoire*, the IR (infrared) fixed point is a set of coupling constants that flow from high energy values to stable, fixed low energy values via the mechanism of the renormalisation group which evolves a system from one energy scale to another.

wouldn't notice the unparticle stuff with current accelerator energies, yielding a reasonable argument for why we haven't detected any.

### 4.2. **Vital ingredients for an Unparticle theory.**

Georgi and Kats state that there are three crucial elements to a theory of unparticles [21]:

(1) The theory must have a scale invariant sector. The scale invariant stuff in this sector is what makes up the unparticles.
(2) This unparticle stuff must somehow couple to the fields of the Standard Model, otherwise, we would never notice it.
(3) We require a BZ theory in which a transition occurs from perturbative physics at high energies to scale invariant physics at low energies. This allows us to use effective field theory and make the scale invariant sector emerge only at low energies. Without this, the high energy regime would be hideously non-linear and very complicated and we want this regime to be our nice and well understood standard model.

### 4.3. **Physics below the scale $\Lambda$.**

So perhaps the first immediate question is, what does unparticle stuff look like? We will address this question by investigating physics below the scale $\Lambda$, which is the scale at which conformal invariance becomes apparent. We will be considering scale invariance in the usual four dimensional space-time.

For the moment we will restrict our discussion to unparticle operators, $\mathcal{O}_U$, of the lowest possible dimension which appear in (4.3). This is because the operators with the smallest dimension effect our theory the most. We actually constrain this further such that $1 < d_U < 2$ for reasons that will become apparent later, for now one can assume that this is for simplicity. In reality the authors of [49] have shown this to be a necessary condition. The unparticle operators produce the familiar states, $|p\rangle$, with 4-momentum $p^\mu$ from the vacuum. We consider the vacuum matrix element,

$$\langle 0|\mathcal{O}_U(x)\mathcal{O}_U^\dagger(0)|0\rangle = \int \frac{d^4p}{(2\pi)^4} e^{-ipx} \left|\langle 0|\mathcal{O}_U(0)|p\rangle\right|^2 \rho(p^2). \tag{4.4}$$

Because this is a CFT the matrix element scales with dimension $2d_U$ which means we can write the phase space of our unparticle stuff as,

$$\left|\langle 0|\mathcal{O}_U(0)|p\rangle\right|^2 \rho(p^2) = A_{d_U}\theta(p^0)\theta(p^2)(p^2)^{d_U-2}, \tag{4.5}$$

where $\theta(p^0)$ is a heavyside step function ensuring only positive energy solutions and $\theta(p^2)$ is a heavyside step function that ensures our unparticle stuff is not tachyonic. Note that we have no on-shell condition (i.e. a delta function that looks like $\delta(p^2 - m^2)$ or as the unparticles must be massless just $\delta(p^2)$). The factor $(p^2)^{d_U-2}$ is enforced by conformal invariance and for now $A_{d_U}$ is just some constant that ensures a good normalisation.

Here lies the first profound implication, spotted by Georgi, of introducing a scale-invariant sector to the standard model; the phase space in (4.5) is the same as that for $n$ massless particles, which is,

$$A_n\theta(p^0)\theta(p^2)(p^2)^{n-2} \quad \text{where} \quad A_n = \frac{16\pi^{5/2}}{(2\pi)^{2n}}\frac{\Gamma(n+\frac{1}{2})}{\Gamma(n-1)\Gamma(2n)}. \tag{4.6}$$

This is why we can dub this 'stuff' as particle like. It can't be made of actual particles because it is scale invariant (see above), and $d_U$ can be fractional, but its phase space is like that of a bunch of massless particles. In Georgi's own words:

> "*Unparticle stuff with scale dimension* $d_U$ *looks like a nonintegral number* $d_U$ *of invisible particles.*"

4.4. **Unparticle Propagators.**

Here we will derive the form of some unparticle propagators which will help us to see what unparticle stuff 'looks like' in the theoretical sense because we can use them to discuss some simple unparticle processes. The term propagator is actually a bit misleading because unparticles don't propagate in the normal sense of the word, we're really just using the same techniques we use for standard particle propagators. There is also a fairly substantial 'but' which we are going to overlook in this section. Grinstein et. al. argue in [23] that the unparticle propagators actually take a slightly different form, to that discussed below, by introducing a subtlety in CFTs which restricts the scaling dimensions of conformal fields from unitarity - a realisation of Mack in [14]. We will discuss this feature in Sec. 5.6, but ignore it for now, although it should be kept in the back of one's mind in the proceeding analysis. One can refer to Appendix D for a summary of the forms of the various unparticle propagators in both position and momentum space representations.

A property of CFTs is that the form of the correlation functions is fixed by scale invariance (see Sec. 2.4). The details and derivation of these properties is discussed by many authors, see [14], [15] and [50], for specifics, and the results have been reproduced in this document in Sec. 2.4. We will now compute some of the correlation functions for unparticle operators by Fourier transforming their structure in position space to momentum space. Firstly lets consider the 2-point function, in 4 spacetime dimensions, of a primary, scalar unparticle operator, $\mathcal{O}$ with a scaling dimension of $d_U$,

$$\begin{aligned}\int d^4x\, e^{ipx}\langle 0|T\mathcal{O}(x)\mathcal{O}^\dagger(0)|0\rangle &= \frac{A_{d_U}}{2\pi}\int_0^\infty dM^2\frac{(M^2)^{d_U-2}}{p^2-M^2+i\epsilon}\\ &= \frac{A_{d_U}}{2\sin(d_U\pi)}\frac{i}{(-p^2-i\epsilon)^{2-d_U}}. \end{aligned} \tag{4.7}$$

This looks rather strange, but seems sensical in that as $d_U \to 1$ we recover normal particle behaviour. If we consider a cut across $p^2 > 0$ and look for the discontinuity, we can expand the factor of $(-p^2 - i\epsilon)^{d_U-2}$ in the following way,

$$\begin{aligned}(-p^2-i\epsilon)^{d_U-2} &= (p^2)^{d_U-2}\big((-1-i\epsilon)^{d-2}-(-1+i\epsilon)^{d_U-2}\big)\\ &\approx (p^2)^{d_U-2}\big(e^{-i(d_U-2)\pi}-e^{i(d_U-2)\pi}\big)\\ &= (p^2)^{d_U-2}(-2i\sin(d_U\pi)). \end{aligned} \tag{4.8}$$

So that (4.7) simplifies to,

$$\int d^4x\, e^{ipx}\langle 0|T\mathcal{O}(x)\mathcal{O}^\dagger(0)|0\rangle = A_{d_U}(p^2)^{d_U-2}. \tag{4.9}$$

We can generalise this to any two primary scalar operators, $\mathcal{O}_i$ and $\mathcal{O}_j$ with dimensions $d_i$ and $d_j$ in $D$ spacetime dimensions. The 2-point function will be zero unless the two operators have the same dimension (this is enforced by a delta function) and the same spins,

$$\langle 0|T\mathcal{O}_i(x)\mathcal{O}_j(0)|0\rangle = \delta_{ij}A_{d_U}\int \frac{d^Dp}{(2\pi)^D}e^{-ipx}(-p^2-i\epsilon)^{d-D/2}, \tag{4.10}$$

where the factor $d - D/2$ in the momentum space propagator is enforced by the scaling dimension of the operators. Each of these $\mathcal{O}_i$ has its own unique phase space, which comes from the imaginary part of the 2-point function, *vis-a-vis* (4.5). This is,

$$d\Phi_j = A_{d_U}\theta(p^0)\theta(p^2)(p^2)^{d-D/2}. \tag{4.11}$$

We can carry this process over to fermionic unparticle operators as well. Take a fermionic operator, $\mathcal{O}$, with a scaling dimension $d_U$, then its propagator in 4 spacetime dimensions will be,

$$\begin{aligned}\int d^4x\, e^{ipx}\langle 0|T\mathcal{O}(x)\mathcal{O}^\dagger(0)|0\rangle &= \frac{A_{d_U-1/2}}{2\pi i}\int_0^\infty dM^2(M^2)^{d_U-5/2}\frac{\not{p}}{p^2-M^2+i\epsilon}\\ &= \frac{A_{d_U-1/2}}{2\cos(d_U\pi)}\frac{-i\not{p}}{(-p^2-i\epsilon)^{5/2-d_U}}.\end{aligned} \tag{4.12}$$

Note that the $1/2$ in the factor $A_{d_U-1/2}$ comes from the spin of the unparticle. This factor is commonly know as the 'twist' of the operator. Using the same process described in (4.8) this simplifies to,

$$\int d^4x\, e^{ipx}\langle 0|T\mathcal{O}(x)\mathcal{O}^\dagger(0)|0\rangle = -A_{d_U-1/2}\not{p}(p^2)^{d_U-5/2}. \tag{4.13}$$

So for two fermionic operators, $\mathcal{O}_i$ and $\mathcal{O}_j$ in $D$ spacetime dimensions,

$$\langle 0|T\mathcal{O}_i(x)\mathcal{O}_j(0)|0\rangle = -\delta_{ij}A_{d_U-1/2}\int \frac{d^Dp}{(2\pi)^D}e^{-ipx}\not{p}(-p^2-i\epsilon)^{d_U-\frac{D+1}{2}}, \tag{4.14}$$

whose phase spaces will arise from the discontinuity in the cut across (4.12) given by,

$$d\Phi_j = A_{d_U-1/2}\theta(p^0)\theta(p^2)\not{p}(p^2)^{d_U-\frac{D+1}{2}} \tag{4.15}$$

Similar restrictions apply to the 2-point function for primary vector operators, $\mathcal{O}_i^\mu$ and $\mathcal{O}_j^\nu$. Firstly lets consider the phase space for primary conformal vector operators, which is,

$$\langle 0|\mathcal{O}^\mu(0)|p\rangle\langle p|\mathcal{O}^\nu(0)|0\rangle\rho(p^2) = A_{d_U}\theta(p^0)\theta(p^2)(-g^{\mu\nu} + \tfrac{p^\mu p^\nu}{p^2})(p^2)^{d_U - D/2}. \tag{4.16}$$

And so the unparticle propagator will look like,

$$\begin{aligned}\int d^Dx\, e^{ipx}\langle 0|T\mathcal{O}_i^\mu(x)\mathcal{O}_j^\nu(0)|0\rangle &= i\delta_{ij}\frac{A_{d_U}}{2\pi}\int_0^\infty dM^2(M^2)^{d_U-D/2}\frac{(-g^{\mu\nu}+\frac{p^\mu p^\nu}{p^2})}{p^2-M^2+i\epsilon}\\ &= i\delta_{ij}\frac{A_{d_U}}{2}\frac{(-g^{\mu\nu}+\frac{p^\mu p^\nu}{p^2})}{sin(d_U\pi)}(-p^2-i\epsilon)^{d_U-D/2}.\end{aligned} \tag{4.17}$$

Once again this appears to be a rather odd result, but if we take the cut across $p^2 > 0$ and look for the discontinuity we find that,

$$\begin{aligned}&= i\delta_{ij}\frac{A_{d_U}}{2}\frac{(-g^{\mu\nu}+\frac{p^\mu p^\nu}{p^2})}{sin(d_U\pi)}(p^2)^{d_U-D/2}\big((-1-i\epsilon)^{d_U-D/2}-(-1+i\epsilon)^{d_U-D/2}\big)\\ &= i\delta_{ij}\frac{A_{d_U}}{2}\frac{(-g^{\mu\nu}+\frac{p^\mu p^\nu}{p^2})}{sin(d_U\pi)}(p^2)^{d_U-D/2}\underbrace{\big(e^{-i(d_U-D/2)\pi}-e^{i(d_U-D/2)\pi}\big)}_{-2isin(d_U\pi)}\\ &= i\delta_{ij}A_{d_U}(-g^{\mu\nu}+\tfrac{p^\mu p^\nu}{p^2})(p^2)^{d_U-D/2}\end{aligned} \tag{4.18}$$

So in summary for two primary vector operators, $\mathcal{O}_i^\mu$ and $\mathcal{O}_j^\nu$, the 2-point function takes the form,

$$\langle 0|T\mathcal{O}_i^\mu(x)\mathcal{O}_j^\nu(0)|0\rangle = \delta_{ij}A_{d_U}\int\frac{d^Dp}{(2\pi)^D}e^{-ipx}\big(-g^{\mu\nu}+\frac{p^\mu p^\nu}{p^2}\big)(-p^2-i\epsilon)^{d_U-D/2}. \tag{4.19}$$

Conformally invariant, two-index tensors are restricted to being either antisymmetric or symmetric and traceless. By similar analysis to that used above for primary vectors, the 2-point functions can be constructed for tensors and higher dimensional representations[xii].

