THE ESSENCE OF AN ACCELERATING UNIVERSE

by

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1 Abstract

The aim of this paper is to review various mechanism and phenomenons that could lead to the current epoch of cosmic acceleration. Our approach is for most part based on the assumption of a homogeneous and isotropic universe, and the validity of the equations of General Relativity. From this viewpoint, cosmic acceleration can only be explained if the universe is currently dominated by an elusive, very smooth form of matter with self repulsive gravity called dark energy. To this end, the standard model of cosmology is developed from first principles. Situations where the model needs to modified in order to be consistent with the observations are highlighted. The development includes discussion and subsequent resolution of the horizon and flatness problem by patching an early epoch of inflation to the standard Big Bang model. Similarities between the two accelerating epochs which live at opposite ends of cosmic history have been explored. The discussion then moves on to represent a brief case for dark matter which if taken together with the CMB results indicating a topologically flat universe, gives a compelling case for a smooth, gravitationally repulsive component whose mass density dominates the universe today at large scales. The discussion of dark energy then takes off with supernova results which offers the most sensitive probe for characterizing dark energy. The leading models of dark energy are presented. The simplest candidate, a true cosmological constant, proves to be the most puzzling candidate as fundamental physics has yet to account for its extremely small value without resorting to extreme fine tunning. The correspond cosmological constant problem that constitutes one the most important unsolved problem in theoretical physics has been extensively discussed. Dynamical dark energy models, initially proposed to dodge this outstanding problem are then presented in detail. In particular Quintessence models in which a canonical scalar field by virtue of slow roll motion along its potential behaves like an apparent cosmological constant are extensively discussed. The recurring problem of fine tunning of parameters that plagues these models is highlighted. Particular attention is given to the tracker models of quintessence which funnel a whole range of initial conditions into a common cosmic history; thereby making cosmology insensitive to initial conditions. The search for a dynamical resolution of the coincidence problem associated with quintessence models takes us to the realm of K-essence; a scalar field whose action contains non canonical kinetic energy terms. Constraints on new long range forces associated with ultra light scalar fields, inspire us to consider the Psudeo- Nambu-Goldstone-Boson-model. Quintessential inflation which provide a bridge between a very early and very late universe is mentioned for its theoretical appeal. We conclude with possible explanation of cosmic acceleration that do not require postulating the existence of dark energy such as modified gravity or the possibility of living in a vast local void.

2 Introduction

An accelerating universe[1, 2] paints an image of an universe that is much more mysterious than ever thought possible. Our basic notion of gravity as an attractive force is manifestly wrong as demonstrated in the biggest laboratory of them all; the universe itself. We are led to believe that on large enough scales the universe is dominated by a component dubbed dark energy whose characteristics goes beyond whatever notion we had previously of matter. In particular we have to envision that space is filled with an exotic type of matter, one which has a repulsive gravitational behavior and acts like a cosmological constant component. As such the implication of cosmic acceleration for fundamental physics is immense; whatever the agent responsible for cosmic acceleration may be, its origin and properties has to be incorporated into the standard model of particle physics and accounted for. Conventionally, the path between particle physics and cosmology used to cross only in the early universe, providing much needed sources of energy density during that period, leading to processes like inflation, baryogenesis, phase transitions etc... Cosmic acceleration, however, forces us to once again look to fundamental physics for answers, this time for phenomenon arising in the late universe. It is safe to say that theorists have shown no lack of enthusiasm for suggesting possible models (some related to and some completely unrelated to particle physics ideas) in this regard as is evident by the enormous variety of models proposed.

The simplest and yet most puzzling explanation for cosmic acceleration involves the addition of a cosmological constant to Einstein's equation. But the value suggested from observation is , in units where $G = c = \hbar = 1$, of order 10^{-120}. From the quantum theory perspective, however, this value, which is conventionally associated with the energy of the quantum vacuum, should be of order unity, or possibly zero, should some symmetry or dynamical mechanism operate. Unfortunately, up to this date, no explanation exists as to why a true cosmological constant should have such a tiny value. As a consequence of this outstanding conundrum, theorist have proposed alternative ideas that dodge this question by postulating that the vacuum itself does not gravitate (by virtue of some unspecified symmetry or dynamical mechanism), but that instead the effect of a cosmological constant like component has been mimicked by a dynamical scalar field that fills the space almost uniformly. [3, 4, 6, 7].

An incomplete list includes: Quintessence models [8] which employs an evolving canonical scalar field with a potential; scalar field models where the small mass of the quintessence field is protected by an approximate global symmetry by making the field a Pseudo-Nambu-Goldstone Boson [15]; a scalar field with a non-canonical kinetic term, known as K-essence [16, 17, 18]. Although these models do away with need for a cosmological constant, but as we shall see in order to be viable dark energy candidates they lead to the introduction of unusually tiny parameters and/or finely-tuned initial conditions. Furthermore, these models also give rise to a new scandal which has come to be known as the coincidence problem: why is the Universe transitioning from deceleration to acceleration so recently?. None of the current models answer this question fully, although some which make use of the scaling properties [10] and tracker nature [11, 12, 13, 14] address it.

Alternatively, one can uphold the familiar notion of matter and instead propose that cosmic acceleration is caused not by an exotic new type of matter, but by the change in the form of gravitational force on cosmological scales[24] or even the existence of extra dimensions[26]. The former approach implies that Einsteins semi-classical theory of general relativity not only breaks down at very short distance; as expected, but also on very long distances; an odd yet interesting point of view. In addition one might also imagine that the accelerated expansion is due to a violation of statistical homogeneity, a fundamental tenet of cosmology.[5] Confirmation of any one of these ideas would have a profound impact on physics. Determining the nature of dark energy is widely regarded as one of the most important problems in physics and astronomy.

This paper is organized as follows. The basic elements of the standard FLRW cosmology are presented in Sec.(3). The need to move beyond this standard model is motivated in Sec.(4), where various shortfalls of the model are discussed. In Sec.(5), we present specific postulates that take us beyond the standard model of both cosmology and particle physics. Particular attention is vested on concepts of inflation, dark matter and dark energy. Sec.(6) is devoted to the discussion of various models of dark energy. Specifically cosmological constant along with various viable scalar field models are extensively discussed. Alternative ideas wherein an accelerated expansion can be realized without recourse to dark energy such as modified gravity or the possibility of living in a local void are very briefly discussed in Sec.(7). We conclude in the final section.

Throughout the paper we adopt natural units $c = \hbar = 1$ and have a metric signature $(-,+,+,+)$. We denote the Planck mass as $m_{\rm pl} = G^{-1/2} = 1.22 \times 10^{19}\,{\rm GeV}$ and the reduced Planck mass as $M_{pl} = (8\pi G)^{-1/2} = 2.44 \times 10^{18}\,{\rm GeV}$. Here G is Newton's gravitational constant. We define $\kappa^2 = 8\pi G = 8\pi m_{\rm pl}^{-2} = M_{\rm pl}^{-2}$.

3 Standard Model Of Cosmology

Modern cosmology is based on the cosmological principle; the idea that our observational place in the universe is in no way special. More formally it states that on sufficiently large scales the observational properties of the universe is the same for all observers. Upholding this principle places two constraints on the structure of the universe on large enough scales; namely isotropy and homogeneity. The latter implies that the same observational evidence is available irrespective of ones location in the universe while the former suggests that the same evidence is obtained by looking in any direction[1]. This principle is so far in accord with the observations of the large scale distribution of galaxies and the near uniformity in the power spectrum of the Cosmic Microwave Background(CMB)radiation. [2]Time, on the other hand, does not enjoy such symmetries as entropy gives it a specific direction; the arrow of time. This was clearly demonstrated

[1] Although the concept are closely related there is a subtle difference. It is possible to construct universes that are homogeneous but anisotropic; the reverse, however, is not possible.
[2] Although the case for homogeneity is, as of yet, not as strong as isotropy which has been verified to within 1 part in 10000.(See Sec.(7.2)).

by the original discovery of the expansion of the universe by Hubble(1922) which could only be interpreted if the in universe were once in a much denser, hotter state. Furthermore the observed isotropy of CMB can only be explained if the universe was once in a much more uniform state with its constituents in thermal equilibrium with each other. In fact a hot early era is essential for the production of light element abundances via the process of nucleosynthesis; with observations in great agreement with theoretically predicted abundances. Together, The Hubble expansion, CMB isotropy and Big Bang Nucleosynthesis(BNN) form the three pillars of the standard Big Bang cosmology. In this section we devolve this model.

The metric that captures the characteristic geometry and casual structure of the spacetime corresponding to an expanding, isotropic and homogeneous universe is the Friedmann-Lemaitre-Robertson-Walker(FLRW) metric whose invariant line element ds^2 is given by [52]

$$ds^2 = -dt^2 + a^2(t)[\frac{dr^2}{1 - Kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)], \tag{1}$$

Here, t is the cosmic time and the expansion of the universe is encoded by the introduction of the cosmic scale factor $a(t)$ which measure the relative expansion of the universe(by convention $a = 1$ today). The coordinates r, θ and ϕ are known as co-moving coordinates and define a frame in which objects that are moving along with the Hubble flow come to rest. Only observers moving on co-moving frames will see an isotropic universe. The geometrical shape of the spatial 3-dimensional space of this universe is quantified by the constant K. In particular constant K can be normalized such that $K = 0$ corresponds to a spatially flat euclidean universe, while $K > 0$ and $K < 0$ correspond to a positively curved(spherical) or a negatively curved(saddle like) global topology, respectively. The co-moving distance between two points on a space-time grid generated by the FLRW metric remains constant with time while the physical distance which is proportional to $a(t)$ increases as the universe expands. We take this to be the defining property of a. In fact it is by the virtue of this relation that the universe preserves its homogeneity as it expands, thus keeping the cosmological principle a valid assumption in an expanding universe. If a was a constant and K vanished this would represent the flat spacetime of special theory of relativity.

In Standard Cosmology, the dynamics of the universe are governed by Einstein theory of General Relativity(GR). According to (GR), gravity is a geometrical property of spacetime. Einstein field equations relate this geometry to the mass/energy content of the universe as follows [52]

$$G^{\mu\nu} \equiv R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R = 8\pi G T^{\mu\nu}, \tag{2}$$

where G^{μ}_{ν} defined above is the Einstein tensor, R^{μ}_{ν} is the Ricci curvature tensor which depends on the metric of the spacetime and its derivatives, R is the Ricci scalar defined as the trace of Ricci tensor and T^{μ}_{ν} is the energy-momentum tensor; the source of curvature in spacetime. In the FLRW background (1) the curvature terms are given by

[58]

$$R^0_0 = \frac{3\ddot{a}}{a}\,, \tag{3}$$

$$R^i_j = \left(\frac{\ddot{a}}{a} + \frac{2\dot{a}^2}{a^2} + \frac{2K}{a^2}\right)\delta^i_j\,, \tag{4}$$

$$R = 6\left(\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{K}{a^2}\right)\,, \tag{5}$$

where a dot denotes a derivative with respect to t. In cosmology, the energy-momentum tensor of the material content is typically modeled as a perfect fluid, specified completely by its rest frame energy density ρ and isotropic pressure p. The corresponding energy-momentum tensor in arbitrary coordinate is given by [52]

$$T^{\mu\nu} = (\rho + p)u^\mu u^\nu - pg^{\mu\nu}. \tag{6}$$

where u^μ is the four velocity of the fluid. The energy-momentum tensor of the material content in a FLRW model is naturally restricted to be isotropic as seen by an observer at rest in the comoving frame. In the comoving frame one has $u^\mu = (1, 0, 0, 0)$ and so (112) reduces to $T^\mu_\nu = Diag(-\rho, p, p, p)$ as a consequence of the symmetry in the model. According to Einstein theory of general relativity then, gravitational field couples both to mass-energy [3] and to pressure. This contrast with the Newtonian theory of gravitation where the coupling is to mass alone. As we shall see it is precisely because of this feature that the concept of accelerated expansion becomes meaningful in the framework of GR. Applying the field equations of GR (2) to the FLRW metric (1), one obtains the following equations for time-time and space-space components, respectively

$$H^2 \equiv (\frac{\dot{a}}{a})^2 = \frac{8\pi G\rho}{3} - \frac{K}{a^2}, \tag{7}$$

$$\dot{H} = -4\pi G(p + \rho) + \frac{K}{a^2}, \tag{8}$$

where H as defined is the Hubble parameter; the constant in the linear expansion law relating velocity of distant galaxies to their separation, and ρ and p denote the total energy density and pressure of all the species present in the universe at a given epoch. The present value of the Hubble parameter is conventionally expressed as $H_0 = 100h$ $km/s/Mpc$, where $h \approx 0.7$ is the dimensionless Hubble parameter containing the error associated with a particular measurement. Here and below, a subscript 0 on a parameter denotes its value at the present epoch. Cosmological quantities are normally expressed with explicit dependence on h, highlighting the uncertainty associated with them. It is worth mentioning that Hubble original value for the expansion rate was $H_0 \approx 570 km/s/Mpc$ which shows not only the huge advances made in observation cosmology over past century, but also the difficulties in measuring cosmological parameters. The difficulty traces back to the oldest and most fundamental problem in astronomy; the distance scale. One can look at distant objects but never touch them.

[3] where mass and energy are interchangeable thanks to Einstein $E = mc^2$

In addition, because the Einstein tensor satisfies the Bianchi identities ($\nabla_\mu G^\mu_\nu = 0$) , it immediately follows from Eq.(2) that $\nabla_\mu T^\mu_\nu = 0$ which in turn implies the conservation of energy-momentum tensor. The corresponding continuity equation is

$$\dot{\rho} + 3H(\rho + p) = 0. \tag{9}$$

Note that this is just the first law of thermodynamics in disguise (ie $d(\rho a^3) = -pd(a^3)$); applied here to the closed system corresponding to the expanding universe.
Another measure of the expansion of the universe related to the stretching of all wavelengths of light as they journey through vast regions of expanding space, is the redshift parameter z defined as

$$1 + z = \frac{\lambda_0}{\lambda} = \frac{a_0}{a}, \tag{10}$$

where on the last equality we have used the defining property of a. The redshift is useful label for epoch in the early universe when z exceeds unity. It is simply the fractional amount by which the wavelength of a photon has been stretched by the expansion between the time the photon is emitted and the time it is received.
Equations 7 and 8 can be combined by eliminating the K/a^2 curvature term, resulting in a differential equation for the scale factor $a(t)$

$$(\frac{\ddot{a}}{a})^2 = -\frac{4\pi G}{3}(\rho_{tot} + 3p_{tot}). \tag{11}$$

Needless to say that only two equations form the set of Eqs.(11),(7) and (8) are in fact independent. It is is more convenient to rewrite this last equation as

$$\Omega + \Omega_K = 1 \tag{12}$$

Where the normalized dimensionless density parameters, Ω and Ω_K, are defined as

$$\Omega \equiv \frac{\rho}{\rho_c} = \frac{8\pi G\rho}{3H^2}, \tag{13}$$

$$\Omega_K = \frac{-K}{(aH)^2}, \tag{14}$$

The quantity ρ_c defined in Eq.(14) by which densities are normalized is called the critical density. The present value of the critical energy density is[60]

$$\rho_c = 3H_0^2 8\pi G = 8.10 \times 10^{-47} h^2 GeV^4, \tag{15}$$

$h \approx .7$, gives a value $\rho_{0c} \approx (.003ev)^4$. As can be seen from Eq.(7), it corresponds to the energy density of a flat universe for a given value of H. Accordingly all cosmic energy densities are often expressed as $\Omega_i = \frac{\rho_i}{\rho_c}$ where i in principle stands for non relativistic matter, radiation or dark energy. In this paper, we Specifically use subtexts r for radiation and m for the non relativistic component. As for the dark energy component, symbols vary according to the specific model under consideration. From Eq.(12), it immediately becomes obvious the matter distribution determines the spatial geometry of the universe. In particular

$$\Omega > 1 \;\text{ or }\; \rho > \rho_c \;\; \rightarrow K = +1\,, \tag{16}$$

$$\Omega = 1 \;\text{ or }\; \rho = \rho_c \;\; \rightarrow K = 0\,, \tag{17}$$

$$\Omega < 1 \;\text{ or }\; \rho < \rho_c \;\; \rightarrow K = -1\,. \tag{18}$$

CMB anisotropy measurement [64] strongly strongly suggests that universe is currently nearly flat with ($\Omega_0 \approx 1$). In fact this turns out to be also the theoretically favored case from the view point of inflation as we shall discuss in Sec.(5.1).

