Effective Field Theories for Inflation

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I thank Professor Carlo Contaldi for proposing a great project to me and for supervising my dissertation. I have discovered a wonderful topic, and going through the study of effective field theory for inflation was both a mind-opening and great experience.

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Contents

Introduction

The description of the origin, the evolution and the fate of our universe becomes more accurate as time passes. Newton described the motion of planets in 1687, Einstein expanded it to the largest scales we could think of in 1916, and we are now able to understand a major part of the evolution of our universe (or at least we have a theory to depict it). Certainly, there are still consequent holes in the standard big bang cosmology, such as the problems of dark matter and dark energy that remain unsolved.

Leaving those difficulties aside, a lot of work has been done concerning the origin of the universe. The big bang model is able to describe in a very accurate way the growth of the spacetime and has been confirmed experimentally at many occasions, most notably by the observation of the CMB by Penzias and Wilson in 1965. However, sticking to this theory lead to, among other things , two issues : the flatness and horizon problems, requiring an incredible fine-tuning of the initial conditions, somehow indicating a limitation of the theory.

Such reservation can be waived by the theory of inflation that introduces a stage of extremely quick expansion of the universe, solving the shortcomings encountered by the hot big bang model. This idea of an inflation theory to solve the horizon problem was first introduced by Guth in 1981 [1]. The universe was supposed to undergo a cooling and to be trapped in a metastable false vacuum. The only way it could leave this state would be by quantum tunnelling to the true vacuum, giving an end to the inflation. But this first version, now referred to as *old inflation*, presented some problems, in particular for the reheating stage.

A revised version, proposed independently by Linde [2], and Albrecht and Steinhardt [3] soon after, solved the issues of Guth through the introduction of *slow-roll inflation* or *new inflation*. In this model, the inflation was driven by a scalar field slowly rolling down a potential energy hill.

Other models have been proposed afterward, such as *chaotic inflation*, or the use of string theory and supergravity in inflation models. In the end, the inflation seems to be a good mechanism to avoid the horizon and flatness problems. While not currently supported by observational data, it is widely accepted as a credible and convincing explanation of one of the first stages of the universe.

In addition to allowing us to solve the difficulties of the hot big bang model, the inflation paradigm also provides an explanation of the generation of the primordial perturbations that will later lead to the formation of large scale structure in the universe. The description of those fluctuations is therefore of prime interest and can also be used to test inflation.

If the derivation of the primordial perturbations has already been largely studied, it suffers from its inherent perturbative treatment, its model dependency and the unknown physics taking place at Planckian scales. Therefore, the application of an effective field theory to the subject of inflation seems natural to consider. It will allow us to describe inflation at some determined energy scales, without having any clue of the actual underlying physical process.

The main objective of this dissertation is to study an effective field theory for inflation that has been developed by Cheung et al. in [4]. In the first section, we will review some general features of the standard cosmological model and point out some issues. Then, we will give a dynamical explication to those problems through the theory of inflation.

The study of primordial fluctuations will be the subject of the second section, where we derive in a canonical way the spectrum of perturbations. We will use it to check the consistency of the effective theory we will describe thereafter.

The effective theory of Cheung et al. [4], or to be more accurate its derivation, is then presented in the third section, while in the fourth part we study some limits that can be compared to the results of the canonical derivation.

1 An Expanding Universe

The current model of cosmology is based upon two main statements, one coming from the observation of the galaxies by Hubble in 1929, the other being more philosophical, first introduced a few centuries ago by Newton in his *Philosophiæ Naturalis Principia Mathematica* (1687) and then adapted following the discoveries made in astronomy :

- The universe is in a period of expansion ;
- When viewed on a sufficiently large scale, every point in the universe shares the same properties as the others (cosmological principle).

Together with the framework of General Relativity, this led to the development of the standard theory describing the evolution of our universe.