[xii]Spinor representations can also be constructed using Dirac $\gamma^\mu$ matrices, as in (4.12).

We will write down the 2-point function for two index tensors and leave it there, this is,

$$\begin{aligned}\langle 0|T\mathcal{O}_i^{\mu\nu}(x)\mathcal{O}_j^{\rho\sigma}(0)|0\rangle = \delta_{ij}A_{d_U}\int\frac{d^Dp}{(2\pi)^D}e^{-ipx}\Big((K^{\mu\rho}(p)K^{\nu\sigma}(p)-\tfrac{1}{4}g_{\mu\nu}g_{\rho\sigma})\pm(\mu\leftrightarrow\nu)\Big)\\ \times(-p^2-i\epsilon)^{d_U-D/2}\\ \text{where}\quad K^{\mu\nu}(x)=g^{\mu\nu}-\frac{x^\mu x^\nu}{x^2}.\end{aligned} \tag{4.20}$$

The structure of the 3-point function is also fixed by scale invariance (see Sec. 2.4) and is produced below. The higher $n$-point functions (for $n > 3$) are particularly restrained by conformal invariance but their precise forms are not completely determined.

$$\begin{aligned}\langle 0|T\mathcal{O}_i(x_i)\mathcal{O}_j(x_j)\mathcal{O}_k(x_k)|0\rangle \propto \frac{1}{(x_{ij})^{(d_i+d_j-d_k)}(x_{ik})^{(d_i+d_k-d_j)}(x_{jk})^{(d_j+d_k-d_i)}}\\ \text{where}\quad x_{ij}=x_i-x_j\end{aligned} \tag{4.21}$$

Using the work published in [22] we can use this formalism to understand some simple processes where unparticle stuff is actually produced as an outgoing state. This is the main way we can analyse what unparticle stuff actually looks like. Our traditional S-matrix formalism is completely useless in CFT because our theory looks the same at all length scales. We cannot have a notion of $|in\rangle$ and $|out\rangle$ states when there is no notion of being infinitely far away. Instead we use a different, yet also familiar approach, by looking at discontinuities across physical cuts in the correlation functions of unparticle operators. The beneficial outcome of this is that we end up with a description analogous to the recognised description of ordinary particle physics (i.e. that of good old quantum field theory). The next section concentrates on how we actually implement this.

### 4.5. **Unparticle self-interactions.**

So what we really want to do is study unparticle processes. That is processes which involve unparticle vertices, either with themselves or with standard model fields. Below we will use some mathematical techniques in conformal correlation functions to help us understand what bits of the new unparticle physics we can relate to scattering amplitudes of a normal particle theory, or at least in analogy to a normal particle theory. We follow much of the ground covered in Sec. 2 of [22] with other relevant contributions from [51] and [52]. We will make explicit use of a generalisation of the operator product expansion (see Sec. 2.6), the *conformal partial wave expansion.* This technique is first discussed in [53–55], and is summarised in [56]. It states that, for two operators with an arbitrary separation, $\mathcal{O}_1(x_1)$ and $\mathcal{O}_2(x_2)$, their time ordered product can be written as,

$$T\mathcal{O}_1(x_1)\mathcal{O}_2(x_2)|0\rangle = \sum_k \int d^D x\, iQ_k(x|x_1,x_2)\mathcal{O}_k(x)|0\rangle, \tag{4.22}$$

where $\mathcal{O}_k$ are the primary operators of the conformal theory and the coefficients $iQ_k(x|x_1,x_2)$ are the 3-point functions of $\mathcal{O}_1(x_1)$, $\mathcal{O}_2(x_2)$ and $\mathcal{O}_k(x)$ with the $\mathcal{O}_k(x)$ leg amputated. In other words, we can express these coefficients, $Q_k$, in terms of amputated 3-point functions,

$$\int d^D x \langle 0|T\mathcal{O}_k(x')\mathcal{O}_k(x)|0\rangle iQ_k(x|x_1,x_2) = \langle 0|T\mathcal{O}_k(x')\mathcal{O}_1(x_1)\mathcal{O}_2(x_2)|0\rangle. \tag{4.23}$$

It becomes obvious that this construction is useful if we consider a 4-point function of two unparticle operators, $\mathcal{O}_1$ and $\mathcal{O}_2$, propagating from positions $x_1$ and $x_2$ to $y_1$ and $y_2$. Using (4.22) twice, this 4-point function can be written as,

$$\begin{aligned}&\langle 0|T\mathcal{O}_2^*(x_2)\mathcal{O}_1^*(x_1)\mathcal{O}_1(y_1)\mathcal{O}_2(y_2)|0\rangle\\ &= \sum_k \int d^D x\, d^D y\, Q_k^*(x|x_1,x_2)\langle 0|T\mathcal{O}_k^*(x)\mathcal{O}_k(y)|0\rangle Q_k(y|y_1,y_2).\end{aligned} \tag{4.24}$$

This allows us to equate process inclusive with unparticle stuff, like the 4-point function on the left hand side of (4.24), to a sum of exclusive process with a propagator, $\mathcal{O}_k$, as on the right hand side of (4.24). For clarity we can show this diagrammatically. Consider a simplistic interaction between standard model fields, $\phi$, and unparticle operators, $\mathcal{O}$, of the form, $\mathcal{L}_{\text{int}} = \phi^2\mathcal{O}$. We arrive at something like Figure 1, where solid lines correspond to SM fields, $\phi$, with dashed lines as unparticles.

So how can we use this construction to describe processes which have outgoing unparticle states? Using the relation (4.24) we can analyse inclusive processes because the discontinuity in the cut of the $n$-point correlator can be expressed as a sum of primary

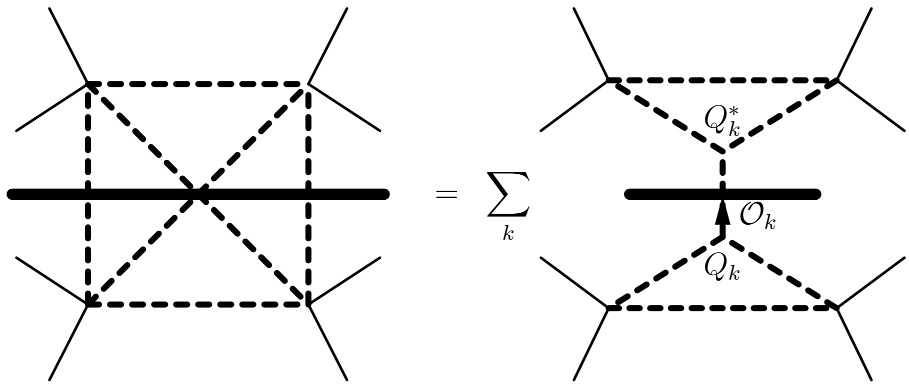



FIGURE 1. Note: time is running up the page. On the left we have a diagrammatic representation of the unparticle 4-point function. There is a physical cut (the dark line) across the unparticle stuff which is the cross section of (4.25) below. The right hand diagram is a sum of two amputated 3-point functions, $Q_k$ with the physical cut across the $\mathcal{O}_k$ propagator giving us a cross section for (4.26).

conformal operators, $\mathcal{O}_k$, of squared amplitudes which in turn are determined by the coefficient functions, $Q_k$, of the conformal partial wave expansion. This is analogous to a normal QFT where the coefficient functions, $Q_k$ play the role of amplitudes and the primary conformal operators, $\mathcal{O}_k$, play the role of particles.

The partial wave expansion is so useful because we can write correlation functions in terms of amputated 3-point functions. These in turn give us the amplitudes for processes like,

$$\phi + \phi \to \phi + \phi + \{\text{unparticle stuff}\}. \tag{4.25}$$

If we take the discontinuity across the cut of the 4-point function (as demonstrated in Figure 1) then we obtain a cross-section for processes of the form,

$$\phi + \phi \to \phi + \phi + \{\mathcal{O}_k \,\text{stuff}\}. \tag{4.26}$$

We arrive at amplitudes for these processes from the amputated 3-point functions, while cutting the operator propagators in (4.23) we get the phase space for these propagators described in (4.11).

4.6. **Distinguishing between particles and unparticles.**

So how do we actually differentiate between real particles and unparticles apart from by playing around with conformal transformations and seeing which fields admit a scale symmetry? In the theoretical sense we can glean information on the type of particle by the form of its propagator. For a standard model particle we can see its mass (massive or not) from the poles in the propagator and determine its spin subject to the involvement of Dirac matrices. For unparticles there are some sublte differences which we can infer from the forms of the propagators given in Sec. 4.4 and summarised in Appendix D. For the case of unparticle propagators with no spin there is a pole providing that $d_U < D/2$, and for the fermionic case, provided that $d_U < \frac{D+1}{2}$. We can use these scaling factors to help us determine between the real stuff and the unparticle stuff. Then when we construct specific scattering problems, we obtain the phase space by taking the discontinuity in the cut across the propagators and can use Fermi's Golden Rule (4.37) to find the cross section for that particular process.

We can simulate unparticle behaviour with normal particle behaviour using the analogy illustrated in the previous section (Sec. 4.5). I.e. we relate 3-point functions to the scattering amplitudes and conformal operators to the propagating particles of a normal theory.

This seems fairly adequate for a theorist, but how can we make the distinction experimentally? This is a much harder question to answer because we cannot detect unparticles directly. We may be able to infer their existence from missing energy in simple quark decays (an example is given below in Sec. 4.8) but the unparticles themselves could also decay into standard model particles. This requires that the conformal symmetry is broken at some scale by a Higgs-like mechanism. In this case we will detect standard model particles from non standard model (i.e. unparticle) processes. A couple of these types of mechanism and their resulting phenomenological impact is discussed in Sec. 5. For now we will suffice to discuss the likelihood of unparticle detection.