The evolution of an expanding universe depends on the properties of its material content. We now know that this includes non relativistic matter, radiation and the elusive dark energy(see Sec.(5.3)). Each component, i, can be classified according to its specific equation of state w_i,

$$w_i = \frac{p_i}{\rho_i} \tag{19}$$

i, in principle, could stand for non relativistic matter, radiation or dark energy. In this paper, we specifically use subtexts b for baryonic matter, r for radiation and m for the non relativistic component. As for the dark energy component, we use abbreviation DE but in general symbols vary according to the specific model under consideration. Non-relativistic matter which includes both non baryonic dark matter(see Sec.(5.2)) and baryonic matter is characterized by the fact that its pressure is negligible compared to its energy density($w_m \approx 0$). Equivalently they can be defined as particles able to cluster by falling into gravitational potential wells, giving rise to the observed structure in the cosmos. Radiation may be used to describe either actual electromagnetic radiation, or massive particles moving at relativistic velocities sufficiently close to the speed of light; usually it refers to photons and neutrinos. From statistical mechanics we know that $w_r = 1/3$. Also, the cosmological constant is defined as a component with an equation of state of w_Λ (see Sec.(6.1)).
In order to realize the the implication of each specific component, lets consider a universe that is dominated by a single perfect fluid with an effective equation of state w. If w is a constant one can find the evolution of the Hubble constant H, the scale factor a and the corresponding energy density ρ. For the observationally and theoretically favored case of a flat universe($K = 0$), these are obtained by solving Einstein Eqs. (7) and (8). The results are

$$H(t) = \frac{2}{3(1+w)(t-t_0)} \tag{20}$$

$$a(t) \propto (t-t_0)^{\frac{2}{3(1+w)}}, \tag{21}$$

$$\rho \propto a^{-3(1+w)}, \tag{22}$$

where t_0 is a constant. It follows that in the radiation dominated epoch $\rho \propto a^{-4}$ and $a(t) \propto (t-t_0)^{1/2}$ while in matter dominated epoch one has $a(t) \propto (t-t_0)^{2/3}$ $\rho \propto a^{-3}$. The condition $\ddot{a} < 0$ is satisfied in both epochs as expected with gravity slowing down the expansion. From Eq.(22), one can deduce cosmic acceleration $\ddot{a} > 0$ is possible only if the following relation is satisfied

$$w < -1/3\,. \tag{23}$$

This condition on w is the fundamental criteria that the total material content of the universe must satisfy in order to give rise to cosmic acceleration. We note that Eq.(22) is not valid if $w = -1$. In this case as a result of the constancy of the Hubble parameter

(Eq.(8) with $K = 0$ and $p = -\rho$), one obtains $\rho = constant$ and $a(t) \propto \exp(Ht)$ form Eq.(9) and (7), respectively. For a flat Universe with matter and cosmological constant, the general solution, which approaches the two cases at early and late times, is $a(t) = (\Omega_m/\Omega_{DE})^{1/3}(\sinh[3\sqrt{\Omega_{DE}}H_0t/2])^{2/3}$ and best describes the current phase.

4 Shortfall's of The Standard Model

The standard Big Bang cosmology suffers from a series of drawbacks which basically stems from the fact that the model is limited to those epochs when the universe is cool enough that the underlying physical processes are well established. Therefore it does not attempt to address the state of the Universe at earlier, hotter, times [33]. Specifically, some features of the model exhibit fine tunning problem; a general class of problems in which no mechanism can be provided to actually account for the required high precision adjustment of initial values of parameters. In addition the model fails to take into account the observed homogeneity and isotropy in the universe. These and other type of problems can only be tackled by borrowing from idea form the realm of high energy physics. This merge of particle physics and cosmology allows one to get insight into the underlying issue. It should be said that this approach does by no means resolves all the outstanding problems. After all there exist limitation on our understanding of very high energy physics and the ultimate goal of unifying all four fundamental interaction has not yet been achieved. In this section we briefly discuss the prominent short comings of the model.

4.1 Singularity Problem

Extrapolation of the expansion of the Universe backwards in time using general relativity yields an infinite density and temperature at a finite time in the past [58]. This singularity signals the breakdown of general relativity which is after all an effective field theory describing a low energy limit of a more fundamental theory. The scale at which this breakdown breakdown occurs is expected to be no larger than the Planck scale. To illustrate the significance of this scales one can imagine performing a measurement with desired resolution Δx, which entails using light with wavelength $\lambda < \Delta x$. In the process the energy $E = hc/\lambda$ [4] of a photon get concentrated in a region with dimension Δx. The Heisenberg uncertainty principle of quantum mechanics implies that this energy is in fact limited within its own Schwarzschild radius If Δx is smaller than the Planck length $\ell_{pl} = \sqrt{(hG/c^3)} \approx 10^{-35}$m (Here we have restored constant h and G to highlight the fact that issue lies in the absence of unified theory of quantum mechanics and gravity.) . Hence If one attempts to probe space with a resolution better than ℓ_{pl}, one can creates a black hole with size larger than ℓ_{pl} thus swallowing the photon. This clearly shows that the concept of spacetime (as we know it) looses its meaning beyond the Planck scale. This is of course the most extreme example of the uncertainty principle but it highlights the fact that only a quantum gravity theory unifying general relativity with quantum mechanics will allow us to better understand the nature of spacetime

[4] Here we have restored constant h to highlight the fact that issue lies in the absence of unified theory of quantum mechanics and gravity.

at this scale. At present, our firm understanding of the elementary particles and their interactions only extends to energies of the order of 100Gev (and very soon 1Tev as underway in LHC at CERN), which corresponds to a time of the order of 10^{-11} seconds or so after the Big Bang while the Planck epoch corresponds to $t_{pl} = 10^{-43}$s. Although advances are constantly being made toward pushing the boundary, but it seems unlikely that we could ever test those theories in laboratories. An example is theory of inflation operating at energy scales near the Planck scale ($m_{pl} \approx 10^{19}$Gev) as discussed in Sec.(5.1) where the predictions agree beautifully with observations. But with our current ignorance of the underlying physics this theory suffers from the same fine tunning of initial conditions that plagues all other theories trying address the very early universe. Although there has been interesting suggestions of possible origin of spacetime such as vacuum polarization[68](see Sec.(5.1)), however a detailed understanding of the origin of sapcetime, might just be to big of the problem to ever be adequately addressed.

4.2 Flatness Problem

As the name suggests the flatness problem arise by the observation[42] that our universe is very nearly flat. In order to motivate the root cause of this problem, we begin by pointing out an interesting implication of the relation introduced in Eq.(14), namely $\Omega_K = -K/(aH)^2$. One can immediately see that if the expansion of the universe is decelerating ($\ddot{a} < 0$), as envisioned in the Standard Big Bang model (where the the material content is dominated by either matter or radiation, both of which slow down the expansion), then Ω_K continues to increase because the term $aH = \dot{a}$ decreases, apart from the case that $K = 0$ from the very beginning. Hence the observational evidence that we are living in a universe that is very close to flat on cosmological scales[42] requires a very precise adjustment of initial value of Ω_K in the very early universe. This is our first encounter with a fine tunning problem which in this case is referred to as the flatness problem. More generally, the flatness problem arises because the theory fails to provide mechanism necessary for the universe to evolve into a flat universe without invoking fine tunning of initial condition. To see this more concretely let $|\Omega - 1|$ be the deviation from critical density in the matter content of the universe. Eq.(12) together with Eq.(22), it is easy to see that: $(\Omega - 1) \propto t$ during the radiation-dominated era and $(\Omega - 1) \propto t^{2/3}$ during the matter-dominated era. As these deviations grow with time in both epochs, one concludes that the flat universe is an unstable solution of possible sets of solutions and thus not likely to have occurred just by mere chance. In order to appreciate the amount of fine tunning required, consider the universe at Planck time ($t_{pl} = 10^{-43}$s). If one demands to get a flat universe today $t = 13.9$Gyr; then the matter content had to have been tunned to an incredible 59 decimal places. We note that Eq.(12) suggests that a phase of cosmic acceleration ($\ddot{a} > 0$) in the past reduces Ω_K. By looking at Eq.(14) One can suspect where the resolution of this problem lies as it demonstrates that Ω_K can decrease provided we have an early epoch of cosmic acceleration ($\ddot{a} > 0$). Equivalently, one can also argue that the flatness problem arises because of the assumption of adiabatic expansion in standard Big Bang model. (i.e., That the entropy in a comoving volume is conserved). It is possible, therefore, that the problem could be resolved if the cosmic expansion was non-adiabatic for some finite time interval during the early history of the universe. Clearly one must look beyond the standard model to get a satisfactory explanation for the flatness problem.

4.3 Horizon Problem

Another hurdle is the so called horizon problem which stems from the existence of particle horizons in FLRW cosmologies. On celestial sphere, we see distant galaxies that according to conventional Big Bang cosmology have not been in causal contact with each other since the singular start of expansion. Why then do apparently causally disconnected parts of space look so similar? After all, information can be passed on only via causal interaction. More technically one should be very surprise to find similarities between regions that are said to be outside each others particle horizon, quantified by the comoving distance the light could have traveled since t=0:

$$\eta = \int_0^{t_0} \frac{d\acute{t}}{a(\acute{t})} \tag{24}$$

Distances separated by larger than this value could have never communicated in the past. This is to be differentiated with the Hubble radius, H_0^{-1}, whose value limits the the current size of the causally connected patches of universes.[5]. In fact the observed uniformity of the CMB radiation implies that the observed universe had become uniform in temperature to an incredible 1 part in 100000 at around $z \approx 1100$[34]. A simple calculation in standard big bang cosmology reveals that the uniformity could be established so quickly only if information could propagate at speeds of more that 100 times the speed of light; a proposition clearly contradicting the law of special relativity. Upholding laws of special relativity forces us again to look to new ideas to resolve this curious issue. [6] Another aspect of horizon problem is related to the fact that the universe is not perfectly homogeneous, but rather it has structure on large scales. These structures presumably formed by the growth of primordial density fluctuation via gravitational instability. As the corresponding perturbation modes enter the horizon only recently, how come they the corresponding structure look so similar to each other? The standard FLRW cosmology does not provide any mechanism for generating such scale invariant initial perturbation and so one has to impose ad-hoc initial conditions for equations of motion in order to find solutions that are compatible with observations. Specifically, theory of structure formation best fits the observations when the initial primordial perturbation are scale invariant, Gaussian and adiabatic.

5 Beyond The Standard Model

Resolution of these drawbacks within the framework of GR forces us to move beyond our familiar notion of both gravity and matter. Ideas from particle physics guide us through this path and make predictions that fit observations beautifully. However, there remains many outstanding problems. For one, in order make sense of observations we need to propose the existence of a truly vast Dark sector which we have so far only been

[5]Since $a = 1$ now, comoving Hubble radius is equivalent to Hubble radius at present

[6]A possible solution though less accepted is that the speed of light has changed over time.[43]

able to detect through its gravitational effects. The astonishing revelation is that the combined dark sector constitute about 96% of the total mass density of our universe[60]. It should become clear by the end of this section that in order to truly understand the nature of difficulties facing the standard model of cosmology, we will most certainly have to move beyond the successful standard model of particle physics; a task not yet achieved.

5.1 Inflation

A mechanism called inflation, originally postulated by physicists working on Grand Unified Theories(GUT) [7] offers explanation not only for the two previous curiosities but also provides the correct initial conditions of the primeval density fluctuation that over time grow to form the large scale structure we observe today[33]. [8]. According to inflationary cosmology the universe expanded exponentially fast for a fraction of a second very early in its history causing the scale factor to increase in magnitude by more than 60-e folds, all within about $10^{-35}s$. According to Eq.(11) such an exponential expansion can occur if the universe is dominated by a high energy component with large negative pressure, $p \approx -1$ which quickly comes to dominate the energy content of the universe. Such properties are characteristics of the vacuum energy, defined as the component with equation of state $w \approx -1$.(see Sec.(6.1)). Ideas from particle physics suggest that the universe is permeated by scalar fields,[9] such as the Higgs field, responsible for the generating masses of all elementary particles. It is possible to construct models such that the scalar field exists in a special state; having a large energy density which can not rapidly decay. Such states are called false vacuum and they are different from true vacuum in that they are only metastable. Once trapped, the field energy density, which is all potential, remains constant with time. As a consequence the field quickly starts to dominate the composition if the universe. The field, as discussed below, also develops cosmological constant (see Sec.(6.1)) like equation of state putting the universe on an inflationary course. Having caused the inflationary growth of the cosmos from a billionth size of an atom to about the size of an orange all within about $10-^35s$, the field then decays through the process of bubble nucleation via quantum tunneling to reach the true vacuum. This was the view according to the original inflationary theory proposed by Guth(1981) whose main motivation was to provide a mechanism to account for the vanishing of the magnetic monopoles number density in the observable universe[33].[10] However it soon became clear that such a scenario is not viable[36]. The many bubble of the vacuum must coalesce in order for the universe as a whole to move to the true vacuum. The ensuing inflation never stops as the bubbles would never merge.