1.1 Motivations and limitations of the Hot Big Bang model

1.1.1 The Standard Cosmological model

The standard big bang model is based on the cosmological principle, which implies that the universe is both *homogeneous*, i.e. is the same at every point, and *isotropic,* i.e. looking the same in every direction. With such a hypothesis, one can derive the FLRW metric :

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = -dt^2 + a^2 \left(\frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right), \tag{1}$$

where $a(t)$ is the scale factor, and k is the curvature of the universe. We can always redefine the coordinates such that k takes only one of the three specific values : $k = -1, 0, 1$, corresponding respectively to a open, flat or closed universe.

The scale factor $a(t)$ will describe the relative expansion of the universe. If it increases, the universe is in expansion, while if it decreases the universe is contracting. A widely used quantity based on the scale factor is the Hubble parameter :

$$H(t) = \frac{\dot{a}(t)}{a(t)},$$

where the dot stands for the time derivative.

The next natural thing to do is to turn to General Relativity to describe the evolution of the universe. Therefore, we start from the Einstein equations :

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G T_{\mu\nu}, \tag{2}$$

and using the specific expression for the metric 1, one can derive the Friedmann equations that will determine the evolution of the scale factor :

$$\begin{aligned} H^2 &= \frac{8\pi G}{3}\rho - \frac{k}{a^2} + \frac{\Lambda}{3}, \\ \frac{\ddot{a}}{a} &= -\frac{4\pi G}{3}(\rho + 3p) + \frac{\Lambda}{3}, \end{aligned} \tag{3}$$

where G is the Newton's constant, ρ is the density, p is the pressure and Λ is the cosmological constant. We will omit that last one in what follows.

The two Friedman equations are sufficient to describe $a(t)$, but we will just give a few more relevant expressions. Defining the ratio between energy density and critical energy

$$\Omega(t) = \frac{\rho(t)}{\rho_{\text{crit}}}, \qquad \text{where } \rho_{\text{crit}} = \frac{3H^2}{8\pi G},$$

we write the first equation of (3) in the following way, which will be useful after :

$$\Omega(t) - 1 = \frac{k}{a^2 H^2}. \tag{4}$$

Also of interest, the continuity equation $\nabla_\mu T^{\mu\nu} = 0$ for the t component gives :

$$\dot{\rho} = -3H(\rho + p). \tag{5}$$

We are now in possession of a certain amount of expressions that will allow us to compute the evolution of the universe through the determination of the scale factor. The solutions of the equations (3) and (5), however, depend on the content of the universe. For a radiation dominated universe, we obtain $a(t) \propto t^{1/2}$ while for a dust dominated universe, we get $a(t) \propto t^{3/2}$. In both cases, when going backwards in time, $a(t)$ decreases and the universe becomes hotter and denser, up until we reach the limit of validity of General Relativity.

This hot big bang model explains the recession of the galaxies observed by Hubble, but it has definitely been recognised since one of its predictions, the CMB, was observed by Penzias and Wilson in 1978. Up to now, it fits admirably the cosmological data, and has been adopted as the standard theory to describe the evolution of the universe.

However, we quickly run into several problems. First of all, the theory may fits the observations really well, but it is at the cost of additional assumptions about the content of the universe. We indeed need to introduce Dark Matter and Dark Energy as major constituents of the universe, even if we do not have a clue as to what they can be. We will not discuss this further, as these issues are still largely unknown today, but we will focus on other limitations of the theory, such as the flatness or the horizon problems, together with their resolution by the theory of inflation.

1.1.2 Flatness problem

The first question that arises is : why is the universe flat ?

Indeed, the measured value of $\Omega(t)$ today is around the unity [5], meaning that the universe is approximately flat. But, since a^2H^2 decreases and considering the expression 4, we see that $\Omega(t)$ increases as time passes. The evolution of Ω will depend on the content of the universe, but typically we have :

$$\begin{aligned} \text{Radiation era} : |\Omega(t) - 1| &\propto t^{2/3}, \\ \text{Matter era} : |\Omega(t) - 1| &\propto t. \end{aligned}$$

Therefore, if we go back in time and, the quantity $|\Omega(t) - 1|$ would have been closer and closer to zero. Here are some values at different stages of the beginning of the universe [6, 7]:

$$\begin{aligned} \text{Nucleosynthesis} : |\Omega(t) - 1| &< \mathcal{O}\left(10^{-16}\right), \\ \text{Electroweak scale} : |\Omega(t) - 1| &< \mathcal{O}\left(10^{-27}\right), \\ \text{Planck scale} : |\Omega(t) - 1| &< \mathcal{O}\left(10^{-64}\right). \end{aligned}$$

Those values show a significant fine-tuning problem of the $\Omega(t)$ parameter and it is clearly a hint that the big bang model is missing a point.