### 4.7. **How far are we from 'seeing' unparticles?**

"*Data, data, data - I cannot make bricks without clay.*"

- The Adventures of Sherlock Holmes

Many phenomenologists suggest that if unparticles do exist and couple with the standard model significantly then observation is well within reach of the LHC. There are some incredibly interesting implications of unparticles on the Higgs sector discussed in Sec. 5.2. Another aspect of the phenomenological side of the argument is briefly presented in Sec. 5.4. But really, as you might expect, it is very hard to say. A range of papers (e.g. [27], [38], [57–61] to name just a few) have discussed what processes we might be able to spot and how they in turn affect what we would expect to see in the SM without a hidden unparticle sector. There are several other proposals of BSM physics which include hidden sectors that could feasibly be mistaken for unparticles. Realistically, if we are going to observe unparticles within the next decade this will have to be at the LHC (or Super LHC). However, this isn't a priority, certainly not an advertised priority, of researchers at CERN. The teams working at ATLAS and CMS are much more concerned with the Higgs search[xiii], SUSY[xiv] and extra dimensions. There is no specific detector or experiment set up to specifically hunt for unparticle stuff, perhaps we won't need one, but in reality we will have to wait until the harem of data from the LHC is collected and analysed before the particle community decide on their next move - whether the theory of unparticles is viable, taking into account what we learn at the LHC. One imagines that first of all the experimentalists will amass evidence which supports more cemented theories, i. e. the Higgs, SUSY etc. and then only afterwards, if they have some bits of the jigsaw left over, will they try and match this evidence to various BSM suggestions. In essence, the theorists have played their shot, the ball is in the court of the experimentalists and whilst new approaches, techniques and subtleties are uncovered we shall have to wait and see

[xiii]A rather elegant suggestion for why it might not be found is presented in [36] and summarised in this document in Sec. 5.2

[xiv]Unparticle physics can also be incorporated here using SQCD (Supersymmetric QCD). See [37] and also Sec. 5.2

if any evidence for unparticles can be found, or at least if unparticles can be linked to unexplainable to data.

So then, perhaps a more sensible question is: 'how *likely* are we to observe unparticle behaviour in the near future?' There is contention on the existence of unparticles between some authors. For example, the existence of an unparticle sector is strongly supported by Kozlov in [62] who argues that a proper description of particle physics at high energy scales ($\approx$TeV) should account for extra degrees of freedom that admit a conformal symmetry and therefore that the unparticle theory is a strong contender. However, he is contradicted by some of Liao's comments in [29] who states that singularities arise in scattering cross sections from unparticle gauge interactions and that the Ward Identities for gauged unparticles are violated. Most seem satisfied to explore the field without commenting on the likelihood that we will observe this sort of behaviour. Perhaps if the subject had more contributors there would be more conflict in opinion. Many authors simply say that *if* the unparticle theory is valid we should see *something* at the LHC because we will be probing energies above the standard model.

The problem is, the unparticle proposition lacks so little evidence of any kind that is impossible to really say anything on its measurement. In particular, it is difficult to make exact predictions without a substantial knowledge of the conformal sector. If unparticles do exist and are not seen, or even inferred, at the LHC then there coupling must be so incredibly weak, at such low energy scales, that there discussion is really futile for a decade or more. Having said that, for a simple process such as the one we are going to discuss in the section below (which is a top quark to up quark decay with unparticle emission), one might conclude that it is fairly likely we would spot this kind of behaviour at LHC. The thing is, this process is a hugely simplified version of anything that we could consider realistic. Never-the-less it at least shows us how missing energy can arise from unparticle interactions.

4.8. **An unparticle example:** $t \to u + U$**.**

Consider a top quark decaying to an up quark and some unparticles, of dimension $d_U$, $t \to u+U$[xv]. For simplicity we will consider this process as involving just the interaction,

$$i\frac{C\Lambda^{d_{BZ}-d_U}}{M^c}\bar{u}\gamma_\mu(1-\gamma_5)t\partial^\mu\mathcal{O}_U + [\text{H.c}], \tag{4.27}$$

analogous to the interaction (4.3), where $\mathcal{O}_U$ is a scalar unparticle operator. We need a dimensionless coupling constant for this interaction vertex, analogous to the $e$ in QED, and we will define this as,

$$\lambda = \frac{C\Lambda^{BZ}}{M^c}, \tag{4.28}$$

so that our interaction looks like,

$$i\frac{\lambda}{\Lambda^{d_U}}\bar{u}\gamma_\mu(1-\gamma_5)t\partial^\mu\mathcal{O}_U + [\text{H.c}]. \tag{4.29}$$

Working in the centre of mass frame, where the top quark approaches with momentum, $P = (m_t, 0)$ and has spin $r$, the outgoing up quark has momentum $p = (E_u, \vec{p})$ and spin $s$, and the unparticle stuff has momentum $p' = (E_U, \vec{p}')$ we can depict our interaction as shown in Figure 2.

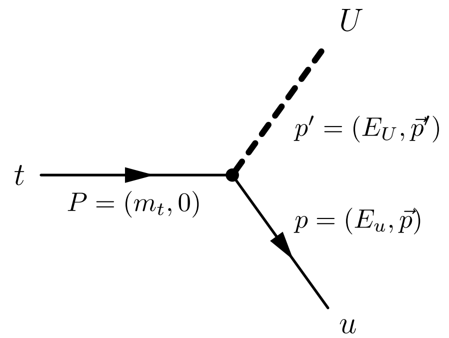



FIGURE 2. The feynman diagram for the $t \to u + U$ process, with the vertex interaction given by (4.29)

The amplitude of this diagram is,

$$i\mathcal{M} = i\frac{\lambda}{\Lambda^{d_U}}\bar{u}_s(p)\gamma_\mu(1-\gamma_5)t_r(P)\partial^\mu\mathcal{O}_U(p'). \tag{4.30}$$

Therefore,

[xv]This is just a test interaction, used for simplicity and instruction. It shouldn't be considered as a realistic unparticle process.

$$
\begin{aligned}
|\mathcal{M}|^2 &= \frac{|\lambda|^2}{\Lambda^{2d_U}}\big(\bar{u}_s(p)\gamma_\mu(1-\gamma_5)t_r(P)\partial^\mu\mathcal{O}_U(p')\big)\big(\partial_\mu\bar{\mathcal{O}}_U(p')\bar{t}_r(P)(1-\gamma_5)\gamma^\mu u_s(p)\big)\\
&= \frac{|\lambda|^2}{\Lambda^{2d_U}}\big(\bar{u}_s(p)\gamma_\mu(1-\gamma_5)t_r(P)\partial^\mu\mathcal{O}_U(p')\partial_\mu\bar{\mathcal{O}}_U(p')\bar{t}_r(P)(1-\gamma_5)\gamma^\mu u_s(p)\big) \qquad (4.31)
\end{aligned}
$$

Averaging over initial spins, summing over final spins and using the familiar relations(xvi),

$$
\sum_s u_s(p)\bar{u}_s(p) = \not{p} + m_u \qquad \sum_r t_t(P)\bar{t}_r(P) = \not{P} + m_t, \qquad (4.32)
$$

whilst assuming something similar for the unparticle operator. Including the gamma matrix relations,

$$
\{\gamma_5,\gamma_\mu\}=0, \qquad \mathrm{tr}(\gamma_5)=0, \qquad \gamma_\mu\gamma^\mu=4, \quad \mathrm{tr}(\gamma_\mu\gamma_\nu)=4\eta_{\mu\nu}, \qquad (4.33)
$$

and ignoring the up quark mass (as it is relatively negligible compared to the top mass), we obtain the matrix element,

$$
\begin{aligned}
|\mathcal{M}|^2 &= \frac{|\lambda|^2}{4\Lambda^{2d_U}}\left[\sum_s u_s(p)\bar{u}_s(p)\gamma_\mu(1-\gamma_5)\sum_r t_r(P)\bar{t}_r(P)(1-\gamma_5)\gamma^\mu\sum_l \partial^\mu\mathcal{O}_U(p')\partial_\mu\bar{\mathcal{O}}_U(p')\right]\\
&= \frac{|\lambda|^2}{4\Lambda^{2d_U}}\mathrm{tr}\left[\not{p}\gamma_\mu\underbrace{(1-\gamma_5)(\not{P}+m_t)(1-\gamma_5)}_{A}\gamma^\mu\not{p}'\right]\\
A &= (1-\gamma_5)(\not{P}+m_t-\not{P}\gamma_5-m_t\gamma_5)\\
&= \not{P}+m_t-2m_t\gamma_5-\not{P}+m_t = 2m_t-2m_t\gamma_5\\
|\mathcal{M}|^2 &= \frac{|\lambda|^2}{4\Lambda^{2d_U}}\mathrm{tr}\left[2m_t\not{p}\gamma_\mu(1-\gamma_5)\gamma^\mu\not{p}'\right]\\
&= \frac{|\lambda|^2}{2\Lambda^{2d_U}}m_t\mathrm{tr}\left[\underbrace{\not{p}\gamma_\mu\gamma^\mu\not{p}'}_{16(p\cdot p')}-\underbrace{\not{p}\gamma_\mu\gamma_5\gamma^\mu\not{p}'}_{0}\right]\\
&= 8\frac{|\lambda|^2}{\Lambda^{d_U}}m_t(p\cdot p'). \qquad (4.34)
\end{aligned}
$$

(xvi)Note: we are working in four dimensions here.