[7] which aims at unifying the strong nuclear force with the electroweak interaction

[8] For an earlier example of an inflationary model with a completely different motivation involving modification of Einstein's equations of general relativity at large curvature (i.e., early times) proposed by Starobinsky see Ref.([59])

[9] A scalar field takes exactly one value at every point in space and time as opposed to vector fields like electromagnetism which carry three distinct components at every point in space and time

[10] Production of magnetic monopoles are predicted in Grand Unified Theories, but so far none of these hypothetically massive particles have been observed. With non inflationary epoch, this very massive particles would very quickly close the universe, setting it up for a Big Crunch

However, one can easily develop a more general description for the inflationary agent, the inflation field, as is done in the new inflation[34] [11] which we now turn to. Consider a spatially homogeneous scalar field, Φ, whose action (in units chosen so Planck's constant $\hbar$ is unity) is given by

$$S = \int d^4x\sqrt{-g}\mathcal{L}, \tag{25}$$

$$\mathcal{L} = \frac{1}{2}g^{\mu\nu}\nabla_\mu\Phi\nabla_\nu\Phi - V(\Phi), \tag{26}$$

where $\mathcal{L}$ is the associated Lagrangian density, g is the determinant of the metric tensor($= a^3$) for the FLRW metric (1) and $V(\Phi)$ is potential of the inflation field. We can then write the energy-momentum tensor of the scalar field

$$T_{\mu\nu} = \nabla_\mu\Phi\nabla_\nu\Phi - g_{\mu\nu}\,\mathcal{L}. \tag{27}$$

The corresponding energy density ρ_Φ and pressure density p_Φ are

$$T_{00} = \rho_\Phi = \frac{\dot{\Phi}^2}{2} + V(\Phi), \tag{28}$$

$$T_{ii} = p_\Phi = \frac{\dot{\Phi}^2}{2} - V(\Phi). \tag{29}$$

where the spatial gradient terms have been eliminated by the assumption of homogeneity ($\partial_i\Phi = 0$).[12] The equation of state of the inflation is thus given by

$$w_\Phi = \frac{p_\Phi}{\rho_\Phi} = \frac{\frac{\dot{\Phi}^2}{2} - V(\Phi)}{\frac{\dot{\Phi}^2}{2} + V(\Phi)} \tag{30}$$

We immediately notice that if dynamics are such that, $\dot{\Phi}^2 \ll V$, then the field energy approximates the effect of vacuum energy, with $p_\Phi \simeq -\rho_\Phi$, as desired. In order to develop the constraint that this slow roll condition places on the potential one should study the field equation of the inflation field which is derived by varying the action with respect to the field Φ

$$\ddot{\Phi} + 3H\dot{\Phi} + \frac{dV}{d\Phi} = 0\,. \tag{31}$$

as well as the Friedmann equation

$$H^2 = \frac{8\pi G}{3}V(\Phi) \tag{32}$$

we have ignored the curvature term, since inflation will flatten the universe any way. Inflation can occur if the evolution of the field is sufficiently gradual that the potential energy dominates over the kinetic energy, and the second derivative of Φ is small enough to allow this states of affairs to be maintained for a sufficient period. This translate into

[11] Guth original inflation theory is referred to as the old inflation

[12] More specifically we assume that the field composes of a homogeneous zero order part,$\Phi^0(t)$ and a first order perturbation, $\delta\Phi(x,t)$. We will work we the zero order part.

requiring $\dot{\Phi}^2 \ll V(\phi)$ and $|\ddot{\Phi}| \ll |3H\dot{\Phi}|, |dV/d\Phi|$. In this case, the above equations reduce to

$$3H\dot{\Phi} + \frac{dV}{d\Phi} = 0\,. \tag{33}$$

$$H^2 = \frac{8\pi G}{3} V(\Phi). \tag{34}$$

Therefore, the slow roll conditions imply

$$\dot{\Phi}^2 \ll V(\Phi) \Longrightarrow \frac{(V')^2}{V} \ll H^2$$

and

$$|\ddot{\Phi}| \ll |3H\dot{\Phi}|, |dV/d\Phi| \Longrightarrow V'' \ll H^2.$$

Accordingly the slow-roll parameters

$$\epsilon = \frac{m_{pl}^2}{16\pi}\left(\frac{1}{V}\frac{dV}{d\Phi}\right)^2 , \quad \eta = \frac{m_{pl}^2}{8\pi}\frac{1}{V}\frac{d^2V}{d\Phi^2}\,, \tag{35}$$

are often used to check the existence of an inflationary solution for the model [36]. Inflation occurs if the slow-roll conditions, $\epsilon \ll 1$ and $|\eta| \ll 1$, are satisfied. The second condition can be viewed as requiring the inflation to last long enough so that every part of observable universe had time to see every other part and learned to relax to almost uniform homogeneity. under slow roll conditions, acceleration(11) and Friedman(32) equations yield an exponential growth of $a(t) \propto \exp(H_{in}t)$ where $H_{in} = \sqrt{\frac{8\pi G\rho_\Phi}{3}}$ and $\rho_\Phi \approx V(\Phi) \approx constant$ during inflation. Inflation dilutes the content of each casually connected patch to negligible amount. Therefore, in order to account for the observed entropy in the universe today($\approx 10^{90}$ particles), any viable inflationary mechanism must address this issue. In fact in most inflationary models this is achieved by rapid oscillation of Φ field at the bottom of its potential. Inflation field through its coupling with fundamental fields, decays into quanta of these fields, thus giving rise to the observed entropy in the universe in forms of field and their excitations. This non-adiabatic phase transition is known as(re)heating where the name signifies the fact that the temperature of the universe following the Big Bang were supercooled as the result of the ensuing exponential expansion of scale factor ($T \propto 1/a$). During reheating the latent heat released in this phase transition gives rise to the usual radiation-dominated phase. The ideas developed here for inflation in the very early universe will prove to be useful in understanding the current late time cosmic acceleration. In particular we will find that some of the most promising models employ scalar fields as the agent responsible. The relevant equations of motion are precisely those that describe dynamical dark energy (See Sec.(6), although the associated energy scales are vastly different. Following reheating (which must generate temperature below GUT scale so it can generate solely the things which we want and not unwanted relics such as monopoles) ρ_Φ is left zero or small so it does not disturb the success of the standard big bang model for the origin of the light elements. With an eye on the other accelerating phase, one can imagine the late time evolution of ρ_Φ to be slow. If slower than the evolution in the mass density in matter, there comes a time when ρ_Φ again dominates, and the universe appears to have a cosmological constant, but one that varies slowly with position and time. For this to be achieved the coupling of inflation to the matter fields must vanish, which means that entropy must be created by other methods than the one mentioned above. We will return to this point in Sec.(6.5) when we consider unifying the two accelerating phase into quintessential inflation model. But first we should see how inflation resolves the problems discussed in Sec.(4).

In an inflationary universe any original curvature initially present have been stretched out to near-flatness as the universe underwent its rapid expansion. During the inflationary period, instead of Ω being driven away from one, Ω is driven towards one,with exponential swiftness:

$$\Omega - 1 \propto e^{-2H_{\rm in}t}, \tag{36}$$

where $H_{\rm in}$ is the, roughly constant, Hubble parameter during inflation. Thus, as long as there is a long enough period of inflation, Ω can start at almost any value, and it will be driven to unity by the exponential expansion. Hence inflation solving the flatness problem by eliminating any need for initial fine tuning of the parameters.[36] Inflation solves the horizon problem by postulating that todays observable universe originated from a much smaller region than that in the noninflationary scenarios. Causal physics operates in this region and so this much smaller region could easily have become uniform before inflation began. Inflation would then stretch this small homogeneous region to encompass the entire observable universe. [35]. In addition the monopoles which are predicted in GUT still remains in possible spectrum of particles, as inflation can dilute the concentration of them to negligible amount in the observable universe. Apart form old inflation(where the scalar field is trapped in the false vacuum and can only decay via quantum tunneling) and the new Inflation (where the scalar field induces inflation by rolling slowly down a potential energy hill as compared to the expansion rate of the universe), there exist another leading model called chaotic inflation proposed by Linde(1986)[35]. According to this latest version of inflation, the universe grew out of a quantum fluctuation in a preexisting region of spacetime and that other universes could do the same from regions within our universe today. Accordingly universe has a fractal geometry on very very large scale and these baby-verses, formed by this budding process, would have their own set of physical laws and material particles.[67] In this way chaotic inflation is a welcome hypothesis for those scientist working on anthropic argument, as it results in many world scenario making these argument work.(for more on anthropic arguments see Sec.(6.1)). Moreover it has been suggested that certain inflationary where the inflation potential does not possess a minimum could result in eternal inflation so that the there is no unique beginning or an ultimate end for the cosmos. For more details on this and other aspect of inflation theory see the review in ([36]).

But perhaps the greatest achievement of inflation is its prediction of primordial density fluctuations that are nearly scale invariant and Gaussian; exactly the initial conditions that is needed to explain the large scale structure of the cosmos. Although the fluctuations are believed to be responsible for the grandest structures of the universe , in inflationary cosmology they arise from quantum fluctuations, usually important only on atomic scales. Inflation field like all other fields, undergoes quantum fluctuations in accord with the Heisenberg uncertainty principle. Inflation takes these quantum fluctuations and stretches them proportional to $a(t)$, magnifying them to macroscopic scales. More precisely, during the period when the scale factor is growing exponentially with time, scalar field fluctuations continuously evolve out of the horizon. These fluctuations reenter the horizon at subsequent epochs and serve as the seeds of all current structure in the Universe[22]. These prediction have been very accurately confirmed by measurements of the CMB.[63]

We end this section by discussing a very intriguing idea regarding the beginning of the universe and its very nature. We begin by noting that quantum uncertainty allows the temporary creation of bubbles of energy, or pairs of particles and anti-particles out of nothing, provided that they disappear in a short time. The less energy is involved, the longer the bubble can exist. The intriguing implication comes when we notice that the energy in a gravitational field is negative, while the energy locked up in matter is positive. But then, if the Universe is exactly flat ,these energies cancel out and the overall energy of the Universe is precisely zero; allowing the initial quantum fluctuation to last forever.! [68] One can then postulate that in the early universe the vacuum (i.e. the various quantum fields that fill space) possessed a large number of symmetries. As the universe expanded and cooled, the vacuum underwent a series of symmetry-breaking phase transitions; separating in the process the four fundamental forces of nature. Gravitation was the first to decouple from the other forces followed by the strong nuclear force and then the weak nuclear force and finally electromagnetism. For example, the electroweak transition broke the SU(2)$\times$U(1) symmetry of the electroweak field into the U(1) symmetry of the present-day electromagnetic field.[4]. This vacuum in fact might be decaying as result of the Hubble friction and its coupling to other matter fields. It maybe that it is only appearing to be a cosmological constant like term today, because the time scale of its variation is very large compared to one expansion time. in this picture the universe is moving toward its true vacuum at ($T = 0$). It is then possible to imagine that the recent phase of cosmic acceleration, signals a new scale at which something new happens. This scale (10^{-33}ev), however, belongs to the very opposite end of spectrum that particle physicist are accustomed to. Having said this, it might well be the case that a true cosmological constant is the responsible agent or the answer might reside in other explanations. Finally we mention that a quantum fluctuation of the vacuum assumes that there was a vacuum of some pre-existing space and so suggestion made above does not really address the problem of origin of universe, although it is an intriguing suggestion.[65]

5.2 Dark Matter

Astrophysical observation require that a large fraction of the gravitationally bound matter associated with the observed large scale structures(LSS) to be nonlunminos[8]. In fact this was first suggested by Zwicky(1933) as a result of studying the orbital velocities of galaxies in the coma cluster which could only be explained if the total mass substantially exceeded that of the visible matter. At first it seems that one can account for this shortfall by considering dark but massive astrophysical object such as black holes, neutron stars or white dwarfs; all of which fit the dark picture. However the shortfall was way more than could be compensated by these object. The situation only deteriorates when one takes into account the constraints provided by BBN for the density of baryonic content of the universe, $\Omega_{b0} \approx 0.04$[6]. Comparing this with the dynamical estimates of($\Omega_{dyn} \approx 0.3$), one is forbidden to even consider such astrophysical candidates as a substitute for the missing mass as all of them are made of baryons and thus must respects the bound given by BBN. So one is actually forced to propose the existence of non-baryonic dark matter in order to fit the observed dynamic of LSS. Subsequent observations of the rotational speeds of galaxies and gravitational lensing of background objects via galaxy clusters such as the Bullet cluster have further solidify the case for the existence of dark matter. Whatever it may be it is clear that it does not interact via electromagnetic force and must interact only very weakly with the standard model particles(or otherwise its existence would have been detected long ago). Although neutrino seems to fit these criteria very well but such a candidate would prevent the growth of structure at appropriate level due to its relativistic nature. In order to find viable candidates one has to move beyond the standard model of particle physics. Leading candidates for non-baryonic dark matter are the weakly interacting massive particles(WIMPs)and axion[45]. WIMPs are hypothesized in supersymmetric extension of the standard model of particle physics with the leading candidate for dark matter being the neutralino (the superpartner of some other neutral particle such as the photino). As a heavy, stable particle, the lightest neutralino is an excellent candidate to comprise the universe's cold dark matter. The axion is a hypothetical particle postulated by the Peccei-Quinn theory in order to resolve the strong-CP problem in quantum chronodynamics(QCD)[36]. It is important to note that both WIPMS and axions belong to a subgroup of Cold Dark Matter(CDM) which means that they were non-relativistic at the time it decoupled form the cosmic plasma. WIMPS are thought to have been in very close contact with rest of cosmic plasma up until when the temperatures drooped below its mass.[13] In most theories Dark matter is produced at temperature of $T = 100Gev$ Alternatively, dark matter that was relativistic at the photon decoupling epoch is called Hot Dark Matter(HDM), whose representative candidate is the neutrino. The solution to the dark matter problem seems within reach: we have a compelling hypothesis, namely that it exists in the form of stable elementary particles left over from the big bang; we know that a small amount of dark matter exists in the form of massive neutrinos; we have two good candidates for the rest of it (the axion and neutralino) and an experimental program to test the particle dark matter hypothesis.

Discussion of dark matter brings us to the realm of structure formation. So far we have assumed that the Universe is perfectly homogeneous, but this is only an approximation. Although we have every indication that the universe at early times was very homogeneous ($\delta\rho \approx 0$) ; however, today inhomogeneity (or structure) is ubiquitous:stars ($\frac{\delta\rho}{\rho} \approx 10^{30}$),galaxies ($\frac{\delta\rho}{\rho} \approx 10^{5}$), clusters of galaxies ($\frac{\delta\rho}{\rho} \approx 10 - 10^{3}$), super clusters, or clusters of clusters ($\frac{\delta\rho}{\rho} \approx 1$), voids ($\frac{\delta\rho}{\rho} \approx -1$)[45]. Theory of structure formation is centered around gravitational instability as the engine behind growth of structure. As uni-

[13]This process is often referred to as freeze-out. Freeze-out is the inability of annihilation to keep the particle in equilibrium.

verse evolves matter accumulates in the initially over dense region, no matter how small the inhomogeneity. Structure is supposed to have formed by the gravitational growth of primeval departures from homogeneity that are adiabatic, scale-invariant, and Gaussian. The paradigm model of structure formation is based on gravitational clustering of CDM and goes by the same name. The CDM model assumes the mass of the universe now is dominated by dark matter that is non-baryonic and acts like a gas of massive, weakly interacting particles with negligibly small primeval velocity dispersion[4]. Once the universe becomes matter dominated (around 1000 yrs after the bang) primeval density inhomogeneities ($\frac{\delta\rho}{\rho} \approx 10^{-5}$) are amplified by gravity and grow into the structure we see today. Because the early universe has to have been very close to homogeneous, and the growing departures from homogeneity at high redshift are well described by linear perturbation theory. The linear density fluctuations may be decomposed into Fourier components. At high enough redshift the wavelength of a mode is much longer than the time-dependent Hubble length H^{-1}, and gravitational instability makes the mode amplitude grow. Adiabatic fluctuations remain adiabatic, because different regions behave as if they were parts of different homogeneous universes. When the Hubble length becomes comparable to the mode proper wavelength, the baryons and radiation, strongly coupled by Thomson scattering at high redshift, oscillate as an acoustic wave and the mode amplitude for the cold dark matter stops growing. At high redshift the dark matter mass density is less than that of the radiation. The radiation thus fixes the expansion rate, which is too rapid for the self-gravity of the dark matter to have any effect on its distribution. The mass densities in dark matter and radiation are equal at redshift $z_{eq} = 2.4 * 10^4 * h^2$[4]; thereafter the dark matter mass density dominates and the fluctuations in its distribution start to grow again.

We point out to an important issue concerning structure formation and that is the baryonic matter by it self can not give rise to observed structure in the universe as it does not cause density power spectrum with sufficient amplitude. This is because the dissipation of the baryon density fluctuations by radiation drag as the primeval plasma combines to neutral hydrogen (at redshift $z >> 1000$) unacceptably suppresses structure formation on the scale of galaxies. Cold dark matter avoids this problem by eliminating radiation drag. This is one of the reasons attention turned to the hypothetical nonbaryonic cold dark matter [45] The key idea here is the fact that once accelerated expansion begins, the growth of linear perturbations effectively ends, since the Hubble damping time becomes shorter than the timescale for perturbation growth. This is to be expected, as dark energy is gravitationally self repulsive, and as long as most of the composition in the universe has this characteristic, matter can not clump. The consensus model had to have been modified after the discovery of cosmic acceleration, and in fact the modification have proved to be a step in the right direction. This consensus model goes by the name ΛCDM and as the name suggest it employs the simplest candidate for dark energy. If dark energy is dynamical, then in principle it can be inhomogeneous, an effect ignored in ΛCDM. In practice, it is expected to be nearly uniform over scales smaller than the present Hubble radius, in sharp contrast to dark matter, which can clump on small scales.