1.1.3 Horizon problem

The second question is : why is the universe so homogeneous ?

We observe today that causally disconnected regions of the universe share the same temperature and other physical properties. For instance, the CMB is remarkably homogeneous and isotropic, the anisotropies being of order $10^{-5}K$ on the whole sky, even if the portions that were in causal contact at its time of emission are of order of the degree.

To convince ourselves, let us compute the size of our universe and the size of a causality patch, following [8]. The size of our observable universe today is of

$$l_0 \sim ct_0 \sim 10^{26}\mathrm{m}.$$

But at an earlier time t_i, this domain was smaller with a ratio a_i/a_0 :

$$l_i \sim ct_0 \frac{a_i}{a_0}.$$

Now, let us compare this length with the size of a causal patch $l_c \sim ct_i$. If we take it at Planckian time, and using

$$\frac{a_i}{a_0} \sim \frac{T_0}{T_i} \sim 10^{-32,}$$

we get

$$\frac{l_i}{l_c} \sim \frac{t_0}{t_i}\frac{a_i}{a_0} \sim \frac{10^{17}}{10^{-43}} 10^{-32} = 10^{28}.$$

The observable universe is therefore much bigger than the part of the sky that is in causal contact, but it is thermalised with a huge precision. This is called the horizon or isotropy problem.

1.2 Inflation

To be perfectly accurate, the flatness and horizon problems of the standard big bang theory are not strictly inconsistencies. The initial conditions leading to such an universe appear to be very unlikely but still possible. However, if an additional theory were to give a dynamical explanation for the homogeneity and the flatness of the universe, one would be tempted to consider it with attention.

The theory of inflation provides such a dynamical mechanism, but it also describes the generation of primordial perturbations, making it even more appealing. We will get back to that later, after the introduction of the general features of inflation.

1.2.1 The comoving horizon

The main idea of inflation is the decrease of the comoving horizon

$$\tau = \int_0^a d\ln a' \; \frac{1}{a' H(a')}.$$

The difference between the comoving horizon and the comoving Hubble radius $(aH)^{-1}$ is worth emphasising. If two particles are separated by a distance greater than $(aH)^{-1}$, they cannot communicate *now.* If two particles are separated by a distance greater than τ, they are causally disconnected ; they could never have communicated at any point in the history of the universe. This nuance is important since τ can be bigger than $(aH)^{-1}$; particles previously in causal contact could not be able to talk to each other today [9] – a subtlety that will be primordial in what follows.

If the comoving Hubble radius was much bigger at a previous time, τ would receive main contributions from earlier times. Therefore, we need the value of $(aH)^{-1}$ to be larger in the past than now. If H is approximately constant while a grows exponentially, we obtain the desired behaviour, as the comoving Hubble horizon will be decreasing during inflation.

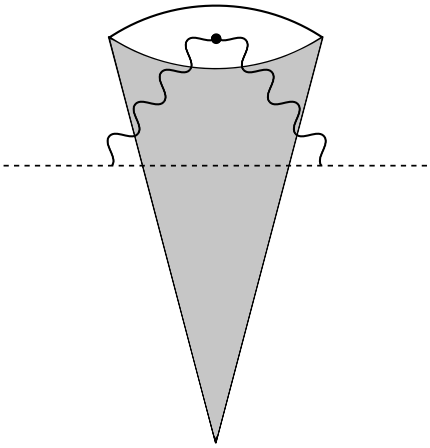
This solves easily the flatness problem ; if $(aH)^{-1}$ decreases sharply, then we see in (4) that the universe is driving toward flatness as Ω quickly approaches the unity. There is no longer any problem of fine-tuning for the value of $|\Omega - 1|$; the universe becomes increasingly flat as inflation expands the spacetime.