We can use the phase space for unparticle stuff as expressed in (4.5) and the phase space for a massless up quark to define the final state densities as,

$$d\Phi(p') = A_{d_U}\theta(p'^0)\theta(p'^2)(p'^2)^{d_U-2} \tag{4.35}$$

$$d\Phi(p) = 2\pi\theta(p^0)\delta(p^2). \tag{4.36}$$

Remember (4.35) has $\theta$-functions which ensure that the unparticle stuff has positive energy and is not tachyonic. It has no on-shell condition and scale invariance is enforced by the factor of $d_U - 2$, in essence dictating that it is an arbitrary amount of conformal *stuff*. The incoming top quark and outgoing up quarks have a definite energy and momentum. We can obtain the differential decay rate[(xvii)],

$$d\Gamma = \frac{|\mathcal{M}|^2}{2M} d\Phi(P), \tag{4.37}$$

by composing the densities, (4.36), as,

$$\begin{aligned} d\Phi(P) &= \int (2\pi)^4 \delta^{(4)}(P - \sum_i p_i) \prod_i \frac{d^4p_i}{(2\pi)^4} d\Phi(p_i) \\ &= \int (2\pi)^4 \delta^{(4)}(P - p - p') d\Phi(p) \frac{d^4p}{(2\pi)^4} d\Phi(p') \frac{d^4p'}{(2\pi)^4}. \end{aligned} \tag{4.38}$$

So using all the information contained within (4.34)-(4.38) we can compute the differential decay rate for such a process,

$$\begin{aligned} d\Gamma &= \frac{4|\lambda|^2}{m_t \Lambda^{2d_U}} m_t (p \cdot p') \int (2\pi)^4 \delta^{(4)}(P - p - p') \\ &\quad \times \frac{d^4p}{(2\pi)^4} \Big[ 2\pi\theta(p^0)\delta(p^2) \Big] \frac{d^4p'}{(2\pi)^4} \Big[ A_{d_U}\theta(p'^0)\theta(p'^2)(p'^2)^{d_U-2} \Big] \\ &= \frac{2|\lambda|^2}{\Lambda^{2d_U}} m_t^2 \int \frac{d^4p}{(2\pi)^4} 2\pi\theta(p^0)\delta(p^2) A_{d_U}\theta(P^0 - p^0)\theta((P-p)^2)((P-p)^2)^{d_U-2} \end{aligned} \tag{4.39}$$

[(xvii)]This is commonly known as 'Fermi's Golden Rule' - see Sec. 6.2 of [63] for simple introduction.

usually we would now split the $d^4p$ integral into $d^3p\,dp^0$, do the $dp^0$ integral and find the differential decay rate with respect to the solid angle, $\frac{d\Gamma}{d\Omega}$. However because our unparticle stuff is non definite we can find the differential decay rate with respect to $E_u$ by doing instead the $d^3p$ integral, using the substitution $r = \vec{p}^2$. Thus,

$$
\begin{aligned}
&= \frac{2|\lambda|^2}{\Lambda^{2d_U}} m_t^2 A_{d_U} \int \frac{d^3p}{(2\pi)^3}\frac{dp^0}{(2\pi)} 2\pi\theta(p^0)\delta(p^{0^2} - \vec{p}^2)\theta(P^0 - p^0)\theta((P-p)^2)((P-p)^2)^{d_U-2} \\
&= \frac{2|\lambda|^2}{\Lambda^{2d_U}} m_t^2 A_{d_U} \int \frac{dp^0}{(2\pi)^3} r dr d\Omega \theta(p^0)\theta(P^0 - p^0)\delta(p^{0^2} - r) \\
&\qquad \times \theta(P^{0^2} - \vec{P}^2 + p^{0^2} - r - 2P^0p^0 + 2\vec{P}\cdot\sqrt{r}) \\
&\qquad \times (P^{0^2} - \vec{P}^2 + p^{0^2} - r - 2P^0p^0 + 2\vec{P}\cdot\sqrt{r})^{d_U-2} \\
&= \frac{2|\lambda|^2}{\Lambda^{2d_U}} m_t^2 A_{d_U} \int \frac{dp^0}{(2\pi)^3} p^{0^2} d\Omega \theta(p^0)\theta(P^0 - p^0)\theta(P^2 - 2P^0p^0)(P^2 - 2P^0p^0)^{d_U-2} \\
&= \frac{4|\lambda|^2}{\Lambda^{2d_U}} \frac{m_t^2 A_{d_U}}{(2\pi)^2} \int dE_u E_u^2 \theta(E_u)\theta(m_t - E_u)\theta(m_t^2 - 2m_tE_u)(m_t^2 - 2m_tE_u)^{d_U-2}
\end{aligned} \tag{4.40}
$$

Now lets inspect these heavyside functions in order to simplify the expression,

$$
\begin{aligned}
&\theta(E_u) && \text{means} \quad E_u > 0 \\
&\theta(m_t - E_u) && \text{means} \quad m_t > E_u > 0 \\
&\theta(m_t^2 - 2m_tE_u) && \text{means} \quad m_t(m_t - 2E_u) > 0 \\
& && \qquad \Rightarrow \text{a)} \;\; m_t < 0,\; E_u > 0 \\
& && \qquad\quad\;\; \text{b)} \;\; m_t > 2E_u > 0
\end{aligned} \tag{4.41}
$$

Clearly, $m_t$ cannot be negative, therefore b) must be correct and so we can simplify these three step functions into one which has the form, $\theta(m_t - 2E_u)$, and therefore,

$$
\begin{aligned}
d\Gamma &= \frac{|\lambda|^2}{\Lambda^{2d_U}} \frac{m_t^2 A_{d_U}}{2\pi^2} \int dE_u E_u^2 \frac{\theta(m_t - 2E_u)}{(m_t^2 - 2m_tE_u)^{2-d_U}} \\
\frac{d\Gamma}{dE_u} &= \frac{|\lambda|^2}{\Lambda^{2d_U}} \frac{A_{d_U} m_t^2 E_u^2}{2\pi^2} \frac{\theta(m_t - 2E_u)}{(m_t^2 - 2m_tE_u)^{2-d_U}}.
\end{aligned} \tag{4.42}
$$

As we are concerned with effects on $E_u$ we can make this a function of $E_u$ by plotting the form of $\frac{d \ln \Gamma}{dE_u}$,

$$\frac{1}{\Gamma}\frac{d\Gamma}{dE_u} = 4d_U(d_U^2 - 1)\left(1 - \frac{2E_u}{m_t}\right)^{d_U - 2}\left(\frac{E_u}{m_t}\right)^2. \tag{4.43}$$

Plotting this for values of $d_U = \frac{4}{3}, \frac{5}{3}, 2, \frac{7}{3}, \frac{8}{3}, 3$ using colours blue, cyan, green, yellow, orange and red respectively, we find that,

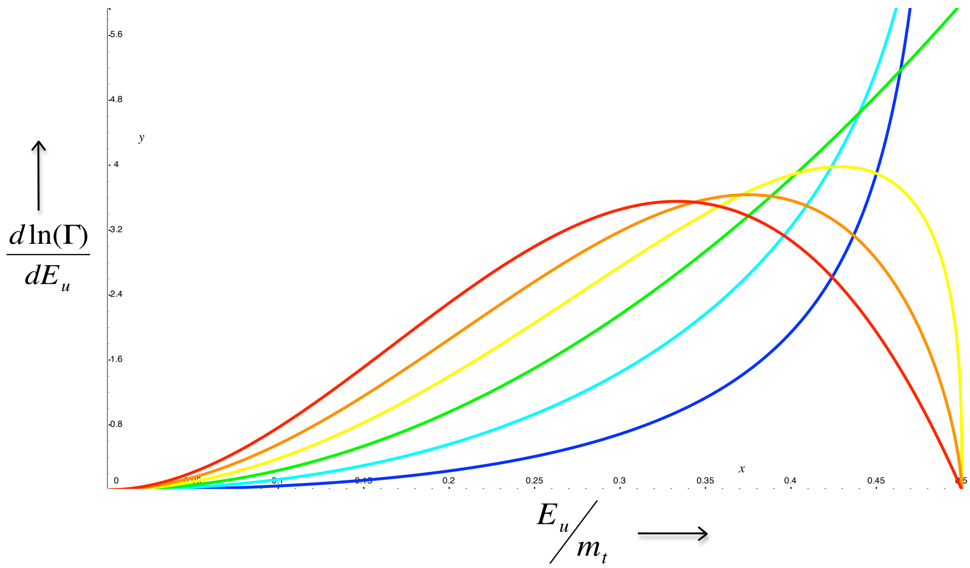



FIGURE 3. An example of missing energy arising from an unparticle process. This is a plot of (4.43) where the lines go through the spectrum from red → blue as the value of $d_U$ increases. If $d_U = 1$, i. e. there is no conformal sector, then $\frac{d \ln \Gamma}{E_u} = 0$ and there would be no missing energy.

Georgi argues that this is an example of how missing energy will arise in the most simple of scattering problems if unparticle behaviour exists.[xviii] It's processes like this that could help us uncover evidence for unparticles.

This concludes our discussion of the most general principles underlying unparticle theory. We will now attempt to expand our discussion by including some extensions to our theory embracing colour charge conservation, a conformal symmetry breaking mechanism and their repercussions on unparticle phenomenology.

[xviii]note that if there was no conformal sector, this graph would simply have a line along $x$-axis where $\frac{d \ln \Gamma}{E_u} = 0$, hence no missing energy.

## 5. Some extensions and potential problems.

### 5.1. Unparticles that carry a colour charge.

In [1] Georgi suggests that it might be interesting to look at unparticle processes which conserve quantum numbers such as the colour charge. This was embraced by Cacciapaglia et. al. in [28] with further expansion and discussion in [29–35]. We will reproduce some of the most interesting findings in this section. A lot of the work from this paper lays the foundations for extensions in unparticle physics, including the Higgs mechanism and the AdS/CFT approach.

Imagine an *unquark* which would carry a colour charge and also couple to a CFT sector. Initially, we introduce some kind of infra-red cutoff because we have to assume that these particles will not effect our low energy phenomenology. In essence, we want to break the conformal symmetry at some scale, $\mu$, so that our continuum spectrum is cut off below this scale. We do this by adjusting the form of (4.10) to,

$$\begin{aligned}\int d^4x\, e^{ipx}\langle 0|T\mathcal{O}(x)\mathcal{O}^\dagger(0)|0\rangle &= i\frac{A_{d_U}}{2\pi}\int_{m^2}^{\infty} dM^2(M^2-\mu^2)^{d_U-2}\frac{1}{p^2-M^2+i\epsilon}\\ &= i\frac{A_{d_U}}{2sin(d\pi)}\frac{1}{(\mu^2-p^2-i\epsilon)^{2-d_U}},\end{aligned} \tag{5.1}$$

which works nicely as the free particle propagator is reproduced as $d\to 1$ and the unparticle propagator (4.10) is reproduced in the limit $\mu\to 0$. We also modify our phase space (taking the discontinuity across the cut) to,

$$d\Phi = A_{d_U}\theta(p^0)\theta(p^2-\mu^2)(p^2-\mu^2)^{d_U-2}. \tag{5.2}$$

So if the propagator (5.1) is correct then we would expect an effective action of the form,

$$S = \frac{2sin(d\pi)}{A_{d_U}}\int \frac{d^4p}{(2\pi)^4}\phi^\dagger(p)(\mu^2-p^2)^{2-d_U}\phi(p). \tag{5.3}$$

The authors of [28] go on to gauge fix this action by introducing a Wilson line between the two unparticle fields at positions $x$ and $y$ and their realisation is that the exponential they introduce includes arbitrarily high powers of the gauge field. This means that there are vertices between two unparticles and *any* number of gauge fields, examples for one and two gauge boson vertices are given in Figure 4.

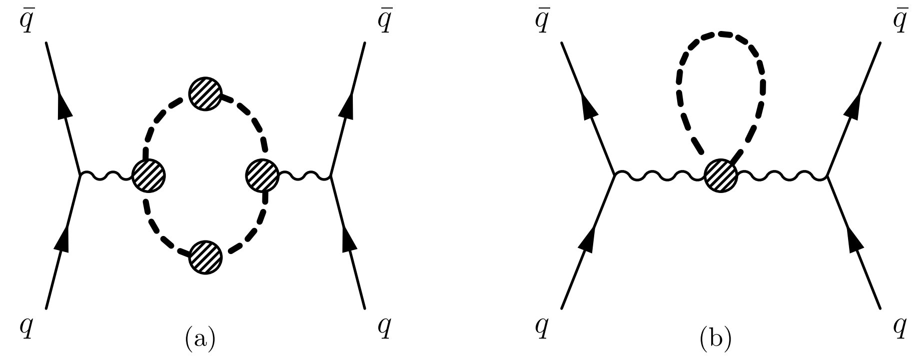



FIGURE 4. Note: time is running from left to right. Scattering of quark anti-quark pairs though a gluon including unparticle vacuum polarisation loops.