5.3 Dark Energy

The need to propose an additional, nonluminous, gravitational self repulsive component that dominates the universe today originated from improved observations of large scale structure dynamics confirming that the total gravitationally bound mass density is probably less than half of the critical density[8]. Also, Combined measurements of the cosmic microwave background(CMB) temperature fluctuations and the distribution of galaxies on large scales began to suggest that the universe is flat as predicted by inflation[66]. The only way to reconcile a flat, low mass universe is to postulate a dark energy component that dominates today, differentiated from dark matter from the fact it resist gravitational collapse or else it would have already been detected as part of the clustered energy in the halos of galaxies. But, as long as most of the energy of the universe resists gravitational collapse, it is impossible for structure to form in the universe. Therefore we must require that the hypothetical dark energy was negligible in the past and became dominant only very recently after galaxies and larger scale structure had time to form. This implies that the energy density of dark energy must evolve with redshift more slowly than matter. According to General relativity, namely equation (22), the only type of energy with this property has negative pressure. Furthermore a sufficiently large negative pressure could result in accelerated expansion according to the cosmic acceleration condition (23). Therefore, some cosmologist[30] in fact anticipated the Supernovae(SNe) result which provided direct evidence for acceleration. Supernovae Ia are produced in the thermonuclear explosion of a carbon-oxygen white dwarf accreting mass from a companion star as it approaches the Chandrasekhar mass[4]. This property along with fact that are about as bright as a typical galaxy upon explosion, recommends their utility as standard candles for cosmology. The studies of supernova Hubble diagrams(redshift vs luminosity) suggested that distant SNe are dimmer than they would be in a decelerating Universe, indicating that the expansion has been speeding up for the past 5 Gyr [1, 2]. Combining these three line of evidence discussed above within the framework of GR, one is forced to postulate that the universe is currently dominated by a a component, dubbed dark energy, which is uniformly distributed at scales larger than the observed universe and whose equation of state satisfies $w < -1/3$.

We should mention that initially a number of concerns were raised about the case SNe result makes for an accelerated universe. It was suggested that distant SNe could appear fainter due to extinction by hypothetical dust rather than acceleration [41]. However subsequent SNe measurements at even higher redshift($z = 1.8$), provided evidence for the expected earlier epoch of deceleration and therefore disfavoring dust extinction as an alternative explanation to acceleration[46]. Observational cosmology today support the case for an accelerating universe to better than 96 part in 100[40].

6 Dark Energy Models

There is no shortage of ideas for what dark energy might be, from the quantum vacuum to a new, ultra-light scalar field. Whatever form dark energy takes, two new cosmological problems arise. First, the component must have a tiny energy density today, roughly $10^{-47} GeV^4$. We refer to the failure to account for this small value on the basis of fundamental physics as the fine tunning problem. A second problem arises because the energy density contribution of dark energy and matter density to the critical density is of the same order of magnitude today. The root cause of the problem is that the energy density of dark energy must decreases at a slower rate than matter density, so that it only emerges at late time to dominate in order to allow for the formation of structure and creation of light elements. But then the ratio shrinks by many orders of magnitude as we extrapolate back in time. As a result, initially the ratio must be set just right so that now, fifteen billion years later, the ratio is of order unity. Accounting for the special ratio in the early universe will be referred to as the coincidence problem [19]. The coincidence problem is a generalization of the flatness problem [61]. Here, we review the leading models.

6.1 Cosmological Constant

The simplest and yet most puzzling explanation of dark energy is a cosmological constant; a component which is constant in both space and time and is characterized by its constant equation of state, $w = -1$. With these characteristic, cosmological constant is the ideal candidate for dark energy and in fact models of structure formation, known as ΛCDM, that incorporate this idea currently fit the observations best[4]. The idea was first proposed by Einstein(1917) in order to construct a static model of the universe; the Einstein's Static universe. In a static universe the repulsive cosmological constant is delicately fine-tuned to balance the gravitational attraction of matter. In this case Einstein field equation (2) is modified by adding the cosmological constant,Λ, in the following way

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}\,. \tag{37}$$

Eqs.(7) and (11) in this case, become

$$H^2 = (\frac{\dot{a}}{a})^2 = \frac{8\pi G\rho}{3} + \frac{\Lambda}{3} - \frac{K}{a^2}, \tag{38}$$

$$(\frac{\ddot{a}}{a})^2 = -\frac{4\pi G}{3}(\rho_{tot} + 3p_{tot}) + \frac{\Lambda}{3}. \tag{39}$$

Einstein main motivation was to incorporate Mach's principle[14], which forbade the idea of empty space, into his theory of gravitation. However, the discovery by Friedmann (1922) of expanding solutions to the Einstein field equations in the absence of Λ, together with the discovery by Hubble(1929) of an expanding universe [15], swept away the need for introducing such a free parameter.

Later, Bondi, Gold and Hoyle (1948) put forth the steady-state cosmology with $\rho_\Lambda > 0$

[14] de Sitter expanding solution of universe with no matter and a positive cosmological constant was the end of Mach's idea as it showed matter is clearly not related to inertia.

[15] In a static universe the number of redshifted galaxies is expected to be the same as the number of blueshifted ones.

and $\rho_m = 0$. In the steady-state model, the dilution of matter due to expansion is counteracted by postulating the continuous creation of matter. The model enjoyed considerable appeal by the aesthetics of an unchanging universe and a serious age problem (the measured value of the Hubble constant at the time, around 500 km/s/Mpc implied an expansion age of only 2 Gyr, less than the age of Earth)and was once considered as a substitute for the Big Bang model. However its firm prediction of an unevolving Universe made it easily falsifiable, and the redshift distribution of radio galaxies, the absence of quasars nearby, and the discovery of the cosmic microwave background radiation did so in the early 1960s. The cosmological constant was briefly resurrected in Lemaitre loitering model during the late 1960s as means of explaining the excess of quasars at redshifts around $z \approx 2$ (as it turns out, this is a real effect: quasar activity peaks around $z \approx 2$).

A fundamental physical basis for the cosmological constant was provided by Zel'dovich (1968)[44]. He realized that one loop quantum vacuum fluctuations gave rise to an energy momentum tensor which, after being suitably regularized for infinities, had exactly the same form as a cosmological constant. Zel'dovich thereby showed that a finite value of the cosmological constant was fully consistent with a field theoretic understanding of zero-point (vacuum) fluctuation. In fact the the zero-point energy of the electromagnetic field at laboratory wavelengths is real and measurable, as in the Casimir (1948) effect.

In order to better understand the concept of vacuum in the context of cosmology we now turn our attention to general relativity. A characteristic feature of general relativity is that the source for gravitation is the entire energy-momentum tensor. So unlike non gravitational field theories where only energy differences matter, in GR it is the absolute value of energy that counts which makes GR a non-renormalizable effective field theory. This behavior opens up the possibility of vacuum energy; an energy density characteristic of empty space. On taking the vacuum to look Lorentz-invariant to a local observer, then by General Covariance its energy-momentum tensor must take on the form of constant vacuum energy density, ρ_{vac}, multiplied by the metric,$g_{\mu\nu}$

$$T^{vac}_{\mu\nu} = -\rho_{vac} g_{\mu\nu} \tag{40}$$

Such an energy-momentum density tensor, is associated with an isotropic pressure(see Eq.(112))

$$p_{vac} = -\rho_{vac}. \tag{41}$$

The energy density, ρ_{vac}, should be constant throughout spacetime, since a gradient would not be Lorentz invariant. Comparing this kind of energy-momentum tensor to the appearance of the cosmological constant in modified Einstein equation (37), we find that they are formally equivalent, as can be seen by moving the $\Lambda g_{\mu\nu}$ term in (37) to the right-hand side and setting

$$\rho_{vac} = \frac{\Lambda}{8\pi G}. \tag{42}$$

This expression illustrates that vacuum energy is mathematically equivalent to a cosmological constant and so could be considered as a source to the effective cosmological constant.[16]

Fundamental physics asserts that empty space is filled with fields that include the gravitational, electromagnetic, Higgs field among many others. According to the rules of quantum mechanics each field undergoes quantum fluctuations even in its ground state as a result of Heisenberg uncertainty principle. In particular in quantum field theory for each mode of a quantum field there is a zero-point energy $\hbar\omega/2$, so that the energy density of the quantum vacuum is given by

$$\rho_{vac} = \frac{1}{2}\sum_{\text{fields}} g_i \int_0^\infty \sqrt{k^2+m^2}\,\frac{d^3k}{(2\pi)^3} \simeq \sum_{\text{fields}} \frac{g_i k_{max}^4}{16\pi^2} \tag{43}$$

where g_i accounts for the degrees of freedom of the field (the sign of g_i is $+$ for bosons and $-$ for fermions), and the sum runs over all quantum fields (quarks, leptons, gauge fields, etc). Here k_{max} is an imposed momentum cutoff, because the sum diverges quadratically. But general relativity as an effective field theory is indeed expected to breakdown at scales above Planck mass and because gravity couples to everything this means that Planck scale provides a natural cutoff for all field theories. Taking the cutoff at Planck scale results in $\rho_{vac} \approx 10^{76} GeV^4$ which is 123 orders of magnitude larger than the currently observed value $\rho_{vac} \approx 10^{-47} GeV^4$. Observationally we know that Λ is of order the present value of the Hubble parameter H_0, that is (Friedmann equation for cosmological constant, Eq.38)

$$\Lambda \approx H_0^2 = (2.13h \times 10^-42 GeV)^2. \tag{44}$$

This corresponds to a critical density ρ_Λ,

$$\rho_\Lambda = \frac{\Lambda}{8\pi G} \approx 10^{-47} GeV^4. \tag{45}$$

A cutoff at the much lower QCD scale does not do much better as it produces a cosmological constant that is forty orders of magnitude larger than the observed value. Again, in the absence of gravity this energy has no effect, and is traditionally discarded (by a process known as normal-ordering). Since according to general relativity all form of energy gravitates and therefore influence the spacetime geometry, an infinite value of Λ is expected to generate an infinitely large space-time curvature, a prospect not favored either by observation or theory. [17] Hence, general relativity and quantum mechanics are strikingly successful on a considerable range of length scales, provided we agree not to use the rules of quantum mechanics to count the zero-point energy density in the vacuum, even though we know we have to count the zero-point energies in all other situations. But when we use general relativity we are really using an effective field theory

[16] This fluid picture is of limited use as it predicts a negative value for the speed of sound c_s^2, making the growth in perturbations unstable.[4]

[17] The situation can be compared to the development of the theory of the weak interactions. The Fermi point-like interaction model is strikingly successful for a considerable range of energies, but it was clear from the start that the model fails at high energy. A fix was discussed – mediate the interaction by an intermediate boson – and eventually incorporated into the even more successful electroweak theory. [8]

to describe a certain limit of quantum gravity. New physics is expected to arise beyond this scale, one that unifies both quantum mechanic and gravitation into quantum gravity.

The discovery of Supersymmetry, the hypothetical symmetry between bosons and fermions, appeared at first to offer a resolution to the problem. In a supersymmetric (SUSY) world, every fermion in the standard model of particle physics has an equal-mass SUSY bosonic partner and vice versa, so that fermionic and bosonic zero-point contributions to ρ_{vac} would exactly cancel. But no supersymmetric particle has been observed in the universe and indeed in largest accelerators which means that supersymmetry (if it exists) is broken at the low temperatures prevailing in the universe today. For a viable supersymmetric scenario, for instance if it is to be relevant to the hierarchy problem, the supersymmetry breaking scale should be around $M_{SUSY} \approx 10^3 GeV$. This leads to a discrepancy of 60 (as opposed to 120) orders of magnitude with observations. The implication for cosmological constant is that it vanishes in the early universe, but reappears during late times when the temperature has dropped below T_{SUSY}. This is clearly an undesirable scenario and almost the very opposite of what one is looking for, since, a large value of Λ at an early time is useful from the viewpoint of inflation, whereas a very small current value of Λ is in agreement with observations. Nonetheless, experiments at the Large Hadron Collider (LHC) at CERN whose one of main goals, apart from observing the Higgs boson, is to find suppersymmetric particles might shed light on the vacuum-energy problem. In addition, also models with spontaneous symmetry breaking [38, 20] contribute to the effective cosmological constant. in these models Lagrangian is given by

$$\mathcal{L} = \frac{1}{2}\, g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi), \tag{46}$$

$$V(\phi) = V_0 - \frac{1}{2}\mu^2\phi^2 + \frac{1}{4}\lambda\phi^4. \tag{47}$$

The symmetric state at $\phi = 0$ is unstable which causes the system to settle in the ground state $\phi = +\sigma$ or $\phi = -\sigma$, where $\sigma = \sqrt{\mu^2/\lambda}$, thus breaking the reflection symmetry $\phi \leftrightarrow -\phi$ present in the Lagrangian. If $V_0 = 0$ then this potential results in a broken symmetry state with a large negative cosmological constant $\Lambda_{eff} = V(\phi = \sigma) = -\mu^4/4\lambda$. For example, In the electroweak model, the phases of broken and unbroken symmetry are distinguished by a potential energy difference of approximately $M_{EW} \approx 200GeV$. The universe is in the broken-symmetry phase during our current low-temperature epoch, and is believed to have been in the symmetric phase at sufficiently high temperatures in the early universe. We thus expect a contribution to the effective cosmological constant of order $(200GeV)^4 \approx 10^{10}GeV^4$. In order to avoid this situation the value of the free parameter V_0 is chosen to counterbalanceΛ_{eff}, as a result one sets $V_0 \sim \mu^4/4\lambda$ so that $\Lambda_{eff}/8\pi G = V_0 - \mu^4/4\lambda \simeq 10^{-47}$GeV ^{4}.

The net cosmological constant, from this point of view, is the sum of a number of apparently disparate contributions, including potential energies from scalar fields and zero-point fluctuations of each field theory degree of freedom, as well as a bare cosmological constant Λ_0. It is possible to imagine That these large and apparently unrelated contributions listed add together, with different signs, to produce a net cosmological constant consistent with the observational limit given in Eq.(45) , But such degree of fine tunning seems just ridiculous. The ensuing regularization of the effective cosmological constant must be done with considerable care, since even small fluctuations in the final value can result in grave consequences for cosmology. For instance if $\Lambda_{eff}/8\pi G < -10^{-43} GeV^4$ the large attractive force exerted by a negative cosmological constant will ensure that the universe re-collapses before it reaches maturity[38, 39]. The age of the universe in this case will be < 1 billion years, far too short for galaxies to form and for life (as we know it) to emerge within the standard big bang scenario. On the other hand if $\Lambda_{eff}/8\pi G > 10^{-43}\mathrm{GeV}^4$, the large repulsive force generated by this will ensure that the universe begins accelerating before gravitationally bound systems have a chance to form. Such a scenario will also clearly preclude the emergence of life[38, 39]. As of yet no special symmetry which could enforce a vanishing vacuum energy while remaining consistent with the known laws of physics has been accounted for; this conundrum is the cosmological constant problem and is one of most significant unsolved problem in fundamental physics. Although the problem was largely ignored in the past because of the success of big bang model with ($\Lambda = 0$); with the discovery of cosmic acceleration, the cosmological constant problem is now front and center and can no longer be ignored.