But it does also solve the horizon problem. The figure 1 qualitatively demonstrates how. The horizon problem comes from the fact that we observe causally disconnected regions that possess the same properties. One of the most glaring example of that, as symbolised on figure 1a, is the CMB, which is highly homogeneous on the whole sky. But this is assuming that the evolution of the scale factor $a(t)$ has always been the same, dictated by the matter or energy content of the universe.

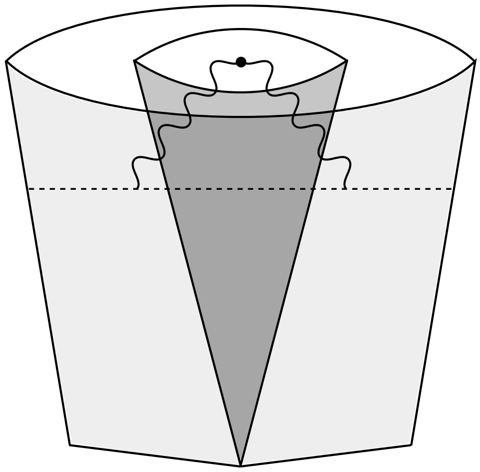
However, if we assume that in extremely early times, the horizon highly expanded, we can see on figure 1b that the causal patch is in fact much greater. The whole CMB we observe today therefore comes from a causally connected region, and it is no longer surprising to see that the temperature is the same everywhere.

We conclude that a first stage of quick expansion at the beginning of the universe would solve the flaws of the big bang model. But the growth of the scale factor $a(t)$ needs to be intense enough. A common way to achieve it is to keep a constant Hubble parameter H together with an exponentially growing scale factor :

$$a(t) \sim e^{Ht}.$$



(a) Without inflation. The observer receives signals from regions that were not in causal contact, therefore it is highly unlikely for those signals to share the same properties.



(b) With a first stage of inflation, the horizon expands quickly in early times and the observer receives signal from regions that were previously in causal contact even if they are not causally connected today. The horizon problem is therefore solved.

Figure 1: The horizon problem. The dashed line represent the last scattering surface, the dot correspond to the observer, while the different cones represent the horizon. Shaded zones were at least once in causal contact. The time is going upwards. Figure based on [10].

This exponential rise of $a(t)$ ensures that the horizon and flatness problems are definitely solved.

1.2.2 The equation of state and the inflaton

The need for a decreasing Hubble comoving radius is not without important implications. Indeed, we have

$$\frac{d}{dt}\left(\frac{1}{Ha}\right) = \frac{d}{dt}\left(\frac{1}{\dot{a}}\right) < 0, \Leftrightarrow \frac{d^2a}{dt^2} > 0 \Leftrightarrow \rho + 3p < 0$$

where we used the acceleration equation (3) and the fact that $a > 0$, $\dot{a}^2 > 0$. So, inflation equivalently corresponds to a decreasing Hubble radius, an accelerated expansion or the relation $p < -\frac{1}{3}\rho$. The last statement seems very unusual ; how can we realise an equation of state $\omega = \frac{p}{\rho} < -\frac{1}{3}$?

This is possible if we consider a single scalar field, called the inflaton. What follows is the simplest model for inflation, but plenty of other models exist with many different features.

The dynamic of the inflaton field is governed by the action

$$S_\phi = \int d^4x\sqrt{-g}\left[\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right].$$

We can derive the energy-momentum tensor

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta S_\phi}{\delta g^{\mu\nu}} = \partial_\mu\phi\partial_\nu\phi - g_{\mu\nu}\left(\frac{1}{2}\partial^\rho\phi\partial_\rho\phi + V(\phi)\right).$$

For a homogeneous field $\phi(x,t) = \phi(t)$, we therefore obtain

$$\begin{aligned} \rho &= \frac{1}{2}\dot{\phi}^2 + V(\phi) \\ p &= \frac{1}{2}\dot{\phi}^2 - V(\phi) \end{aligned} \tag{6}$$

which gives

$$\omega = \frac{p}{\rho} = \frac{\frac{1}{2}\dot{\phi}^2 - V(\phi)}{\frac{1}{2}\dot{\phi}^2 + V(\phi)} < -\frac{1}{3}. \tag{7}$$