Using this approach one can compute the Feynman vertices shown in Figures 4(a) and 4(b), something we will not do here (although one can refer Appendix C for the results), and hence find cross sections for these processes. The result is that these diagrams' contribution to scalar unparticle production is suppressed (to first order) compared to that of normal scalar particle production in the following way,

$$\text{Fig. 4(a) vertex:} \qquad \frac{\sigma_d}{\sigma_1} = \frac{d(2-d)^2(4-d)}{3} \tag{5.4}$$

$$\text{Fig. 4(b) vertex:} \qquad \frac{\sigma_d}{\sigma_1} = \frac{(d-1)(d-2)(d^2-5d+3)}{3} \tag{5.5}$$

$$\sum_{\text{diags}} \qquad \frac{\sigma_d}{\sigma_1} = (2-d) \tag{5.6}$$

This means that if these sort of processes occur at sufficiently high energies, we will observe a suppression in the cross section of normal quark scattering which would supply us with evidence for unparticles. Note that the contribution of both the diagrams combined (5.6) becomes negative if $d > 2$! This is a considerable problem - our cross section becomes non-sensical if the scaling dimension is greater than 2. Even so, do not fear, there are ways of getting around problems like this if we use a field theory in anti de Sitter space as a way of describing our CFT. This is the celebrated AdS/CFT correspondence, [64] and presumably if we can describe a CFT with unparticles and also with a field theory in AdS space then we should be able to describe unparticles with AdS theory. The advantage of this is that problems which are tricky from one standpoint tend to be easier from the other(xix). The authors of [49] explain how the problem of negative crosse sections if $d_U > 2$ can be justified from an AdS approach. We will discuss this a bit further in Sec. 5.5.

### 5.2. **Higgs coupling.**

In this section we discuss a proposal where the usual conformal unparticle sector is coupled to the Higgs sector of the standard model. This was first considered in [37] but a handful of subsequent publications have followed, taking a variety of different approaches: general QFT/electroweak breaking [25] - [38], supersymmetric [24], sector mixing [36] and dark matter [65].

As discussed in Sec. 5.4 the unparticle sector implies a spectrum with no mass gap. We can justify the inclusion of a theory without a mass gap by saying that above a certain energy scale our theory flows away from its conformal fixed point, becoming non-conformal, bringing us back to a normal particle model at high energies. But how do we actually implement this transition and can we incorporate it into the physics that we already know and love? Consider a coupling between the Higgs sector of the standard model and an unparticle sector,

$$\frac{1}{M^{d_{UV}-2}}|H|^2\mathcal{O}_{UV} \tag{5.7}$$

where $\mathcal{O}_{UV}$ (dimension, $d_{UV}$) represents the unparticle fields in the ultraviolet sector (which we have previously called the Banks-Zaks fields, $\mathcal{O}_{BZ}$). Below the coupling scale, $\Lambda$, these fields flow to conformally invariant couplings in the infrared,

$$\frac{C\Lambda^{d_{UV}-d_{IR}}}{M^{d_U-2}}|H|^2\mathcal{O}_{IR} = \kappa|H|^2\mathcal{O}_{IR} \tag{5.8}$$

in complete analogy with (4.2) and (4.3), and assuming, as mentioned previously, that $\mathcal{O}_{IR}$ has dimension $1 < d_U < 2$.

(xix)Typically, further difficulties will arise because finding the AdS dual of a CFT isn't usually a trivial exercise.

We work with the familiar tree level[(xx)] Higgs potential,

$$V_0 = m^2|H|^2 + \lambda|H|^4. \tag{5.9}$$

The effect of ensuring $1 < d_U < 2$ is that when the Higgs acquires a vev (*a lá* normal spontaneous symmetry breaking) the coupling described in (5.8) is imprinted with a scale. The unparticle sector flows away from its conformal fixed point (at a scale $\Lambda_B$) and thus presumably becomes a traditional particle sector where,

$$\Lambda_B^{4-d_U} = \left(\frac{\Lambda}{M}\right)^{d_{UV}-d_U} M^{2-d_U} v^2. \tag{5.10}$$

To be consistent and to maintain an unquestionably conformal theory, $\Lambda > \Lambda_B$. The consequence of this is that if our accelerator energy is not above the scale, $\Lambda_B$, then unparticle behaviour will just mimic that of ordinary particles (because it has flowed from its conformal fixed point and acquired a mass gap). This puts a bound on where we can start observing unparticle physics - i.e. only in the regime, $E$, where $E > \Lambda_B$. The conclusion of [37] is that unparticle phenomena are dependent only on the experiment energy, the electroweak symmetry breaking scale and the standard model/unparticle interaction scale. Significantly, there is no dependence on the scaling dimension of the conformal operator.

Actually we can propose a simple model in which our previous definition of the vacuum matrix element, (4.5), is adjusted to,

$$|\langle 0|\mathcal{O}_U|p\rangle|^2\rho(p^2) = A_{d_U}\theta(p^0)\theta(p^2-\mu)(p^2-\mu)^{d_U-2} \tag{5.11}$$

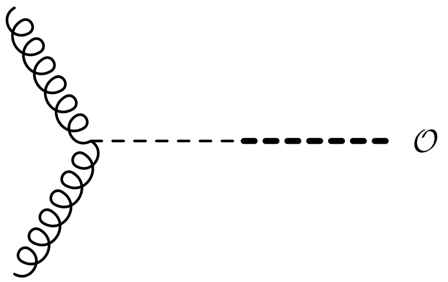
where $\mu$ is the energy at which conformal invariance is broken. Our previously continuous spectrum for unparticles has now acquired a mass gap and the scale invariance is broken.

Furthermore, Delgado et. al. suggest in [36] that electroweak breaking is actually influenced by the unparticle sector. Their argument is that when the Higgs acquires a vev it

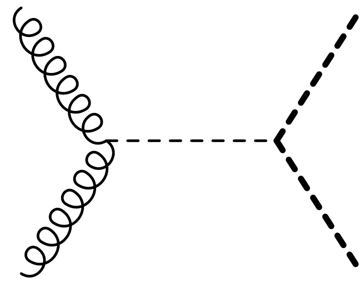
---

[(xx)]Feynman diagrams with no loops.

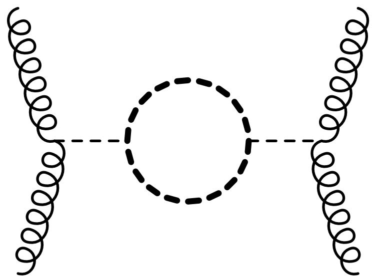
mixes with the unparticle operator $\mathcal{O}_{IR}$ in (5.8) and a tadpole appears for $\mathcal{O}_{IR}$. Therefore it also acquires a vev giving us a term like $v^2\mathcal{O}$. Furthermore, we have this mixing term, something like $vH\mathcal{O}$, which allows unparticle 'production' via an off-shell Higgs boson, as shown in Figure 5. The free $\phi$ fields radiated in 5(b) are just the imaginary part of the full unparticle propagator which we would only expect to see in loops like 5(c).




(a) $gg \to H \to \mathcal{O}$ for an unparticle operator, $\mathcal{O}$



(b) $gg \to H \to \phi^\dagger\phi$ for hidden sector scalars, $\phi$



(c) The unparticle propagator is a $\phi$ loop with its imaginary part containing the process 5(b), $gg \to \phi^\dagger\phi$.

FIGURE 5. Unparticle-Higgs mixing produced from gluon fusion.

To explicitly calculate the effect of this on the Higgs potential a deconstructed version of the unparticle sector is required. This is explained in [26] and [66] and is beyond the realms of this document. To summarie the findings of Delgado et. al. we simply write down the result of the unparticle sector gaining a vev on the Higgs potential,

$$\lambda = -\frac{m^2}{v^2} + \lambda_U(\mu^2)^{2-d_U}v^{2(d_U-2)}. \tag{5.12}$$

The mathematical details of this are not particularly relevant. The point is that we have a mixed spectrum, the spectral function of which has a single pole. This pole clearly corresponds to the Higgs field but its mass is no longer the familiar $\sqrt{2\lambda v^2}$ but actually entirely different, (5.12). The Higgs mass is no longer what we would expect from a normal standard model construction, providing another area in which we could amass evidence to support the unparticle proposition. This has profound implications on unparticle and Higgs phenomenology (see Sec. 5.4) on the whole, but particularly for the Higgs search if its mass is greater than the unparticle mass gap, see Sec. 5.6.2.

### 5.3. **The Unhiggs.**

This little extension to our discussion is an absolutely fascinating one. First discussed in a superb paper by Stancato and Terning, [40], and only braved by a couple of other authors in [41], [42] and [67]. Where in the previous section we explored the consequences of coupling unparticles to a standard model Higgs sector, we now look at the ramifications of making the Higgs itself an unparticle, i.e. a situation where the Higgs emerges from an (approximately) conformal sector. Stancato and Terning show that such an 'Unhiggs' performs similarly to our SM Higgs in that it breaks the electroweak symmetry and can unitarize the $WW$ scattering. We will work through some of the simple proposals of the Unhiggs as a Higgs-like mechanism and describe the conclusions of Stancato et. al.

Lets consider an unparticle field (call it $H$ as it will be our Unhiggs) with scaling dimension $d$ and an infrared cutoff scale, $\mu$, as in Sec. 5.2 and also (5.3). It will have an effective action in momentum space of[(xxi)],

$$S_p = -\int \frac{d^4p}{(2\pi)^4} H^\dagger(-p^2+\mu^2)^{2-d}H. \tag{5.13}$$

Using this we can easily write down the propagator for our unhiggs field as,

$$\Delta(p) = \frac{-i}{(-p^2+\mu^2-i\epsilon)^{2-d}} \tag{5.14}$$

which has the nice property that as $d \to 1$ we recover usual particle behavior.