To see the coincidence problem of relatively similar contributions from two seemingly unrelated components to the total energy density of the universe, we note that the best-fit universe model has $\Omega_{\Lambda 0} \approx 0.7$ and $\Omega_{m0} \approx 0.3$. But, the relative balance of vacuum and matter changes rapidly as the universe expands:

$$\frac{\Omega_\Lambda}{\Omega_m} = \frac{\rho_\Lambda}{\rho_m} \propto a^3 \,. \tag{48}$$

As a consequence, at early times the cosmological constant was negligible in comparison to matter and radiation, while at late times matter and radiation are negligible. There is only a brief epoch of the universe's history during which it would be possible to witness the transition from domination by one type of component to another. It seems remarkable that we live during the short transitional period between these two eras. The approximate coincidence between matter and cosmological constant in the current universe is one of several puzzling features of the composition of the total energy density[29].

The rather small window permitted for life to emerge in the presence of Λ has led several researchers [38, 39] to develop anthropic arguments for the existence of a small cosmological constant. Because a value much larger than that needed to explain the observed cosmic acceleration would preclude the formation of galaxies, we could not find ourselves in a region with such large ρ_{vac}. This anthropic approach finds a possible home in the landscape version of string theory, in which the number of different vacuum states is very large and essentially all values of the cosmological constant are possible[31]. Furthermore such landscape scenario boosts the eternal inflation theory since it can provide a mechanism to populate the landscape of string vacua. While all of these vacua are described by the same fundamental string theory, the apparent laws of physics at low energies could differ dramatically from one vacuum to another. In particular, the value of the cosmological constant (e.g., the vacuum energy density) would be expected to have different values for different vacua. The combination of the string landscape with eternal inflation has in turn led to a markedly increased interest in anthropic reasoning, since we now have a respectable set of theoretical ideas that provide a setting for such reasoning. In the multiverse, life will evolve only in very rare regions where the local laws of physics just happen to have the properties needed for life, giving a simple explanation for why the observed universe appears to have just the right properties for the evolution of life. The incredibly small value of the cosmological constant is a telling example of a feature that seems to be needed for life, but for which an explanation from fundamental physics is plainfully lacking[36, 38].

6.2 Scalar Field Models of Dark Energy

If the cosmological constant problem is solved in a way that it completely vanishes we need to find alternative models of dark energy. Cosmological constant by definition does not vary with space or time; its value is set once and for all at the beginning of the universe. However, by introducing a new degree of freedom, a scalar field ϕ, one can mimic the effect of vacuum energy and make vacuum energy effectively dynamical [15, 22, 8, 12]. Cosmic acceleration is then attributable to the fact that the Universe has not yet reached its true vacuum state, for dynamical reasons. Unlike vacuum energy, scalar-field energy clusters gravitationally, but only on the largest scales and with a very small amplitude. Considering dynamical dark energy as an alternative to an honest cosmological constant has several benefits. First of all a dynamical energy density can be evolving slowly to zero, allowing for a solution to the cosmological constant problem which makes the ultimate vacuum energy vanish exactly. Second, it provides an observational opportunity to study the evolution of the dark energy from which one might learn something about the underlying physical mechanism. Finally allowing the dark energy to evolve opens the possibility of finding a dynamical solution to the coincidence problem. This is done if the recent take over by dark energy is triggered independently of (or at least for a wide range of) the parameters in the theory.

It needs to be stressed that this approach has an effect of a double edged sword: although it allows for richer dynamics at the same time it gives rise to additional parameters whose value need to be fine tunned to be compatible with observational cosmology. Ideally, the theory would have order-unity parameters at, say, the Planck scale, and the dark-energy density today would be insensitive to the field's initial conditions.

The canonical scalar filed model for dynamical dark energy is the quintessence[11] which consists of an inhomogeneous scalar field ρ slowly rolling down a potential. There exist also scalar field models with non canonical kinetic energy term known as k-essence[16]. Phantom [50] is another type of scalar field models which has a negative kinetic energy term. In this case null energy condition is violated and the corresponding dark energy component is referred to as phantom energy. This means that the energy density grows, rather than decays, with time. In a phantom-dominated Universe, the scale factor and expansion rate diverge in finite time, ripping apart everything; galaxies, stars, atoms; before the Universe terminates in a Big Rip singularity. Such theories are typically unstable and may arise via quantum fluctuations of the scalar field. We will not explore this case in this paper. For more on this topic see Ref.[50] and [51]. In what follows, we shall discuss Quintessence and K-essence models in detail. We also explore PNGB [15] models and talk about quintessential inflation as well. See [7], for a more comprehensive discussion.

First, we give a flavor of what is about about to come. Scalar field models can be classified dynamically as thawing or freezing [21]. In freezing models, the field rolls more slowly as time progresses which happens if the slope of the potential drops more rapidly than the Hubble friction term $3H\dot{\phi}$. So in this case the equation of state approaches -1 asymptotically in time. The canonical examples are exponential and inverse power-law potentials. In thawing models the field is frozen by the Hubble friction term until recently and acts as an effective vacuum energy but when the expansion rate drops below $H^2 = d^2V(\phi)/d\phi^2$, the field begins to roll and w evolves away from -1. The simplest example of a thawing model is a scalar field of mass m_ϕ, with $V(\phi) = m_\phi^2 \frac{\phi^2}{2}$. Pseudo Nambu Goldstone Boson(PNGB) also belongs to this class and represents a viable candidate of dark energy based on particle physics ideas. We will discuss each specific model shortly, but first we shall develop some techniques that enables us to further study the evolution of universe. In particular, we study the dynamics of a general scalar field model in the presence of a background barotropic fluid with a constant equation of state, w_B that could be either radiation or matter. The field energy density and its pressure are denoted by ρ_ϕ and p_ϕ, respectively. It is understood that scalar field energy density must remain sub dominant in both radiation and matter dominated epoch to be consistent with Big Bang Nucleosynthesis results for light element abundance and formation of large scale structure, only to emerge recently causing the expansion of the universe to accelerate. Since the light-element abundances depend on the expansion rate during a relatively brief period (rather than on the behavior of perturbations, or an an integral of the expansion rate over a long period), it is very sensitive to presence of a self repulsive dark energy component. As such BNN produces the tightest bound on dark energy density at nucleosynthesis epoch (3min$\leq t \leq$17min)[12].

From Eqs.(7) and (8), we obtain

$$H^2 = \frac{\kappa^2}{3}\left(\rho_B + \rho_\phi\right) \tag{49}$$

$$\dot{H} = -\frac{\kappa^2}{2}\left[\rho_\phi + \rho_B\right] , \tag{50}$$

$$\tag{51}$$

The energy density ρ_ϕ and ρ_B each separately satisfy the continuity equation (9)

$$\dot{\rho_\phi} + 3H(\rho_\phi + p_\phi) = 0 \tag{52}$$

$$\dot{\rho_B} + 3H(\rho_B + p_B) = 0 \tag{53}$$

$$\tag{54}$$

The background component energy density evolves as $\rho_B = \rho_0 a^{-3(1+w_B)}$ as w_B is a constant while w_ϕ dynamically varies in time.

6.3 Quintessence

Quintessence is a canonical scalar field ϕ with a potential V(ϕ) responsible for the late-time cosmic acceleration.[11]. The name quintessence derives from the medieval word for fifth element; according to some meta-physicians at the time, the universe consisted of earth, air, fire and water, plus an additional all-pervasive, component that accounted for the motion of the Moon and planets. In the current context, quintessence would be the fifth dynamical component that has influenced the evolution of the universe, in addition to the previously known baryons, leptons, photons, and dark matter [19]. The potential $V(\phi)$ has to have a particular form in order to cause the current phase of cosmic acceleration and this form as we shall constraint the mass of the scalar field to be ultra light (10^{-42}Gev). In addition there must be a value such that $V(\phi)$ is equal to todays dark energy density ($10^{-47}GeV^4$). One must also explain the fact that the field has that particular value, today. In general, this is not simply a matter of constructing the potential, but also a matter of carefully choosing the initial value of the field and its time-derivatives. So, instead of tuning one parameter as in the cosmological constant case, one must now tune the parameters of the potential and the initial conditions in the field. It is emphasized once again that the scalar field models do not address the cosmological constant problem: they simply assume that the minimum value of $V(\phi)$ is zero or very small.

Because the quintessence component evolves in time, it is, by general covariance, necessarily spatially inhomogeneous as it has to respond to the changing gravitational potential of the background matter. Quintessence is described by scalar field ϕ that is minimally coupled to gravity whose action is given by

$$S = \int d^4x\sqrt{-g}(-\frac{1}{2}g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi - V(\phi)). \tag{55}$$

The energy momentum tensor of the field is given by

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta S}{\delta g^{\mu\nu}}. \tag{56}$$

Using the fact that $\delta\sqrt{-g} = -(1/2)\sqrt{-g}g_{\mu\nu}\delta g^{\mu\nu}$, we find

$$T_{\mu\nu} = \nabla_\mu\phi\nabla_\nu\phi - g_{\mu\nu}\left[\frac{1}{2}g^{\alpha\beta}\nabla_\alpha\phi\nabla_\beta\phi + V(\phi)\right]. \tag{57}$$

In the flat FLRW background we obtain the following energy density and pressure density for a spatially homogeneous scalar field (i.e.,$\partial_i = 0$)

$$\rho = T_{00} = \frac{1}{2}\dot{\phi}^2 + V(\phi) \tag{58}$$

$$, p = T_{ii} = \frac{1}{2}\dot{\phi}^2 - V(\phi). \tag{59}$$

Then Eqs. (7) and (11) yield

$$H^2 = \frac{8\pi G}{3}[\frac{1}{2}\dot{\phi}^2 + V(\phi)], \tag{60}$$

$$\frac{\ddot{a}}{a} = -\frac{8\pi G}{3}[\dot{\phi}^2 - V(\phi)]. \tag{61}$$

The equation of state for the field ϕ is given by

$$w_\phi = \frac{p}{\rho} = \frac{\dot{\phi}^2 - 2V(\phi)}{\dot{\phi}^2 + 2V(\phi)}. \tag{62}$$

Note that the equation of state for the field ϕ ranges in the region $-1 \leq w_\phi \leq 1$[18] We also see that the universe accelerates for $\dot{\phi}^2 < V(\phi)$(see Eq.(23). In fact the field acts just like a cosmological constant with $w_\phi \approx -1$ provided that $\dot{\phi}^2 \ll V(\phi)$. In the case of a stiff matter characterized by $\dot{\phi}^2 \gg V(\phi)$ one gets $w_\phi = 1$. In a flat FLRW spacetime the variation of the action (55) with respect to ϕ gives[19]

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi} = 0\,. \tag{63}$$

where we have assumed based on observation that the field is homogeneous, at least on scales smaller than our the observable universe, thus eliminating any gradient term. Also, note the presence of the Hubble friction term $(3H\dot{\phi})$ which slows down the field motion as it rolls down the potential. In the context of late time acceleration the slow-roll parameters defined in Eq.(35) are not completely trustworthy, since both dark matter and dark energy contribute to the late time dynamics. However they still provide a good measure for solutions that lead to accelerated expansion. Writing them in terms of the Hubble parameters is a better choice as it takes into account both dark energy and dark matter contributions. For example $\epsilon = -\dot{H}/H^2$.

Specializing The analysis carried out in Sec.(6.2) to the case of canonical quintessence model, we obtain the following set of dynamical equation

$$H^2 = \frac{\kappa^2}{3}\left(\rho_B + \frac{1}{2}\dot{\phi}^2 + V\right), \tag{64}$$

$$\dot{H} = -\frac{\kappa^2}{2}\left[\rho_B + P_B + \dot{\phi}^2\right], \tag{65}$$

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi} = 0 \tag{66}$$

where $\kappa^2 = 8\pi G$. Next define the following dimensionless parameters

$$\begin{aligned} x &\equiv \frac{\kappa\dot{\phi}}{\sqrt{6}H}\,, \quad y \equiv \frac{\kappa\sqrt{V}}{\sqrt{3}H}\,, \\ \lambda &\equiv -\frac{V'}{\kappa V}\,, \quad \Gamma \equiv \frac{VV''^2}{V'}\,, \end{aligned} \tag{67}$$

[18] It is also possible to have $w < -1$, though at the expense of ghosts, by changing the sign of the kinetic energy term Eq.(55). In this case null energy condition is violated and the corresponding dark energy is referred to as phantom energy. Such theories are typically unstable and may arise via quantum fluctuations of the scalar field. For more on this topic see Ref.([50]) and ([51]).

[19] This is, of course, just the Klein Gordon equation with spatial derivatives neglected.

where the prime indicates differentiation with respect to ϕ. Then the above equations can be written in the following form [7, 14]:

$$\frac{dx}{dN} = -3x + \frac{\sqrt{6}}{2}\lambda y^2 + \frac{3}{2}x\left[(1-w_B)x^2 + (1+w_B)(1-y^2)\right], \tag{68}$$

$$\frac{dy}{dN} = -\frac{\sqrt{6}}{2}\lambda xy + \frac{3}{2}y\left[(1-w_B)x^2 + (1+w_B)(1-y^2)\right], \tag{69}$$

$$\frac{d\lambda}{dN} = -\sqrt{6}\lambda^2(\Gamma-1)x, \tag{70}$$

together with the Friedmann constraint equation

$$x^2 + y^2 + \frac{\kappa^2\rho_B}{3H^2} = 1. \tag{71}$$

where $N = ln(a)$ is the number of e-foldings. The equation of state w_ϕ and the fraction of the energy density Ω_ϕ and Ω_B for the field ϕ and the background fluid B are given by

$$w_\phi \equiv \frac{p_\phi}{\rho_\phi} = \frac{x^2 - y^2}{x^2 + y^2}, \tag{72}$$

$$\Omega_\phi \equiv \frac{\kappa^2\rho_\phi}{3H^2} = x^2 + y^2, \tag{73}$$

$$\Omega_B \equiv \frac{\kappa^2\rho_B}{3H^2} = 1 - x^2 + y^2. \tag{74}$$

which satisfy $\Omega_\phi + \Omega_B = 1$ as expected for a flat universe. Ω_ϕ is bounded, $0 \le x^2+y^2 \le 1$, for a non-negative fluid density, $\rho_B \ge 0$, and so the evolution of this system is completely described by trajectories within the unit disc. The lower half-disc, $y < 0$, corresponds to contracting universes. As the system is symmetric under the reflection $(x, y) \to (x, -y)$ and time reversal $t \to -t$, we only consider the upper half-disc, $y \ge 0$ in the following discussion. Let us also define the total effective equation of state:

$$w_{\text{eff}} \equiv \frac{p_\phi + p_B}{\rho_\phi + \rho_B} = w_B + (1-w_B)x^2 - (1+w_B)y^2. \tag{75}$$

An accelerated expansion occurs for $w_{\text{eff}} < -1/3$ (see Eq.(23)).