In order for the condition (7) to be met, the potential needs to be always positive. Therefore,

$$\begin{aligned}
& \frac{\frac{1}{2}\dot{\phi}^2 - V(\phi)}{\frac{1}{2}\dot{\phi}^2 + V(\phi)} < -\frac{1}{3} \\
\Leftrightarrow \quad & \frac{1}{2}\dot{\phi}^2 - V(\phi) < -\frac{1}{3}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right) \\
\Leftrightarrow \quad & \frac{1}{2}\dot{\phi}^2 + \frac{1}{6}\dot{\phi}^2 < \frac{2}{3}V(\phi) \\
\Leftrightarrow \quad & \dot{\phi}^2 < V(\phi) \qquad (8)
\end{aligned}$$

we see that the potential must dominate over the kinetic energy. Under this condition, the universe is in accelerated expansion.

Before going on with this inequality, we adapt the Friedman equation (3) to our present case using the expressions (6) for the pressure and the energy density, where we choose the specific values $k = \Lambda = 0$:

$$H^2 = \frac{8\pi G}{3}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right). \qquad (9)$$

Similarly, using

$$\dot{\rho} = \frac{\partial}{\partial t}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right) = \frac{1}{2}\left(2\dot{\phi}\ddot{\phi}\right) + \frac{\partial V}{\partial t} = \dot{\phi}\ddot{\phi} + V_{,\phi}\,\dot{\phi},$$

the continuity equation (5) is

$$\dot{\rho} = 3H(p+\rho) = 3H\dot{\phi}^2$$

which gives

$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi} = 0. \qquad (10)$$

This last expression is the equation for a harmonic oscillator, with a damping term proportional to H.

1.2.3 The slow-roll regime

We know that, in order to have an accelerating expansion of the universe, the potential energy must be greater than the kinetic energy. In the limit of vanishing kinetic energy, the acceleration is exponential and never-ending.

Even if we want inflation to have an end, this gives us a good hint of a limit we could consider :

$$\dot{\phi}^2 \ll V.$$

Moreover, if the damping in the equation (10) is large, i.e. if the friction is important, the acceleration term can be neglected when compared to the friction term :

$$\ddot{\phi} \ll 3H\dot{\phi}.$$

This enforces a slow-roll regime, where the expressions (9) and (10) simplify to

$$H^2 \simeq \frac{8\pi G}{3} V(\phi)\,, \tag{11}$$

$$3H\dot{\phi} \simeq -V_{,\phi}\,. \tag{12}$$

Let us examine how the above expressions can be recast. Replacing (12)

$$\dot{\phi} \simeq -\frac{V_{,\phi}}{3H} \tag{13}$$

in the condition $\dot{\phi}^2 \ll V$, we get

$$\begin{aligned} & \left(\frac{V_{,\phi}}{3H}\right)^2 \ll V \\ \Leftrightarrow \quad & \frac{V_{,\phi}^2}{9\left(\frac{8\pi G}{3}V\right)} \ll V \\ \Leftrightarrow \quad & \frac{M_{\mathrm{Pl}}^2}{3}\frac{V_{,\phi}^2}{V^2} \ll 1. \end{aligned}$$

Similarly, the condition on the acceleration $\ddot{\phi} \ll 3H\dot{\phi}$ is written as

$$\ddot{\phi} = \partial_t\left(\dot{\phi}\right) \simeq \partial_t\left(\frac{V_{,\phi}}{3H}\right) = \frac{V_{,\phi\phi}}{3H}\dot{\phi},$$

yielding the following inequality :

$$\begin{aligned} & \frac{V_{,\phi\phi}}{3H}\dot{\phi} \ll 3H\dot{\phi} \\ \Leftrightarrow \quad & \frac{1}{9H^2}V_{,\phi\phi} \ll 1 \\ \Leftrightarrow \quad & \frac{1}{3\left(8\pi G\right)V}V_{,\phi\phi} \ll 1 \\ \Leftrightarrow \quad & \frac{M_{\mathrm{Pl}}^2}{3}\frac{V_{,\phi\phi}}{V} \ll 1. \end{aligned}$$