We want our Unhiggs effect to locally break the electroweak symmetry so we introduce a gauge coupling of our Unhiggs field to the electroweak group. We also cannot allow these new states to be of an arbitrarily small mass otherwise we would have already seen them at experiments like the LEP and Tetravon. So we conclude that the conformal

(xxi)ignoring a normalisation constant.

symmetry breaking of our new approximately scale invariant Higgs sector must be broken just below the weak scale. This threshold actually isn't enough to account for the vev of the Unhiggs so we also include coupling to SM fields which dictates further conformal breaking and a higher vev. For simplicity we do this by introducing a Yukawa coupling to the top quark (as this will be the most significant) with a cutoff scale of $\Lambda$ and ignore contributions from the lighter quarks. As such the action becomes,

$$S = \int d^4x \quad \underbrace{-H^\dagger (D^2 + \mu^2)^{2-d} H}_{\text{Unhiggs-electroweak coupling}} \quad - \quad \underbrace{\lambda_t \bar{t}_R \frac{H^\dagger}{\Lambda^{d-1}} \begin{pmatrix} t \\ b \end{pmatrix}_L}_{\text{Unhiggs-top Yukawa coupling}} \quad + \text{ h.c.} \tag{5.15}$$

When we include loop corrections, these couplings to the standard model will now break the scale invariance and some additional terms arise,

$$S_{\text{loop}} = \int d^4x \frac{C}{\Lambda^{2d-2}} D_\mu H^\dagger D^\mu H - \lambda \left( \frac{H^\dagger H}{\Lambda^{2d-2}} - \frac{V^2}{2} \right)^2. \tag{5.16}$$

We now have a renormalised action which is $S + S_{\text{loop}}$. We have a potential with an instability which will give rise to a non-zero vev for the Unhiggs. This potential is given by,

$$\tilde{V} = -(\mu^2)^{2-d} H^\dagger H - \lambda \left( \frac{H^\dagger H}{\Lambda^{2d-2}} - \frac{V^2}{2} \right)^2. \tag{5.17}$$

If we make the reparameterisations, $\tilde{\mu}^2 = (\mu^2)^{2-d}$ and $\tilde{\Lambda} = \Lambda^{2d-2}$, then we can rewrite this potential as,

$$\tilde{V} = -\frac{\lambda}{\tilde{\Lambda}^2} (H^\dagger H)^2 + \left( \frac{\lambda V^2}{\tilde{\Lambda}} - \tilde{\mu}^2 \right) (H^\dagger H) - \lambda \left( \frac{V^2}{2} \right)^2. \tag{5.18}$$

Now we follow the normal steps for spontaneous symmetry breaking where we find the minimum of this potential by differentiating with respect to the field, $H$,

$$\frac{\partial \tilde{V}}{\partial H} = -\frac{2\lambda}{\tilde{\Lambda}^2} (H^\dagger H) H^\dagger + \left( \frac{\lambda V^2}{\tilde{\Lambda}} - \tilde{\mu}^2 \right) H^\dagger. \tag{5.19}$$

So either $H^\dagger = 0$ or,

$$(H^\dagger H) = -\frac{\tilde{\mu}^2 \tilde{\Lambda}^2}{2\lambda} + \frac{V^2 \tilde{\Lambda}}{2} = \frac{\lambda V^2 \Lambda^{2d-2} - \mu^{4-2d} \Lambda^{4d-4}}{2\lambda}. \tag{5.20}$$

Using the standard form of picking a direction for the symmetry breaking we now define the expectation value of $H$ as,

$$\langle H\rangle = \begin{pmatrix} 0 \\ \left(\frac{\lambda V^2\Lambda^{2d-2}-\mu^{4-2d}\Lambda^{4d-4}}{2\lambda}\right)^{1/2} \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v^d \end{pmatrix}. \tag{5.21}$$

And thus in the usual way we can decompose this into Goldstone and physical modes using generators $T^a$,

$$\begin{aligned} H &= \tfrac{1}{\sqrt{2}}\exp\left(\tfrac{i}{v^d}(T^a\zeta_a)\right)\begin{pmatrix} 0 \\ v^d+\eta \end{pmatrix} \\ &= \langle H\rangle + \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ \eta \end{pmatrix} + \dots \end{aligned} \tag{5.22}$$

and hence $H$ acts in the usual way *vis-a-vis* breaking the electroweak symmetry. Analysis of the pure derivative kinetic term concludes that there can be vertices of two unhiggs' with any number of gauge bosons, as we have already concluded for unparticle fields which carry a colour charge in Sec. 5.1. For two simple examples see Figure 6. Further extensions of this formalism surmise that the Unhiggs is also still able to unitarise $WW$ scattering, as in Figure 7, and so it seems a good contender for the SM Higgs. Another very interesting feature of the Unhiggs is that it weakens the little hierachy problem(xxii) but this is beyond the discussion of this dissertation. Refer to [40] and [42] for further reading.

(xxii)The Higgs mass must be relatively light in order for the electroweak theory to hold.

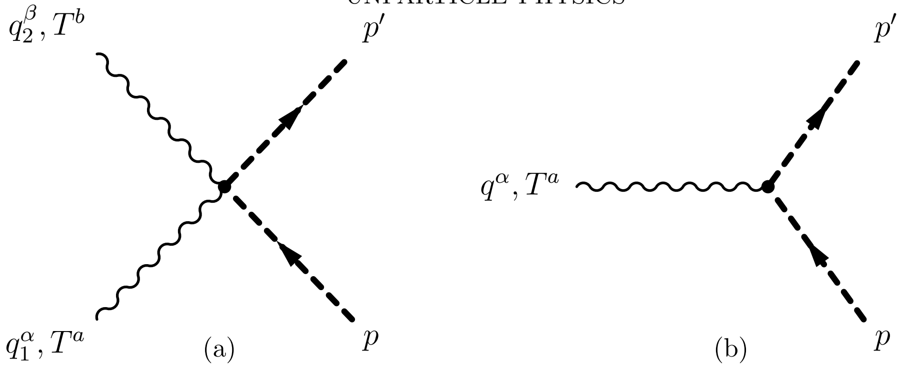



FIGURE 6. Example Feynman diagrams for the two Unhiggs' - two gauge boson interaction 6(a) and the two Unhiggs' - one gauge boson interaction 6(b).

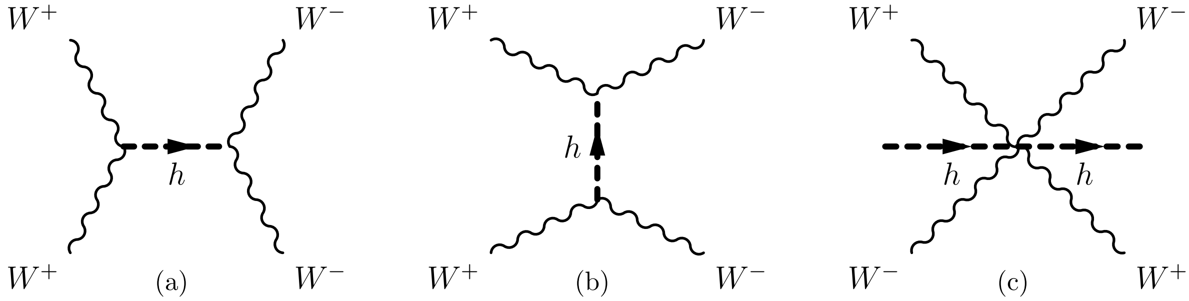



FIGURE 7. $WW$ scattering for two gauge bosons and one Unhiggs vertex, 7(a) and 7(b). The four boson two unhiggs contribution to $WW$ scattering, 7(c).

### 5.4. **The Unparticle spectrum and the hidden valley link.**

The unparticle sector has a continuous mass spectrum - it is conformally invariant, any mass will break the conformal symmetry and therefore its spectrum must be continuous. This is only true of the unparticle operators, not the SM operators to which it couples. There is some discussion of Higgs mechanisms and unparticles above, in Secs. 5.2 and 5.3, whereupon the conformal symmetry can be broken and the unparticle obtains a mass gap and even where the Higgs itself can be considered as approximately conformal invariant and thus itself is an unparticle. In this section we will discuss the link between unparticles and hidden valleys and the consequences on observations at the LHC (some of which are quite striking) - it is a brief review of the work of Strassler in [27] which builds on [43].

So what is a hidden valley scenario?

- A hidden sector weakly coupled to the standard model with some mediator.
- It must contain a mechanism for multiparticle production.
- It requires a mass gap.

As a consequence of these conditions, a hidden valley scenario describes almost any gauge theory with a mass gap where particles produced in the hidden sectors can decay back into SM particles.

Similarities with the unparticle scenario,

- A hidden sector weakly coupled to the standard model with a mediator.
- Conformal dynamics which is a multiparticle production mechanism.
- However, no mass gap. So we will require a Higgs coupling which breaks the conformal symmetry and induces a mass gap in the unparticle spectrum.

If we include a symmetry breaking mechanism in the unparticle model, as we have done above in Sec. 5.2, then we induce a mass gap and the unparticle model becomes a hidden valley scenario. In fact, we only require a mass 'ledge' - where a particle gets stuck because it cannot decay via the hidden sector, thus is forced to decay via the SM sector - for the previous statement to hold. The central concept of the hidden valley scenario is just this, hidden sector particles are forced to decay via the SM model producing new phenomenology. So the dominant question here is, what does this then imply for unparticle phenomenology?

Hidden valley scenarios often produce spectacular phenomenology and the extent of this is vast, making it very hard to predict using just the dimension of the conformal (aka unparticle) operators in theory above the mass gap. We actually require a lot of detail of the hidden sector to determine whether we can predict this phenomenology and whether it will be visible at the LHC. For example, the Higgs-unparticle interplay we have already discussed produces some dramatic Higgs decays (to four leptons, eight or more partons etc.) but the details we cannot know without more specific knowledge of how the conformal symmetry is broken in the hidden sector.

There are many different types of hidden valley scenario which include many classes of models: supersymmetric, extra-dimensional, little-Higgs, compactification, confinement etc. The general structure of these hidden valley models predict some or all of the following:

- Previously unseen neutral light states which then decay to SM modes.
- These new states will have very long lifetimes therefore inducing substantial missing energy.
- Lots of high-multiplicity final states.
- Non-standard decays for the Higgs.

Already there seem to be lots of analogies to the unparticle model we have been discussing, although remember in the unparticle case we require the hidden sector to be conformal which isn't the case for a general hidden valley.

Strassler argues that what we end up with is some very interesting phenomenology where the invisble, hidden sector particles can decay into visible stuff, see Figure 8.

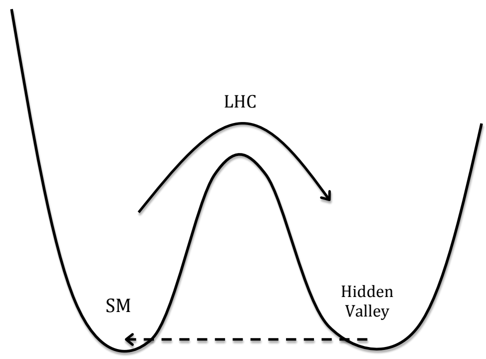



FIGURE 8. A pictorial representation of a simple hidden valley scenario. The hidden valley is obscured from the standard model sector via a barrier which could be overcome with higher energy accelerators. These hidden valleys allow for decays from the hidden sector to the standard model culminating in some spectacular phenomenology. For an unparticle scenario, the hidden valley will be a conformal sector. Decays from the hidden valley to the SM produce new phenomenology.