6.3.1 Scaling Solutions

Scaling solutions are defined as attracting solution whereby the evolution of the field approaches, and then locks onto, a universal track for a broad range of initial conditions. In particular, scaling solutions are characterized by the relation

$$\rho_\phi/\rho_B = C, \tag{76}$$

where C is a nonzero constant. These solutions arise if λ as defined in Eq.(67) is constant. From Eq.(67). In this case (70) is trivially satisfied and so the condition $\Gamma = 1$

Name	x	y	Existence	Stability	Ω_ϕ	γ_ϕ
(a)	0	0	All λ and γ	Saddle point for $0 < \gamma < 2$	0	–
(b1)	1	0	All λ and γ	Unstable node for $\lambda < \sqrt{6}$ Saddle point for $\lambda > \sqrt{6}$	1	2
(b2)	-1	0	All λ and γ	Unstable node for $\lambda > -\sqrt{6}$ Saddle point for $\lambda < -\sqrt{6}$	1	2
(c)	$\lambda/\sqrt{6}$	$[1-\lambda^2/6]^{1/2}$	$\lambda^2 < 6$	Stable node for $\lambda^2 < 3\gamma$ Saddle point for $3\gamma < \lambda^2 < 6$	1	$\lambda^2/3$
(d)	$(3/2)^{1/2}\gamma/\lambda$	$[3(2-\gamma)\gamma/2\lambda^2]^{1/2}$	$\lambda^2 > 3\gamma$	Stable node for $3\gamma < \lambda^2 < 24\gamma^2/(9\gamma-2)$ Stable spiral for $\lambda^2 > 24\gamma^2/(9\gamma-2)$	$3\gamma/\lambda^2$	γ

Table 1: The properties of the critical points. for the exponential potential given by Eq.(77).

can also be used to characterise these particular solutions. Integrating Eq.(67), we find the following exponential form for the scalar field potential $V(\phi)$

$$V(\phi) = V_0 e^{-\kappa\lambda\phi} \,. \tag{77}$$

where V_0 is the integration constant with dimension of mass to the power o four. For convenience we also define $\gamma = 1 + w_B$ and $\gamma_\phi = 1 + w_\phi$ so that matter and radiation correspond to ($\gamma = 1$) and ($\gamma = 4/3$), respectively. The effective equation of state for the scalar field at any point is given by

$$\gamma_\phi \equiv \frac{\rho_\phi + p_\phi}{\rho_\phi} = \frac{\dot{\phi}^2}{V(\phi) + \dot{\phi}^2/2} = \frac{2x^2}{x^2+y^2} \tag{78}$$

Fixed points at finite values of x and y in the phase-plane correspond to solutions where the scalar field has a barotropic equation of state and the scale factor of the universe evolves as $a \propto t^p$ where $p = 2/3\gamma_\phi$. Depending on the values of γ and λ, we have up to five fixed points (critical points), obtained by setting $dx/dN = 0$ and $dy/dN = 0$ in Equations (68) and (69). The corresponding fixed points and their stabilities for the realistic case of $0 \leq \gamma \leq 2$ are summarized in TABLE I. (see Sec.(10.1.2) for details) The eigenvalues of the corresponding Jacobian matrix $\mathcal{M}$ for the linearized system given in Eq. (136) are as follows.

- Point (a):

$$\mu_1 = -\frac{3}{2}(2-\gamma)\,, \quad \mu_2 = \frac{3}{2}\gamma\,. \tag{79}$$

- Point (b1):

$$\mu_1 = 3 - \frac{\sqrt{6}}{2}\lambda\,, \quad \mu_2 = 3(2-\gamma)\,. \tag{80}$$

- Point (b2):

$$\mu_1 = 3 + \frac{\sqrt{6}}{2}\lambda\,, \quad \mu_2 = 3(2-\gamma)\,. \tag{81}$$

- Point (c):

$$\mu_1 = \frac{1}{2}(\lambda^2 - 6)\,, \quad \mu_2 = \lambda^2 - 3\gamma\,. \tag{82}$$

- Point (d):

$$\mu_{1,2} = -\frac{3(2-\gamma)}{4}\left[1 \pm \sqrt{1 - \frac{8\gamma(\lambda^2 - 3\gamma)}{\lambda^2(2-\gamma)}}\right] . \quad (83)$$

In what follows we clarify the properties of the five fixed points given in TABLE I. The point (a) corresponds to a fluid dominated solution where $\Omega_\phi = 0$ and is an unstable saddle point since $\mu_1 < 0$ and $\mu_2 > 0$. The points (b1) and (b2) are either an unstable node or a saddle point depending upon the value of λ and correspond to solutions where dynamics are dominated by the kinetic energy of a scalar field with a stiff equation of state, $\gamma_\phi = 2$. As expected these solutions are unstable and are only expected to be relevant at early times. We are left with only two possible late-time attractor solutions. Point (c) corresponds to the scalar field dominated solution ($\Omega_\phi = 1$) which exists for sufficiently flat potentials, $\lambda^2 < 6$. The scalar field has an effective barotropic index $\gamma_\phi = \lambda^2/3$ giving rise to accelerating expansion ($\ddot{a} > 0$) for $\lambda^2 < 2$. This point is a stable node for $\lambda^2 < 3\gamma$, whereas it is a saddle point for $3\gamma < \lambda^2 < 6$. For $\lambda^2 > 3\gamma$ we find a different late-time attractor in point (d) where neither the scalar-field nor the barotropic fluid entirely dominates the evolution. Instead we have a scaling solution where the energy density of the scalar field remains proportional to that of the barotropic fluid with $\Omega_\phi = 3\gamma/\lambda^2$. Since both μ_1 and μ_2 are negative for $\lambda^2 > 3\gamma$ from Eq. (83), the point (d) is stable in this case. Meanwhile point (d) is a saddle point for $\lambda^2 < 3\gamma$, but this case is not physical because the condition, $\Omega_\phi \le 1$, is not satisfied. We note that the point (d) becomes a stable spiral for $\lambda^2 > 24\gamma^2/(9\gamma + 7)$. One therefore sees that the fixed point (d), due to its attractive behavior, allows for a very wide range of initial conditions for ϕ and $\dot{\phi}$ to rapidly approach a common evolutionary track, so that the cosmology is insensitive to the initial conditions. In Fig. 1 we present the phase space plot for $\lambda = 2$ and $\gamma = 1$. As already noted, the trajectories are confined within the circle given by $x^2 + y^2 = 1$ with $y \ge 0$. In this particular case the point (c) is a saddle point, whereas the point (d) is a stable spiral. Hence the late-time attractor is the scaling solution (d) with $x = y = \sqrt{3/8}$.

The above analysis of the critical points shows that one can obtain an accelerated expansion provided that the solutions approach the fixed point (c) with $\lambda^2 < 2$, in which case the final state of the universe is the scalar-field dominated one ($\Omega_\phi = 1$). The scaling solution (d) is not viable to explain late-time acceleration as the solution does not exit from the scaling regime ($\Omega_\phi = constant$) to connect to the accelerated epoch. So long as the field tracks the equation-of-state of the background, it cannot overtake the matter-density and induce cosmic acceleration. Hence, this is an unacceptable candidate for the dark energy component. In order to allow Ω_ϕ to catchup with Ω_m and thus put the universe into an accelerating phase, the slope of the field potential must gradually decreases and become shallow at late time as compared to the one corresponding to the exponential case. This can be realized by the field potential in which λ decreases with time. In fact, the scaling solutions live right on the the border between accelerating and decelerating cosmic behavior.

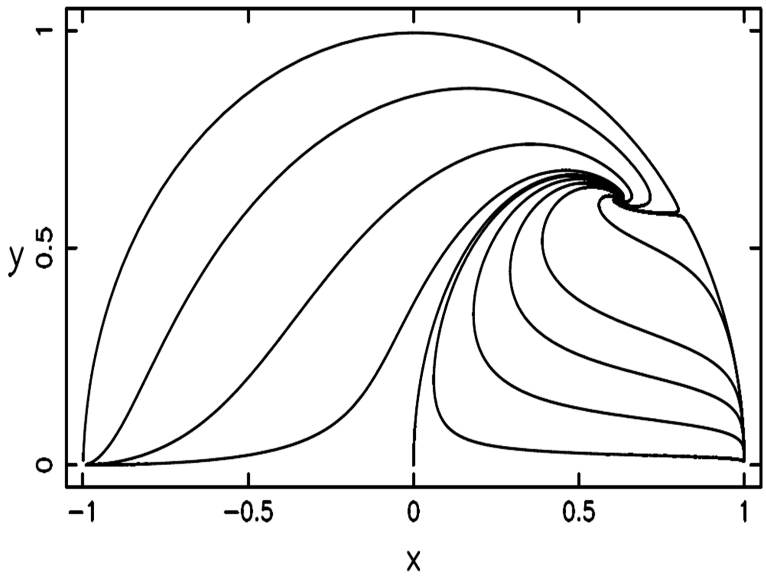

Figure 1: The phase space for $\lambda = 2$ and $\gamma = 1$. The scalar field dominated solution (c) is a saddle point with $x = (2/3)^{1/2}$ and $y = (1/3)^{1/2}$. The point (d) is a stable spiral in this case; Hence, the late-time attractor corresponds to the scaling solution with $x = y = (3/8)^{1/2}$. From Ref. [10].

6.3.2 Tracker Solutions

Tracker models aim to improve on the scaling model and address the problem faced by them; the inability of the field to exit the scaling regime and catchup to the background energy density, and to ultimately dominate the universe. In tracker models, the problem is addressed by demanding the curvature of the potential to ultimately fall below a critically small value once the field passes a particular value. As we shall see this causes the field to become frozen in its track and begins to act like a cosmological constant.

Recall that the exponential potentials correspond to constant λ and $\Gamma = 1$. Consider a potential $V(\phi)$ along which the field rolls down toward plus infinity with $\dot{\phi} > 0$. This means that $x > 0$ in Eq. (70). Then if the condition,

$$\Gamma > 1\,, \tag{84}$$

is satisfied, λ decreases toward 0. Hence the slope of the potential defined by Eq.(67) becomes even more flat, thereby allowing the the field to overtake the matter density and cause an accelerated expansion at late times. condition (84) is regarded as the tracking condition under which the energy density of ϕ eventually catches up that of the fluid [11, 14]. The tracker models are similar to inflation in that they funnel a diverse range of initial conditions into a common final state. In order to construct viable quintessence models, we require that the potential to satisfy this condition. [20]
In order to study the evolution of the scalar field when the slope of the potential is

[20]When $\Gamma < 1$ the quantity λ increases towards infinity. Since the potential is steeper than the one corresponding to scaling solutions, the energy density of the scalar field becomes negligible compared to that of the fluid. Then we do not have an accelerated expansion at late times.

varying (i.e.,λ is changing with time), we rewrite equations of motion by defining $\epsilon = 1/\lambda$, $x = \epsilon X$, and $y = \epsilon Y$. We then have[14],

$$\frac{dX}{dN} = \sqrt{6}(\Gamma - 1)X^2 - 3X + \sqrt{\frac{3}{2}}Y^2 + \frac{3}{2}X[2\epsilon^2 X^2 + \gamma(1 - \epsilon^2 X^2 - \epsilon^2 Y^2)], \quad (85)$$

$$\frac{dY}{dN} = \sqrt{6}(\Gamma - 1)XY - \sqrt{\frac{3}{2}}XY + \frac{3}{2}Y[2\epsilon^2 X^2 + \gamma(1 - \epsilon^2 X^2 - \epsilon^2 Y^2)], , \quad (86)$$

$$\frac{d\epsilon}{dN} = \sqrt{6}\epsilon(\Gamma - 1)X\,. \quad (87)$$

For ϵ small (λ large) or $\Gamma \approx 1$, ϵ becomes nearly constant. Moreover, if Γ is also nearly constant we can solve $dX/dN = dY/dN = 0$ and find the following instantaneous critical points

$$x_c(\lambda) = \sqrt{\frac{3}{2}}\frac{\gamma_\phi}{\lambda}, \quad (88)$$

$$y_c^2(\lambda) = \frac{3}{2}\frac{\gamma_\phi}{\lambda^2}(2 - \gamma_\phi)\,, \quad (89)$$

where the equation of state of the scalar field is

$$\gamma_\phi = \frac{1}{2}[\gamma + (2\Gamma - 1)\lambda^2/3] \pm \frac{1}{2}\sqrt{[-\gamma + (2\Gamma - 1)\lambda^2/3]^2 + 8\gamma(\Gamma - 1)\lambda^2/3}\,. \quad (90)$$

We only consider the negative root as the plus root lead to unphysical results. The contribution of the field to the total energy density is $\Omega_\phi = 3\gamma_\phi/\lambda^2$. Note that in the limit when $\Gamma - 1 \ll 1$ one has

$$\gamma_\phi = \gamma[1 - \frac{2(\Gamma - 1)}{1 - 3\gamma/\lambda^2}]\,. \quad (91)$$

In other words, for potentials with small curvature, the equation of state of the scalar field is very close(but not the same, as in the scaling case) to the equation of state of the background fluid, and it is said that the field tracks the background fluid. This property allows the possibility for the field to dominates at some point in the future and this is exactly what one wants for obtaining late time acceleration. When $\Gamma = 1$ then $\gamma_\phi = \gamma$ as it should since this is just the scaling case considered in the last section. In the limit of large λ, when the background fluid is completely dominating, from Eq.(90), one gets the following relation for the equation of state of scalar field

$$\gamma_\phi = \frac{\gamma}{2\Gamma - 1}. \quad (92)$$

So if a potential satisfies the tracking condition $\Gamma > 1$, then one can see that in the case of matter dominated background($\gamma = 2$) the equation of state of the field($w_\phi = \gamma_\phi - 1$) is necessarily negative. Hence allowing for the field to catch up ad exit the scaling regime. In the limit when $\Gamma - 1 \approx 0$, the above expression is equivalent to Eq.(91) in the limit of large λ. The eigenvalues for the Jacobian matrix of linearized perturbation matrix in

Eq.(137) are

$$m_{\pm} = -\frac{3}{4\lambda^2}\left[(\gamma-\gamma_\phi)(3\gamma_\phi+\lambda^2)+(2-\gamma_\phi)\lambda^2\right]\times$$
$$\left[1\pm\sqrt{1-\frac{8\,\lambda^2(2-\gamma_\phi)(\gamma\lambda^2-3\gamma_\phi^2)}{[(\gamma-\gamma_\phi)(3\gamma_\phi+\lambda^2)+(2-\gamma_\phi)\lambda^2]^2}}\,\right]. \tag{93}$$

These eigenvalues reduce to those in Eq.(83) when $\gamma = \gamma_\phi$ (the pure exponential case) as they should. The system is stable if the real part of both eigenvalues is negative. From Eq.(93), this is true if,

$$\begin{aligned}\gamma_\phi &< \sqrt{\frac{\gamma\lambda^2}{3}}, \quad \text{and}\\ \gamma_\phi &< \frac{3\gamma-2\lambda^2}{6}\left(1-\sqrt{1+\frac{12\lambda^2(2+\gamma)}{(3\gamma-2\lambda^2)^2}}\right).\end{aligned} \tag{94}$$

Moreover, if the quantity under the square root of Eq. (93) is negative, the critical points are a stable spiral; and a stable node, otherwise. The first of the conditions in Eqs. (94) marks the point for which only one of the real parts of the eigenvalues becomes positive, and the second condition the point where both real parts of the eigenvalues become positive. In the limit of large λ the latter is $\gamma_\phi = 1+\gamma/2$. For a large class of scalar potentials $V(\phi)$, stability is possible for a large number of e-folds allowing a scalar field sub-dominance during a long period of time as we will see in the applications below. We should point out that so far these results are general since we have not assumed, yet, any type of potential $V(\phi)$.

But in order to obtain viable dark energy models, we must further show that the scalar field dominated solution in an attractor. That is, a viable tracking model must give rise to accelerated universe at late time. To this end we now turn to analysis for the scalar field dominated universe which corresponds to the case when evolution of the scalar field approaches the unit circle $x^2+y^2=1$. Accordingly, we introduce new variables $x=\lambda X$ and $y^2 = 1-\lambda^2 Y^2$. Then, the dynamical equations become,

$$\begin{aligned}\frac{dX}{dN} &= \sqrt{6}\lambda^2(\Gamma-1)X^2-3X+\\ &\quad\sqrt{\frac{3}{2}}(1-\lambda^2Y^2)^2+\frac{3}{2}\lambda^2X[2X^2+\gamma(-X^2+Y^2)]\end{aligned} \tag{95}$$

$$\begin{aligned}Y\frac{dY}{dN} &= \sqrt{6}\lambda^2(\Gamma-1)XY^2+\\ &\quad\sqrt{\frac{3}{2}}X(1-\lambda^2Y^2)-\frac{3}{2}(1-\lambda^2Y^2)[2X^2+\gamma(-X^2+Y^2)].\end{aligned} \tag{96}$$

In order make the analysis simpler here, we note that because at late time field should freeze in its track, one can apply the slow roll conditions discussed earlier in Sec.6.3. These conditions then imply that λ is approaching zero and Γ is nearly constant. Accordingly, we take $\lambda \approx 0$ and and assume $\lambda^2(\Gamma-1)$ nearly constant.