Defining the so-called *potential slow-roll parameters*

$$\varepsilon_V \equiv \frac{M_{\rm Pl}^2}{3}\left(\frac{V_{,\phi}}{V}\right)^2 \qquad \text{and} \qquad \eta_V \equiv \frac{M_{\rm Pl}^2}{3}\frac{V_{,\phi\phi}}{V},$$

the conditions determining the range of validity of the slow-roll regime are simply

$$\varepsilon_V \ll 1, \qquad |\eta_V| \ll 1.$$

The parameters constrain the potential to be almost flat, such that the inflaton field slowly roll down the potential. As long as those parameters are small, the spacetime inflates[1] and is approximately de Sitter

$$a\left(t\right) \sim e^{Ht}.$$

Some others parameters, called *Hubble slow-roll parameters*, are also used

$$\varepsilon_H \equiv -\frac{\dot{H}}{H^2} \qquad \text{and} \qquad \eta_H \equiv -\frac{1}{2}\frac{\ddot{H}}{\dot{H}H}, \tag{14}$$

with the correspondence

$$\varepsilon_V \simeq \varepsilon_H \qquad \text{and} \qquad \eta_V \simeq \eta_H + \varepsilon_H$$

which is only valid when the parameters are small and when the potential is very flat.

As long as the slow-roll parameters are small enough, the universe is in accelerated expansion. Once they get a value close to one and the inflaton rolled down the potential to the minimum, inflation ends, leaving the room for others mechanisms (such as reheating) leading to a FLRW universe.

The inflation needs to persist long enough to solve the horizon and flatness problems. A quantity often used to count the amount of inflation is the number of e-foldings N before the end of inflation :

$$N\left(\phi\right) \equiv \ln\left(\frac{a_{\rm end}}{a}\right).$$

As time goes by, N decreases and is equal to zero when inflation ends. It measures the number of times the space grew. The amount of e-folds necessary to solve the horizon and flatness problem is of at least $N \sim 60 - 75$

[1]To be more accurate, the universe will inflates if the condition $\varepsilon_V < 1$ is met. But the slow-roll regime we are interested in requires both $\varepsilon_V \ll 1$ and $\eta_V \ll 1$.

[8, 9], depending on the models and on the estimations of current parameters.

The quantity N can be related to the Hubble parameter :

$$\frac{dN}{dt} = \frac{d\ln a}{dt} = H. \tag{15}$$

and the following expression will be useful later :

$$\varepsilon_H = -\frac{\dot{H}}{H^2} = -\frac{1}{H^2}\frac{dH}{dN}\frac{dN}{dt} = -\frac{d\ln H}{dN}. \tag{16}$$

1.3 Reheating

The reheating mechanism is an important step in the history of the universe.

As we previously saw, inflation is not eternal but must end at some point, were it only to allow the universe to evolve into a FLRW space which we observe today. In the slow-roll approximation, inflation ends when the scalar field reaches the point where the potential is no longer flat enough, i.e. when the slow-roll parameters cannot be neglected anymore. From this point on, the inflaton scalar field will start to oscillate around the minimum of the potential, and a major part of the kinetic energy is stored in the oscillations of ϕ [11]. But if it were all, the universe would be empty of all the conventional matter we are made of. We also need to explain how the universe, left in a state of almost zero temperature at the end of inflation, can be reheated to connect smoothly to the high temperatures required for a hot Big Bang model [12].

The mechanism that allows the conversion of all the scalar field energy into the conventional matter of the Standard Model is called reheating. It happens through the coupling of the inflaton field to conventional matter. It was first thought as being quite simple but now the mechanism as a whole appears to be very rich and complicated, involving many different steps, such as the creation of particles, thermalisation, parametric resonance, and so on.

2 Cosmological Perturbations : A Canonical Approach

The inflation model solves the problems highlighted above. But it is also believed to explain the origin of primordial perturbations. In a few words, those come from the inflaton quantum fluctuations that have been stretched to galactic scales during the inflation stage, while the amplitude was kept almost unchanged. Such perturbations, by dint of gravitational instability, were the seeds for the formation of large-scale structures such as galaxies and clusters we observe today.