The ingredients he uses are that of an interacting hidden conformal sector with scalar fields, $\phi$, a hidden gauge boson, $Y$, and a Higgs mechanism similar to the one we discussed in Sec. 5.2 in which processes like that shown in Figure 5 are allowed. The Higgs can produce pairs of the hidden fields $\phi$ (as shown in Figure 5) but also pairs of the gauge boson $Y$ (just as it can decay to WW and ZZ pairs). He proposes some of the following processes which might be of interest.

$$\left.\begin{aligned} &gg \to h \to \hat{\phi}_b\hat{\phi}_b \to (b\bar{b})(b\bar{b}) \\ &gg \to h \to YY \to (l^+l^-)(l^+l^-) \\ &gg \to h \to YY \to (q\bar{q})(l^+l^-) \end{aligned}\right\} \text{Figure 9} \tag{5.23}$$

$$\left.\begin{aligned} &gg \to h \to \hat{\phi}_b\hat{\phi}_b \to (YY)(YY) \to (q\bar{q})(l^+l^-)(\nu\bar{\nu})(l^+l^-) \\ &gg \to h \to \hat{\phi}_b\hat{\phi}_c \to (YY)(b\hat{b}) \to (q\bar{q})(l^+l^-)(b\bar{b}) \\ &gg \to h \to \hat{\phi}_b\hat{\phi}_c \to (YY)(\hat{\phi}_d\hat{\phi}_d) \to (q\bar{q})(l^+l^-)(b\bar{b})(b\bar{b}) \end{aligned}\right\} \text{Figure 10} \tag{5.24}$$

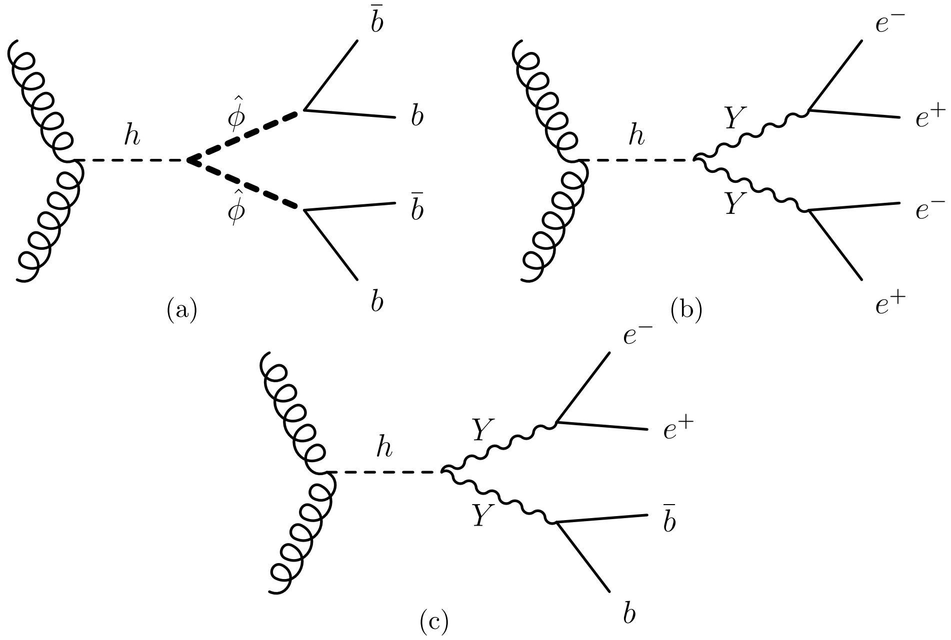



FIGURE 9. The Higgs in general can decay into a pair of hidden sector fields, $\phi$, or into a pair of the hidden sector gauge bosons, $Y$. These will in turn then decay into the lighter quarks and leptons.

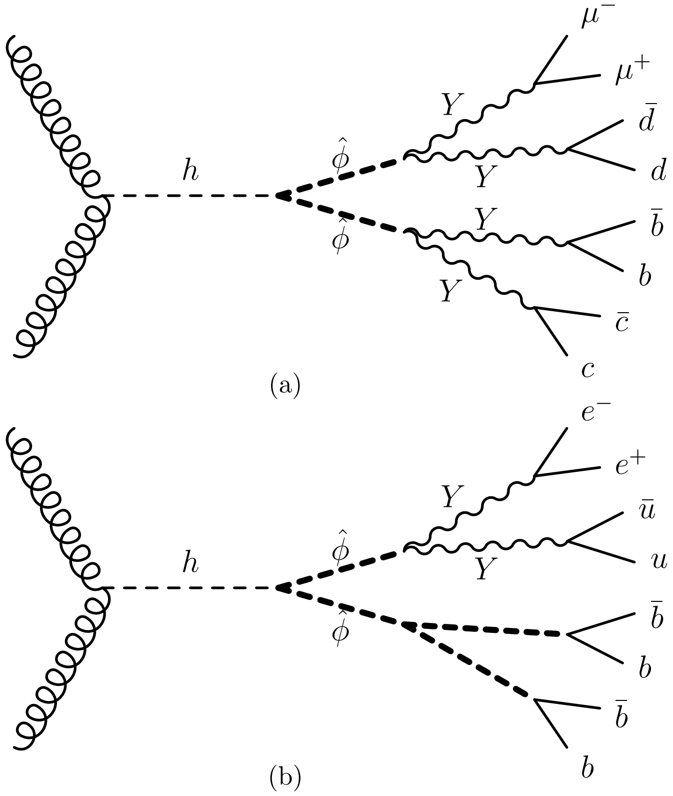



FIGURE 10. In these process the $\phi$ fields themselves decay into pairs of the $Y$ boson which will have sufficient energy to produce some of the heavier quark and lepton states.

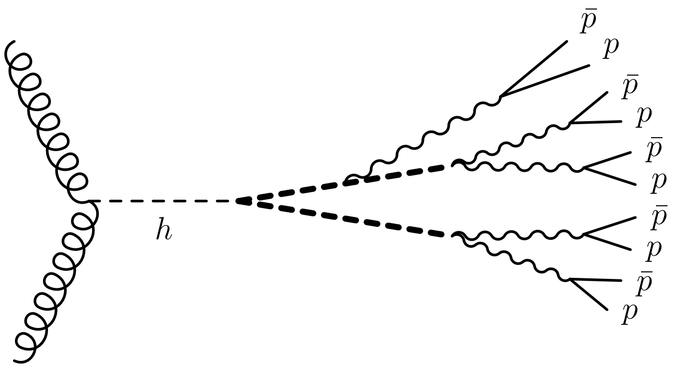



FIGURE 11. The most spectacular type of phenomenology for hidden valley scenarios, where the states $p$ ($\bar{p}$) can represent any standard model fermion (anti-fermion), $u, d, s, c, t, b, e, \mu, \tau, \nu$

### 5.5. **The AdS/CFT approach.**

In 1997 Maldacena showed that some conformal field theories are dual to graviational theories in one extra dimension in anti-de Sitter space, [64]. This AdS/CFT correspondence can provide a useful approach to studying unparticle physics from the perspective of gravitational theories. A few authors examine the compatibility of the AdS illustration and the unparticle illustration of CFTs and go on to use this correspondence to avoid, or give an explanation for, problems that arise in the unparticle approach by using the AdS approach.

For instance, unparticle propagators have been shown, in certain cases (e.g. [60]), to give divergent results for an operator with a scaling dimension greater than 2. As we have already seen in Sec. 5.1 the cross section for colour conserving gauge boson unparticle scattering is negative for a scaling dimension greater than 2. Regarding the former issue, if one uses the AdS approach the divergences can be cancelled by UV-dependent terms in the propagator that don't decouple when the UV cutoff is performed (e.g. [68]). As for the latter problem, the authors of [49] show that we can associate this difficulty with a physical explanation. They state that scalar unparticle processes with $d_U < 2$ (and fermion unparticle processes with $d_U < 5/2$) are insensitive to UV cutoff effects. They show that the holographic boundary conditions of the AdS theory produce the condition that $1 < d_U < 2$ for scalar unparticles (and $3/2 < d_U < 5/2$ for fermionic unparticles), which is why we have restricted our attention to these ranges thus far.

The great advantage of this approach is that problems which are typically very difficult in CFTs can turn out to be much simpler from the AdS stand point, and vice-versa. However, there are considerable challenges to this approach, particularly that not all CFTs have AdS duals, and even if they do finding them is not a trivial task. It is also very hard to exactly define the Lagrangian description of the physics using AdS/CFT. Typically one has to construct the Lagrangian in AdS space without knowing the exact 'look' of the theory being described. And although there are a handful of examples from string theory this often makes it hard to differentiate between actual physical effects and side effects of the construction. As Georgi himself says, "While AdS based models can provide very useful guidance and examples, their ability to describe realistic unparticle physics scenarios is limited." [22].

### 5.6. **Problems and adjustments.**

In this section we will mention a couple of side issues that might be of interest but don't bear much significance on our previous discussions.

#### 5.6.1. *Adjustments to Unparticle propagators.*

Using Mack's work on CFT's, [14], the authors of [23] realise that the unparticle propagators defined in (4.10)-(4.21) need slight adjustments, due to the following condition.

**Condition 2.** *A property of gauge invariant primary operators in CFTs is that they must have scaling dimension, $d \geq j_1 + j_2 + 2 - \delta_{j_1,j_2,0}$, where $(j_1, j_2)$ are the Lorentz spins of the operator.*

As a result, vector operators(xxiii) must have $d \geq 3$ and can only have $d = 3$ if they are a conserved current, i.e. that $\partial_\mu \mathcal{O}^\mu = 0$. So the unparticle propagator needs to be rearranged so that it accounts for the fact that $\partial_\mu \mathcal{O}^\mu \neq 0$ if $d \neq 3$. For example the propagator (4.17) must instead take the form,

$$\int d^D x\, e^{ipx} \langle 0|T\mathcal{O}^\mu(x)\mathcal{O}^\nu(0)|0\rangle = iC(-g^{\mu\nu} + \tfrac{2(d-2)}{d-1}\tfrac{p^\mu p^\nu}{p^2})(-p^2 - i\epsilon)^{d-D/2}. \tag{5.25}$$

As such, scattering problems which involve the vector unparticles have to be adjusted.

#### 5.6.2. *What happens if we can't find the Higgs at the LHC?.*

The standard model needs a mechanism for giving the gauge bosons a mass. The celebrated suggestion of Peter Higgs, of spontaneous symmetry breaking, provides an elegant solution for making the standard model a polished theory of particle physics. But how would the physics community explain, to Europe's funding big wigs, a situation in which we cannot find it? In Sec. 5.2 we saw that a single pole arose from a mixed sector of unparticle fields with the Higgs, which is clearly the Higgs itself with an adjusted mass from that of the usual, $2\lambda v^2$, see (5.12). In addition, the conformal symmetry was broken and the continuous unparticle spectrum obtains a mass gap. Now, if the unparticle mass gap is less than that of the Higgs mass there are profound implications on the Higgs search.

(xxiii)Note that these operators *must be* gauge invariant and primary.

In such a situation the Higgs, in principle, can decay into unparticles, which might be very difficult to explicitly detect. We could find ourselves in a situation where the Higgs doesn't decay through the channels we expect, but instead decays via unparticles into other standard model particles.

# 6. Conclusions

By weakly coupling a scale invariant sector to the standard model, which emerges at energies below the scale $\Lambda$, we produce new physics below this scale. This is the physics of unparticles which describes a fractional number of massless fields. The conformal invariance inherent within unparticle physics restricts the correlation functions and propagators of unparticle stuff to a particular form, as summarised in Appendix D.