Eq. (95) then reads,

$$\begin{aligned} X' &= \sqrt{6}\lambda^2(\Gamma-1)X^2 - 3X + \sqrt{\frac{3}{2}}\,, \\ YY' &= \sqrt{6}\lambda^2(\Gamma-1)XY^2 + \sqrt{\frac{3}{2}}X - \frac{3}{2}[2X^2 + \gamma(-X^2 - Y^2)]\,. \end{aligned}$$

The system has critical points in,

$$x_c(\lambda) = \frac{\lambda_\phi}{\sqrt{6}}\,, \quad y_c^2(\lambda) = 1 - \frac{\lambda_\phi^2}{6}\,. \tag{97}$$

Hence, the scalar field is dominant, $\Omega_\phi = 1$ and $\gamma_\phi = \lambda_\phi^2/3$, where we have defined

$$\lambda_\phi = \frac{3}{2}\left[\frac{1 \pm \sqrt{1 - 4(\Gamma-1)\lambda^2/3}}{(\Gamma-1)\lambda}\right]\,, \tag{98}$$

for $\Gamma \neq 1$, and $\lambda_\phi = \lambda$ otherwise. As before, only the minus solution has physical meaning. Expanding λ_ϕ we can approximate the solution by

$$\lambda_\phi = \lambda\left[1 + \frac{1}{3}(\Gamma-1)\lambda^2\right]\,. \tag{99}$$

As we did for the tracker solution, we perturb the solutions around the critical points to study their stability. Expanding Eq. (95) and using Eq. (98) to write $\Gamma - 1$ in terms of λ and λ_ϕ, we find the following eigenvalues:

$$\begin{aligned} m_+ &= 6\,(\lambda_\phi - \lambda)\frac{1}{\lambda_\phi} + \frac{1}{2}(\lambda_\phi^2 + \lambda\lambda_\phi - 6\gamma) \\ m_- &= 3 - 6\,\frac{\lambda}{\lambda_\phi} + \frac{1}{2}\,\lambda_\phi(3\lambda_\phi - 2\lambda)\,. \end{aligned}$$

For $\Gamma = 1$ this expression reduces to the eigenvalues found in Eq.(82), as we would expect.

The prime example of a potential giving rise to this tracking behavior is the inverse power-law potential[22] motivated by models of dynamical breaking of supersymmetry

$$V(\phi) = \frac{M^{4+\alpha}}{\phi^\alpha}\,. \tag{100}$$

for which the relevant quantities are

$$\lambda = -\frac{\alpha}{\kappa\phi}, \quad \Gamma - 1 = \frac{1}{\alpha}\,. \tag{101}$$

We have seen in Sec.(6.3.2) that when the background fluid is dominant the equation of state of the scalar field can be well approximated by Eq.(92). Therefore, in this case the equation of state is a constant and is given by

$$\gamma_\phi = \frac{\gamma\alpha}{2+\alpha}. \tag{102}$$

It is easy too see that Eq.(102) still holds for negative α provided $\alpha < -2$. Moreover, we have seen that for large λ, the solutions are stable provided $\gamma_\phi < 1 + \gamma/2$, which yields,

$$\begin{aligned} \alpha &> 2\frac{\gamma+2}{\gamma-2}, \qquad \alpha > 0\,, \\ \alpha &< 2\frac{\gamma+2}{\gamma-2}, \qquad \alpha < 0\,. \end{aligned} \tag{103}$$

For $\alpha > 0$ the condition is always true, however, for $\alpha < 0$ it imposes the bounds $\alpha < -6$ and $\alpha < -10$ for matter and radiation dominated fluids, respectively.

To sum up, the interesting and significant features of tracking are that as for the scaling case, a wide range of initial conditions are drawn towards a common cosmic history. However, the tracking solutions do not self-adjust to the background equation-of-state, but, instead, maintain some finite difference in the equation-of-state such that the scalar field ultimately dominates and the universe enters a period of acceleration. These properties are attractive as means to alleviate the coincidence problem, however they do not entirely resolve the problem: Although Ω_ϕ grows in the future to unity it remains to be explained why it roughly equals Ω_m near the present epoch and not any other time. Indeed the value of the potential energy at which the field starts to freeze in its track is inputed into our theory such that acceleration happens only recently. On the other hand, The tracker solutions do alleviate some aspects of the fine tunning problem mentioned earlier. To see this consider the free parameter M associated with the inverse power law potential in Eq.(102). The value of this parameter is determined by the observational constraint that $\Omega_\phi \approx 0.7$ today. From the Friedmann constraint, one can estimate the present potential energy of quintessence which is itself roughly equal to energy density of the field in the slow roll limit (see (Eq.59)). Hence one has

$$\rho_\phi^{(0)} \approx M_{\rm pl}^2 H_0^2 \approx 10^{-47}\,{\rm GeV}^4\,. \tag{104}$$

where $M_{\rm pl}$ is the reduced Planck mass. The mass squared of the field ϕ is given by $M_\phi^2 = \frac{d^2V}{d\phi^2} \approx \rho_\phi/\phi^2$, whereas the Hubble expansion rate is given by $H^2 \approx \rho_\phi/M_{pl}^2$. The universe enters a tracking regime in which the energy density of the field ϕ catches up that of the background fluid when m_ϕ^2 decreases to of order H^2 which is the characteristic property of all freezing models including the inverse power potential under consideration. [21]. This shows that the field value at present is of order the Planck mass ($\phi_0 \sim m_{\rm pl}$). Since $\rho_\phi^{(0)} \approx V(\phi_0)$, we obtain the mass scale

$$M = \left(\rho_\phi^{(0)} m_{pl}^\alpha\right)^{\frac{1}{4+\alpha}}\,. \tag{105}$$

This then constrains the allowed combination of n and M. For example the constraint implies $M = 1GeV$ for $n = 2$ [4]. This energy scale in turn can be compatible with the one in particle physics, which means that the severe fine-tuning problem of the cosmological constant is alleviated.

6.3.3 Pseudo Nambu Goldstone Bosons

We have seen that in order to roll slowly enough to produce accelerated expansion, the effective mass of the scalar field must be very light compared to other mass scales in particle physics, $m_\phi \approx 10^{-42}$ GeV, even though the field amplitude is typically of order the Planck scale, $\phi \approx 10^{19}$ GeV. Although up to now we have always set the coupling of quintessence to ordinary matter to zero, but in principle, Quintessence field must couple to ordinary matter, which even if suppressed by the Planck scale will lead to long range forces and time dependence of the constants of nature[9]. It is hard to conceive of a scalar field that is realistically embedded in a model of physics beyond the Standard Model with such such small mass. One way out of this problem is to consider models in which the light mass of the Quintessence field is protected from large quantum corrections by an underlying symmetry. This happens in the case of Pseudo Nambu Goldstone Boson(PNGB)[15] which provides a natural way to have an extremely low mass scalar and at the same time keep it dark or non-interacting with the Standard Model of particles physics. The best example of PNGB from particle physics is the π meson.[21]. An example of very light PNGB is the hypothetical axion associated with Peccei-Quinn symmetry introduced to solve the strong CP problem[49]. The key ingredients of the theory are the scale of spontaneous symmetry breaking f (at which the effective Lagrangian still retains the symmetry) and a scale of explicit symmetry breaking μ(at which the effective Lagrangian contains the symmetry breaking term)[15]. For generic PNGB when the symmetry breaking scale μ is set to zero, the symmetry becomes exact and radiative correction do not yield an explicit symmetry breaking term(radiative correction are multiplicative of scale μ)In order to keep the mass of the field small it must be protected by symmetries from large quantum corrections.[22] The associated potential is

$$V(\phi) = \mu^4(1 + cos(\frac{\phi}{f}) \tag{106}$$

The shift symmetry $\phi \to \phi + 2\pi f$ disables couplings to Standard Model fields that would otherwise spoil the darkness[6]. The mass of the field at the potential maximum is $m_\phi = \sqrt{V''} = -\mu^2/f$, where prime indicates differentiation with respect to the field ϕ. If this energy at potential maximum is responsible for the current accelerated expansion, from Friedmann constraint we have $3H^2 \approx (8\pi G)\mu^4$. Form this we see that $\mu^4 \approx \Lambda \approx (.003ev)^4$ and so $\mu \approx .003ev$. Then when f is of order of m_{pl} as required for the slow roll condition, we get

$$m_\phi = \frac{\mu^2}{f} = -3H_0^2 \tag{107}$$

The field has been frozen by Hubble friction through most of cosmic history when $|m_\phi^2|$ is smaller than H^2. It is currently relaxing to its ground state by rolling down the potential when $|m_\phi^2| \approx H_0^2$. In the future, the field will oscillate rapidly in the bottom of the

[21] longitudinal W and Z are exact goldstone bosons associated with the gauge fields

[22] This can be compared to the smallness of the mass of Higgs boson which must be protected from radiative correction. The resulting Hierarchy problem is postulated to be resolved through emergence of suppersymmetry.

potential, redshifting like non- relativistic matter[9].

This model offers no explanation for the coincidence problem; the coincidence is indeed a very brief moment in the course of evolution and just reflects the mass scale of the scalar field. However because the scales f and μ are derived from the energy scale of other physics this alleviates some need to explain the coincidence problem. The fine-tuning problem also remains unsolved as the early-time values of ϕ directly determine the present-day properties of the dark energy. However, the fine-tuning problem is eased because the PNGB potential is periodic; the range of starting values of ϕ that produces a viable scenario is a non-negligible portion of the allowed range $\phi \in [2\pi f]$.[6]

6.4 K-essence

Up to now we have considered only those Quintessence models that rely on the potential energy of scalar fields to lead to the late time acceleration of the universe. It is possible to have a situation where the accelerated expansion arises out of modifications to the kinetic energy of the scalar fields. The model has its roots in kinetic energy driven inflation, called K-inflation[23] but, the same principle can be used on modeling late time acceleration and the corresponding scenario is called K-essence[16, 17, 18]. K-essence is characterized by a scalar field with a non-canonical kinetic energy. In order to develop the theory we write the most general scalar-field action

$$S = \int \mathrm{d}^4x\sqrt{-g}\mathcal{L}(\phi, X)\,, \tag{108}$$

where

$$X = -(1/2)(\nabla\phi)^2\,, \tag{109}$$

is the canonical kinetic energy. We point out that the action (108) includes quintessence models as special cases. The Energy-momentum tensor obtained by varying the action (108) with respect to metric $g_{\mu\nu}$ is given by

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta S}{\delta g^{\mu\nu}} = \left(\frac{\partial\mathcal{L}(\phi, X)}{\partial X}\nabla_\mu\phi\nabla_\nu\phi - \mathcal{L}(\phi, X)g_{\mu\nu}\right) \tag{110}$$

If we introduce an effective 4-velocity

$$u^\mu = \sigma\frac{\nabla_\mu u}{\sqrt{2X}} \tag{111}$$

where$\sigma = sign(\partial_0\phi)$ (we assume $\nabla\phi$ to be time like.that is $X > 0$) by using Eq.111, the energy-momentum tensor 110 can be rewritten in the perfect fluid form corresponding to Eq.(112)

$$T^{\mu\nu} = (\rho + p)u^\mu u^\nu - pg^{\mu\nu}. \tag{112}$$

where the pressure and energy density are given by

$$p = \mathcal{L}(\phi, X)\rho = 2X\frac{\partial\mathcal{L}}{\partial X} - \mathcal{L} \tag{113}$$

The field equation for K-essence is obtained in the usual manner; by varying the action(**??**actionK) with respect to the field ϕ

$$\frac{1}{\sqrt{-g}}\frac{\delta S}{\delta\phi} = \frac{\partial\mathcal{L}}{\partial X}g^{\mu\nu} + (\frac{\partial^2\mathcal{L}}{\partial^2 X}\nabla^\mu\phi\nabla^\nu\phi)\nabla_\mu\nabla_\nu\phi + 2X\frac{\partial^2\mathcal{L}}{\partial X\partial\phi} - \frac{\partial\mathcal{L}}{\partial\phi} \tag{114}$$

Assuming as always that the field is homogeneous through space($\partial_i = 0$), In a flat FLRW universe, the relevant dynamical equation for the K-essence field become

$$H^2 = \frac{\kappa^2}{3}(\rho_B + \rho_\phi) = \frac{\kappa^2}{3}\left(\rho_B + 2X\frac{\partial\mathcal{L}}{\partial X} - \mathcal{L}\right), \tag{115}$$

$$\frac{\ddot{a}}{a} = -\frac{\kappa^2}{6}(\rho_B + 3p_B + \rho_\phi + 3p_\phi), \tag{116}$$

$$\dot{\rho}_B = -3H(1+w_B)\rho_B, \tag{117}$$

$$\ddot{\phi}\left(\frac{\partial p}{\partial X} + \dot{\phi}^2\frac{\partial^2 p}{\partial X^2}\right) + 3H\frac{\partial p}{\partial X}\dot{\phi} + \frac{\partial^2 p}{\partial X\partial\phi}\dot{\phi}^2 - \frac{\partial p}{\partial\phi} = 0 \tag{118}$$

where ρ_B and p_B are the energy density and the pressure of the background matter and/or radiation, respectively. Although k-essence models arise from any choice of a lagrangian $\mathcal{L}(\phi, X)$ with a nonlinear dependence on X, Usually K-essence models are restricted to the Lagrangian density of the form [16, 17, 18]:

$$\mathcal{L}(\phi, X) = f(\phi)(-X + X^2)\,. \tag{119}$$

The central point is that the cosmic acceleration can be realized by the kinetic energy X of the field. One of the motivations to consider this type of Lagrangian originates from string theory where non-standard kinetic terms appear generically in the effective action describing the massless scalar degrees of freedom.[23] Normally, the non-linear terms are ignored because they are assumed to be small and irrelevant. This is a reasonable expectation since the Hubble expansion damps the kinetic energy density over time. However, one case in which the non-linear terms cannot be ignored is if there is an attractor solution which forces the non-linear terms to remain non-negligible. This is precisely what occurs in the dynamics of Eq.119. It is the dynamical attractor behavior that makes such a model attractive for our porpuse. In this case, the pressure density and the energy density are given by

$$p = f(\phi)(-X + X^2)\rho = 2X\frac{\partial p}{\partial X} - p = f(\phi)(-X + 3X^2)\,. \tag{120}$$

Then the equation of state of the field is given by

$$w_\phi = \frac{p}{\rho} = \frac{1-X}{1-3X}\,. \tag{121}$$

Recall that scaling solutions keep w_ϕ constant. The above relation shows that in this case X is also a constant. Furthermore, recall that the energy density ρ satisfies the continuity equation (9). During the radiation or matter dominant epoch($\rho \ll \rho_B$), from

Eq. (21) the evolution of the Hubble rate is given by $H = 2/[3(1+w_B)(t-t_0)]$. Thus, Eq.(117) becomes

$$\dot{\rho} = -\frac{2(1+w_\phi)}{(1+w_B)(t-t_0)}\rho\,. \tag{122}$$

Substituting Eq.(121 and (120) in the above relation, we thus obtain the form of $f(\phi)$ that gives rise to scaling solution.