A scattering theory for unparticles is constructed by using the conformal partial wave expansion to relate $n$-point functions ($n$ even) for unparticle operators, $\mathcal{O}_n$, to 3-point functions of conformal operators, $\mathcal{O}_k$, with combinations of the $\mathcal{O}_n$'s. The theory becomes analogous to that of a normal QFT with 3-point functions playing the role of scattering amplitudes and the operators $\mathcal{O}_K$ playing the role of particles.

We discuss a collection of processes and phenomenologies that demonstrate how evidence for unparticle behaviour may arise in the future. These are summarised below.

The existence of unparticle stuff is betrayed by missing energy in the most simple of fundamental processes. We have shown an example of this in a conjectural decay of a top quark into an up quark with unparticle emission, Figures 2 and 3.

Under the introduction of a colour charge to the conformal sector, unparticle-gauge boson interactions of any order are allowed. An unparticle-boson vertex can contain an arbitary number of bosons. Furthermore, unparticles with a scaling dimension of $d$ will suppress the $q\bar{q} \to q\bar{q}$ cross section by a factor of $(2-d)$, Figure 4 and (5.6).

Introduction of a conformal breaking Higgs mecahnism provides the Higgs with a different vev to that expected from the standard model, (5.12). In this construction the unparticle field also acquires a vev providing the theory with a mass gap. When this mass gap is less that the mass of the Higgs, then decays are allowed from the Higgs boson into unparticle stuff.

When the Higgs itself is conformally invariant (at least approximately), named the Unhiggs, it still breaks the electroweak symmetry, all be it with a supressed vev, and also unitises the $WW$ scattering in the normal way, Figures 6 and 7.

Unparticle models with mass gaps are examples of hidden valleys. This results in varied and spectacular phenomenology under which gluon fusion can produce any number of standard model fermion-antifermion pairs, Figures 9 - 11.

Studying unparticle theory using an anti de Sitter duality allows complex unparticle issues to be resolved straight-forwardly. It imposes the condition that the scaling dimension of unparticle operators must lie in the range, $1 < d_U < 2$ for scalars, and $3/2 < d_U < 5/2$ for fermions.

It is certainly viable that we will find BSM physics at the LHC as we explore energies in the TeV regime. Unparticle physics provides an explanation for extra degrees of freedom, at these energies, which are conformally invariant. Existence of these extra degrees of freedom will precipitate missing energy in standard model processes as well as triggering new and dramatic phenomenology.

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## Appendix A. Proof that $e^{i\alpha D}P^2e^{-i\alpha D} = e^{2\alpha}P^2$

Using the fact that, $[P_\mu, D] = iP_\mu$ we find,

$$[P^2, D] = 2iP^2 \tag{A.1}$$

$$[P^2, D^2] = 4(iD-1)P^2 \tag{A.2}$$

$$[P^2, D^3] = (6iD^2 - 12D - 8i)P^2 \tag{A.3}$$

Therefore we can rearrange any term so that all the $P^2$'s are on the right hand side,

$$P_\mu D = (i+D)P_\mu \tag{A.4}$$

$$P^2 D = (2i+D)P^2 \tag{A.5}$$

$$P^2 D^2 = (D^2 + 4iD - 4)P^2 \tag{A.6}$$

$$P^2 D^3 = (D^3 + 6iD^2 - 12D - 8i)P^2 \tag{A.7}$$

Expanding the exponentials, working to third order in $\alpha$ we write,

$$\begin{aligned}
e^{i\alpha D}P^2e^{-i\alpha D} &= \left(1+i\alpha D+\frac{(i\alpha D)^2}{2}+\frac{(i\alpha D)^3}{3!}+\dots\right)P^2\left(1-i\alpha D+\frac{(i\alpha D)^2}{2}-\frac{(i\alpha D)^3}{3!}+\dots\right)\\
&= P^2 + (i\alpha)(DP^2 - P^2D) + \frac{(i\alpha)^2}{2}(D^2P^2 + P^2D^2 - 2DP^2D) + \\
&\quad + \frac{(i\alpha)^3}{3!}(D^3P^2 - P^2D^3 + 3DP^2D^2 - 3D^2P^2D) + \dots\\
&= P^2 + i\alpha[D, P^2] + \frac{(i\alpha)^2}{2}(D^2 + 4iD - 4 + D^2 - 4iD - 2D^2)P^2 + \\
&\quad + \frac{(i\alpha)^3}{3!}\big(D^3 - (D^3 + 6iD^2 - 12D - 8i) + 3D(D^2 + 4iD - 4) - 3D^2(2i+D)\big)P^2 + \dots\\
&= P^2 + 2\alpha P^2 + \frac{(2\alpha)^2}{2}P^2 + \frac{(2\alpha)^3}{3!}P^2 + \dots\\
&= \left(1 + 2\alpha + \frac{(2\alpha)^2}{2} + \frac{(2\alpha)^3}{3!} + \dots\right)P^2\\
&= e^{2\alpha}P^2
\end{aligned} \tag{A.8}$$

## Appendix B. Proof of O'Raifeteighs theorem

In the specific context of this dissertation we want to prove that the sum,

$$\sum_{n=0}^{\infty} \frac{i^n}{n!} x^{\nu_1} \ldots x^{\nu_n} \big[P_{\nu_1}, [\ldots [P_{\nu_n}, X] \ldots]\big], \tag{B.1}$$

is finite.

This is a consequence of a general theorem of O'Raifertaigh expressed in [69] and summarised by Mack and Salaam in [10]. Let **L** be any finite dimensional lie algebra which contains the Poincaré algebra, $SO(3,1) \otimes T$. There will be a finite number $n_0$ such that,

$$\big[T, [T, [\ldots T, X] \ldots]\big] = 0 \;\text{ for }\; n \geq n_0 \;\text{ and any }\; X \in \mathbf{L}. \tag{B.2}$$

## Appendix C. Results of gauge boson-unparticle Feynman vertices

These are the results of Cacciapaglia et. al. in [28]. As mentioned in Section 5.1 we have an effective action of the form,

$$S = \frac{2sin(d\pi)}{A_{d_U}} \int \frac{d^4p}{(2\pi)^4} \phi^\dagger(p)(m^2 - p^2)^{2-d_U} \phi(p). \tag{C.1}$$

To add in a global symmetry we can Fourier transform this to position space so that,

$$S = \int d^4x d^4y \phi^\dagger(y) F(x-y) \phi(x), \tag{C.2}$$

and then gauge fix this by introducing a Wilson line,

$$W(x,y) = P \exp\big[-igT^a \int_x^y A^a_\mu dw^\mu, \tag{C.3}$$

using Mandelstam's method. This 'path ordered exponential' includes unlimited powers of the gauge field, so we can have any number of gauge bosons at the vertex. The results for one gauge boson and two gauge bosons are shown below, with Feynman diagrams for these processes given in the main document (Figure 4),

$$ig\Gamma^{a\mu}(p,q) = igT^a \frac{2sin(d\pi)}{A_{d_U}} \frac{2p^\mu + q^\mu}{2p \cdot q + q^2} \big[(m^2 - (p+q)^2)^{2-d} - (m^2 - p^2)^{2-d}\big] \tag{C.4}$$

$$\begin{aligned} g^2\Gamma^{ab\mu\nu}(p,q_1,q_2) = -g^2 \Big[ &(T^aT^b + T^bT^a) g^{\mu\nu} \mathcal{F}(q_1+q_2) \\ &+ T^aT^b \frac{(2p+q_2)^\nu (2p+2q_2+q_1)^\mu}{q_1^2 + 2(p+q_2)\cdot q_1} \big(\mathcal{F}(q_1+q_2) - \mathcal{F}(q_2)\big) \\ &+ T^bT^a \frac{(2p+q_1)^\mu (2p+2q_1+q_2)^\nu}{q_2^2 + 2(p+q_1)\cdot q_2} \big(\mathcal{F}(q_1+q_2) - \mathcal{F}(q_1)\big)\Big] \end{aligned} \tag{C.5}$$

$$\text{where} \quad \mathcal{F}(q) = \frac{2sin(d\pi)}{A_{d_U}} \frac{(m^2 - (p+q)^2)^{2-d_U} - (m^2 - p^2)^{2-d_U}}{q^2 + 2p \cdot q}$$

## Appendix D. Summary of Unparticle propagators

The following table gives a summary of the unparticle propagators *without* including the conditions imposed in Sec. 5.6.1.

Table 2. **Summary of Unparticle propagators**

Scalar: $$\langle 0|T\mathcal{O}_i(x)\mathcal{O}_j(0)|0\rangle = \delta_{ij}A_{d_U}\frac{1}{(-x^2+i\epsilon)^{d_U}} \tag{D.1}$$

$$= \delta_{ij}A_{d_U}\int\frac{d^Dp}{(2\pi)^D}e^{-ipx}\frac{1}{(-p^2-i\epsilon)^{d_U-D/2}} \tag{D.2}$$

Fermion: $$\langle 0|T\mathcal{O}_i(x)\mathcal{O}_j(0)|0\rangle = -\delta_{ij}A_{d_U-1/2}\frac{\not{x}}{(-x^2+i\epsilon)^{d_U}} \tag{D.3}$$

$$= -\delta_{ij}A_{d_U-1/2}\int\frac{d^Dp}{(2\pi)^D}e^{-ipx}\not{p}(-p^2-i\epsilon)^{d_U-\frac{D+1}{2}} \tag{D.4}$$

Vector: $$\langle 0|T\mathcal{O}_i^{\mu}(x)\mathcal{O}_j^{\nu}(0)|0\rangle = \delta_{ij}A_{d_U}\frac{1}{(-x^2+i\epsilon)^{d_U}}(g^{\mu\nu}-\frac{2x^\mu x^\nu}{x^2}) \tag{D.5}$$

$$= \delta_{ij}A_{d_U}\int\frac{d^Dp}{(2\pi)^D}e^{-ipx}(-g^{\mu\nu}+\frac{p^\mu p^\nu}{p^2}) \times(-p^2-i\epsilon)^{d_U-D/2} \tag{D.6}$$

Tensor: $$\langle 0|T\mathcal{O}_i^{\mu\nu}(x)\mathcal{O}_j^{\rho\sigma}(0)|0\rangle = \delta_{ij}A_{d_U}\frac{1}{(-x^2+i\epsilon)^{d_U}} \times\Big((K^{\mu\rho}(x)K^{\nu\sigma}(x)-\tfrac{1}{4}g_{\mu\nu}g_{\rho\sigma})\pm(\mu\leftrightarrow\nu)\Big) \tag{D.7}$$

$$= \delta_{ij}A_{d_U}\int\frac{d^Dp}{(2\pi)^D}e^{-ipx} \times\Big((K^{\mu\rho}(p)K^{\nu\sigma}(p)-\tfrac{1}{4}g_{\mu\nu}g_{\rho\sigma})\pm(\mu\leftrightarrow\nu)\Big) \times(-p^2-i\epsilon)^{d_U-D/2} \tag{D.8}$$

where $$K^{\mu\nu}(x)=g^{\mu\nu}-\frac{x^\mu x^\nu}{x^2}$$