$$f(\phi) \propto (\phi - \phi_0)^{-\alpha}\,, \quad \alpha = \frac{2(1+w_\phi)}{1+w_B}\,, \tag{123}$$

Moreover, during scaling era $w_\phi = w_B$ and so the function $f(\phi)$ behaves as $f(\phi) \propto (\phi - \phi_0)^{-2}$ in the radiation or matter dominant era. From the above one can see that if, on the other hand, the function $f(\phi)$ is given by

$$f(\phi) \propto \phi^{-\alpha}, \tag{124}$$

then there exists a solution such that the equation of state is characterized by

$$w_\phi = \frac{(1+w_B)\alpha}{2} - 1. \tag{125}$$

This corresponds to the tracker solution for the specific model of K-essence under consideration. Hence if we require that $w_\phi < 0$ during the matter dominated epoch, then the exponent α should satisfy

$$\alpha < 2. \tag{126}$$

In this case the field will catchup with the background matter. Note that the weak energy condition ($w_\phi \geq -1$) can be violated if $\alpha < 0$. The stability against perturbations is signified by the speed of sound, defined by[4]

$$c_s^2 = \frac{\frac{\partial p}{\partial X}}{\frac{\partial \rho}{\partial X}} \tag{127}$$

For the model in Eq.(119), one finds $c_s^2 = (1+w_\phi)/(5-3w_\phi)$, and the model is unstable for perturbations on all length scales if the weak energy condition is violated: $w_phi < -1$. Hence we see that K-essence model can give rise to phantom fields. It goes without saying that the quintessence models do not share this instability property, in fact $c = 1$ inthat case. The solution is found to be an attractor, by the same method that we applied in the previous sections[17, 18]. The energy density of the k-essence field tracks the radiation density throughout the radiation-dominated epoch. But, at the onset of matter dominated era, the radiation-like attractor solution becomes unstable, and the energy density in the k-essence field begins to drop several orders of magnitude until a new matter-dominated attractor solution is found at which equation of state of K-essence resembles that of a cosmological constant. The drop in energy density means that the k-essence cannot dominate immediately. But, it is only a matter of time before it dominates the universe and throws it into a phase of cosmic acceleration[19] The debate is still on, whether k-essence models actually resolve the coincidence problem. A better understanding of non-linear attractor behavior is needed to see if simple, plausible examples can be found. Furthermore,, In order to apply this to dark energy we need to fine-tune $f(\phi)$ to be of order the present energy density of the universe.

6.5 Quintessential Inflation

Quintessential inflation[28] refers to a theory where the same field whose energy-stress tensor was responsible for the inflationary era in the very early universe, is in fact the agent causing the late time acceleration of cosmos. After all as we have seen both eras can be modeled by scalar fields slowly rolling down their potentials. In order to achieve this though, we must propose alternate mechanism for the creation of entropy at the end of inflation. As we mentioned in Sec.(5.1), in the usual inflationary scenario the inflation field is supposed to decay into particles(excitations of fields)via its coupling with the fields of fundamental physics. The couplings must be weak to protect the flatness of the potential which as we have seen is essential for inflationary process to be realized. on the other hand, they have to be strong enough to heat up the universe to such temperatures that usual FLRW cosmology takes over. It should be clear then, that in this case the same field can not act as a quintessence in the late time universe as it completely decays at the bottom of its potential. The Alternative suggestion for production of entropy is through gravitational particle production. In this case the inflation field does not interact with matter and therefore its energy density can roll monotonically toward zero; resembling an effective cosmological constant at late times. To this it is essential the inflation potential does not possess a minimum so that it does not completely decay. A model for quintessential is given by [28]

$$\begin{aligned} V(\Phi) &= \lambda(\Phi^4 + M^4) \quad \text{for } \Phi < 0\,, \\ &= \frac{\lambda M^4}{1 + (\Phi/M)^\alpha} \quad \text{for } \Phi \geq 0\,. \end{aligned} \tag{128}$$

For $\Phi < 0$ we have ordinary chaotic inflation.(see Sec.(5.1)) Much later on, for $\Phi > 0$ the universe once again begins to inflate but this time at the lower energy scale associated with quintessence. Following inflation the remnant of inflation field energy density dominates the universe, but because most of its potential energy has been converted to kinetic energy, the $\dot{\Phi}^2/2$ term in Eq.(30) dominates over the V(Φ) term resembling a stiff matter with equation of state parameter, $w_\Phi = 1$. From Eq.(22), this in turn implies that $\rho_\Phi \propto a^{-6}$. Hence the expansion could in principle become radiation dominated before BBN. But the gravitational particle production is indeed very inefficient at producing entropy. This problem may be alleviated in the instant preheating scenario. in the presence of an interaction $(1/2)g^2\phi^2\chi^2$ between the inflation ϕ and another field χ;where g is the coupling constant. Needless to say quintessential inflation also requires a degree of fine tuning, in fact perhaps even more than before as there are no tracker solutions we can rely on for the initial conditions.

The quintessential inflation model simplifies the role of the inflation by decoupling it from the matter. It remains to be seen whether this will aid the search for a believable physical basis for the inflation. Detection of gravity waves which are predicted in this theory, offers a great experimental tool to test this theory.

7 Alternative Ideas

Having discussed scalar field models of dark energy, we now turn to some alternative ideas that dispense with the idea of dark energy, altogether. Rather, they postulate other mechanism that could explain the current observed cosmic acceleration. Here, we present two very different alternate approaches.

7.1 Modified Gravity

The necessity to introduce some form of dark energy to explain the late time acceleration is based on validity of general relativity. Rather than to alter the energy content of the universe, it could be entirely possible that cosmic acceleration arises from new gravitational physics. More precisely one might suspect that the relation between stress-energy tensor and spacetime geometry is given by a different theory of gravitation on cosmological moderate scales than one proposed by Einstein in Eq.(2). General relativity precision tests regarding the solar system and binary pulsars correspond to length scales characteristic of order of 10^{13} cm. But, an length important scale for cosmology is the Hubble length, $H_0^{-1} \sim 5000$ Mpc $\sim 10^{28}$ cm, fifteen orders of magnitude larger[4]. An extrapolation of fifteen orders of magnitude in energy from that achieved at the largest accelerators, $\sim 10^{12}$ ev, brings us to the very different world of the Planck energy. Are we justifies in making such enormous extrapolation? From a fundamental physics point view, we indeed are as all the known open issues of physics have to do with small length scales. Also, if the physics of cosmology were very different from general relativity it would have already been manifest in serious problems with the cosmological tests. Having said this, we emphasize once again that in the standard cosmology the two dominant contributions to the stress-energy tensor, dark energy and dark matter, are hypothetical, introduced to make the theories fit the observations. Moreover, our most basic notion about how gravity works is violated as the cosmic acceleration seems to imply that distant galaxies repel rather than attract each other. To this end many alternative theory have been proposed such as scalar-tensor gravity[24], Dvali-Gabadadze-Porrati (DGP) [25] or brane-world motivated scenarios [26]. But non of them provides as simple a description of gravitational phenomena as does GR. This failure of alternative gravitational theory stems from the fact they must be able to reproduce success of conventional GR at solar system scale. Consequently, describing alternative theory of gravitation does not result in any savings in terms of economy of description. Nevertheless, the absentee of dark energy in these models as the agent responsible for cosmic acceleration, make them theoretically appealing.
General relativity is based on the Einstein Hilbert action which takes the following

$$S = \int d^4x \sqrt{g}\, R \,. \tag{129}$$

Among Extended Theories of Gravity, f(R) gravity[24] represents a viable alternative to dark energy and naturally gives rise to accelerating singularity-free solutions in early and late cosmic epochs. These models propose adding terms to the action that are proportional to R^n. It is known that, for $n > 1$, such terms lead to modifications of the standard cosmology at early times which lead to de Sitter behavior (e.g.,Starobinsky inflation [59]). that, for $n < 0$, such corrections become important in the late Universe and can lead to self-accelerating vacuum solutions, providing a purely gravitational alternative to dark energy[53]. In this view, the simplest to modify gravitational theory when the curvature becomes very small (at late times in the universe) is to simply add a piece proportional to $1/R$,

$$S = \int d^4x\sqrt{-g}\left(R - \frac{\mu^4}{R}\right) , \tag{130}$$

where μ is a parameter with dimensions of mass. Although this model gives rise to accelerating vacuum solution[24], However it was shown that this simple model is plagued by a matter instability[54] as well as by a difficulty to satisfy local gravity constraints[55]. These results show how non-trivial it is to obtain a viable f(R) model. Furthermore these models give rise to a new massive scalar degree of freedom in addition to the conventional massless gravitons. As a result of the coupling of this field to matter, gravitational constant, G, is time and scale dependent.

Investigations are still ongoing to see whether modification of GR could explain the acceleration of the universe while remaining consistent with experimental tests; in the meantime, the difficulty in finding a simple extension of GR that does away with the cosmological constant provides yet more support for the standard scenario.

7.2 Inhomogeneous Universe

Instead of modifying the material content or Equations of General Relativity to explain the supernova observations, a third logical possibility is to drop the assumption that the Universe is spatially homogeneous on large scales. Although the Universe has been observed to be very nearly isotropic on our celestial sphere, on the basis of the near-isotropy of the CMB temperature pattern [63], But the evidence for the other fundamental tenet of modern cosmology, homogeneity, is weaker[57]. While the Universe has been observed to be approximately homogeneous across the distances probed by large-scale structures, but radial homogeneity on scales $\geq 1Gpc$, remains to be proven. It has been argued that the non-linear gravitational effects of spatial density perturbations, when averaged over large scales, could yield a distance-redshift relation in our observable patch of the Universe that is very similar to that for an accelerating, homogeneous Universe [56], eliminating the need for either dark energy or modified gravity. Such scenarios can be studied using Lemaitre-Tolman-Bondi(LTB) model for a spherically symmetric inhomogeneous expanding universe filled with dust. The LTB line-element, in co-moving coordinates (r, θ, φ) and cosmic time t, is

$$ds^2 = -dt^2 + S^2(r,t)dr^2 + R^2(r,t)(d\theta^2 + \sin^2\theta d\varphi^2), \tag{131}$$

in units $c = 1$. Note that the model is spatially isotropic as functions S and R depend on position only via the radial coordinate r.One would also wants the models to be very similar to an FLRW one at the beginning of the expansion, but become observably different at later times. In this manner, we could retain the accomplishments of the FRW cosmology in dealing with early epochs. In fact, models of the universe described by a

LTB spacetime, have been shown to produce a Hubble diagram that is consistent with observations[57]. These models require no cosmological constant or other form of dark energy, and locally resemble a matter-dominated, low-density universe or void. The major failure of these models is that in order to be consistent be consistent with the SN data and not conflict with the isotropy of the CMB, our observational place needs to occupy a very special location, at or near the center of the void, in violation of the Copernican Principle[21].
Whether or not such models can be made consistent with the wealth of precision cosmological data remains to be seen; moreover, requiring our galaxy to occupy a privileged location, in violation of the spirit of the Copernican principle, is not, as of yet, theoretically favored[6].

8 Conclusion

The discovery of cosmic acceleration has uncovered a deep dark secret that that will revolutionize our whole view of the universe and our place in it. Due to the enormous length scales associated with this phenomenon, Only, the universe itself is the appropriate laboratory for unwinding this secret. Awaiting future high precision measurements that could help us to get a better understanding of its causal agent, mind can run wild in coming up with possible explanations. Indeed, there is no shortage of ideas concerning this very issue in literature and in this paper we just touched on the surface. We saw that although a cosmological constant provides the simplest explanation, but at the same time accepting such an explanation entails to come to terms with the idea that for no (as of yet) fundamental reason we live in a highly fine tunned universe where even a small deviation of this constant would preclude our very existence. Of course this would not be the only constant whose value needs to fine tunned in order to explain the observed universe. But physicist, being physicist, have instead come up with a diverse set of ideas ranging from proposing a new scalar degree of freedom, to more radical ideas such as modification of gravity or even possibility of living a vast local void. But to this date, all of the proposed models require fine tunning of their parameters if they are to explain the current phase of cosmic acceleration; some more than others. Perhaps, our inability to come up with a definite answer, stems from the fact that our current understanding of fundamental physics has its limitation. In particular it seems very likely that in order to understand the universe at very large scales, we need to know the behavior of spacetime on the scale of very small. This in turn would imply that perhaps a unified theory of all fundamental interaction, would give our universe as its unique solution and thus doing away with all the fine tunning that plague all our current cosmological models. Or perhaps this is just to much to expect, and we may have to live with the fact that we live in finely tunned universe.

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10 Appendix

10.1 Autonomous system of scalar-field dark energy models

We briefly present some basic definitions related to dynamical systems. For simplicity we shall study the system of two first-order differential equations, but the analysis can be extended to a system of any number of equations. Let us consider the following coupled differential equations for two variables $x(t)$ and $y(t)$:

$$\dot{x} = f(x, y, t)\,, \quad \dot{y} = g(x, y, t)\,, \tag{132}$$

where f and g are the functions in terms of x, y and t. This system is said to be autonomous if f and g do not contain explicit time-dependent terms. The dynamics of the autonomous systems can be analyzed in the following way.

10.1.1 Fixed or critical points

A point (x_c, y_c) is said to be a *fixed point* or a *critical point* of the autonomous system if it satisfies

$$(f, g)|_{(x_c, y_c)} = 0\,. \tag{133}$$

A critical point (x_c, y_c) is called an *attractor* when it satisfies the condition

$$(x(t), y(t)) \to (x_c, y_c) \;\text{ for }\; t \to \infty\,. \tag{134}$$

10.1.2 Stability around the fixed points

In order to determine the stability of fixed points we consider perturbation δx and δy around the critical point (x_c, y_c),

$$x = x_c + \delta x\,, y = y_c + \delta y\,. \tag{135}$$

Then substituting into equation (132) leads to the first-order differential equations:

$$\frac{d}{dN}\begin{pmatrix} \delta x \\ \delta y \end{pmatrix} = \mathcal{M}\begin{pmatrix} \delta x \\ \delta y \end{pmatrix}, \tag{136}$$

where $N = \ln(a)$ is the number of e-foldings which is convenient to use for the dynamics of dark energy. The Jacobian matrix $\mathcal{M}$ depends upon x_c and y_c, and is given by

$$\mathcal{M} = \begin{pmatrix} \frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} \\ \frac{\partial g}{\partial x} & \frac{\partial g}{\partial y} \end{pmatrix}_{(x=x_c, y=y_c)}\,. \tag{137}$$

This has two eigenvalues μ_1 and μ_2. The general linearized solution around each fixed point can be written as

$$\delta x = C_1 e^{\mu_1 N} + C_2 e^{\mu_2 N}\,, \tag{138}$$

$$\delta y = C_3 e^{\mu_1 N} + C_4 e^{\mu_2 N}\,, \tag{139}$$

where C_1, C_2, C_3, C_4 are integration constants. The stability around the fixed points depends on the nature of eigenvalues and is classified according to the following [10]:

- (i) Stable node: $\mu_1 < 0$ and $\mu_2 < 0$.
- (ii) Unstable node: $\mu_1 > 0$ and $\mu_2 > 0$.
- (iii) Saddle point: $\mu_1 < 0$ and $\mu_2 > 0$ (or $\mu_1 > 0$ and $\mu_2 < 0$).
- (iv) Stable spiral: The determinant of the matrix $\mathcal{M}$ is negative and the real parts of μ_1 and μ_2 are negative.

A fixed point is an attractor in the cases (i) and (iv), but it is not so in the cases (ii) and (iii).

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