The spectrum of these perturbations is a prediction of the inflation theory, and it can be tested with the observations of CMB anisotropies and galaxies cluster surveys [13]. While the CMB fluctuations provide a powerful observational test, one should however be more careful about large-scale structures analysis, since the linear treatment usually used to derive the perturbations spectrum breaks down in the recent past of the history of the universe[14].

As usual in perturbation theory, the variables are split into a background term that is only time dependant, and the perturbation part

$$\begin{aligned}\phi\left(\boldsymbol{x},t\right) &= \phi_0\left(t\right)+\delta\phi\left(\boldsymbol{x},t\right)\\ \rho\left(\boldsymbol{x},t\right) &= \rho_0\left(t\right)+\delta\rho\left(\boldsymbol{x},t\right)\\ g^{\mu\nu} &= \overline{g}^{\mu\nu}+\delta g^{\mu\nu}...\end{aligned}$$

Since the perturbations are small, we can also expand the Einstein's equations at linear order in the perturbations

$$\delta G_{\mu\nu} = 8\pi G\delta T_{\mu\nu}.$$

2.1 Statistical tools for perturbation theory

The aim of this section is to review the derivation of the power spectrum $P_s\left(k\right)$ for the scalar perturbations. There also exist tensor perturbations, but we will not consider them here.

Let us introduce a few concepts used in this context of cosmological perturbations. We consider a random variable $X\left(x\right)$ representing the perturbations. In Fourier space, it can be written as :

$$X\left(\boldsymbol{k}\right) = \int d^3\boldsymbol{x}X\left(\boldsymbol{x}\right)e^{-i\boldsymbol{k}\boldsymbol{x}}.$$

An important statistical tool used in cosmology in order to study a random variable is the power spectrum $P_X\left(\boldsymbol{k}\right)$:

$$\left\langle X\left(\boldsymbol{k}\right)X\left(\boldsymbol{k}'\right)\right\rangle = 2\pi^2\delta\left(\boldsymbol{k}+\boldsymbol{k}'\right)P_X\left(k\right) \tag{17}$$

where the brackets $\langle\ldots\rangle$ denote the ensemble average of the fluctuations and $k=|\boldsymbol{k}|$. The dimensionless equivalent expression is

$$\Delta_X^2 = \frac{k^3}{2\pi^2}P_X\left(k\right).$$

It is worth noting that if X is a gaussian variable, all the statistical information is contained in the two points correlation function (17) and, as we are in an isotropic universe, in the power spectrum. The observation of a non-vanishing three-points function would be a powerful tool to discriminate different models of inflation and is an active field of research (see [15, 16, 17]for reviews), even if the last Planck results do not seem to favour large non-gaussianities.

We also define the spectral index n_s that encompasses the scale-dependence of the power spectrum :

$$n_s - 1 = \frac{d \ln \Delta_s^2}{d \ln k}.$$

where Δ_s^2 is the dimensionless power spectrum associated to $P_{\mathcal{R}}(k)$ and $\mathcal{R}$ is a gauge invariant quantity that will be defined later. When $n_s = 1$, the power spectrum is scale invariant.

2.2 Gauge-invariant variables

When dealing with the background evolution of the inflaton, there is an obvious choice of coordinates, from which all physical quantities can be easily defined. But when we consider perturbations from this background, there is no longer any such straightforward coordinate system, because there exist multiple ways to separate the background and the perturbations. That can lead to confusions on the nature of the perturbations. A solution is obviously to work with gauge invariant variables, as they will not be affected by a coordinate transformation. Some of them will even display useful features.

Let us therefore introduce the *curvature perturbation on uniform-density hypersurfaces*

$$-\zeta = \Psi + \frac{H}{\dot{\rho}_0}\delta\rho, \tag{18}$$

and the *comoving curvature perturbation*

$$\mathcal{R} = -\frac{H}{\dot{\phi}_0}\delta\phi.$$

Both those quantities are gauge invariant, and they remain constant after horizon crossing. On superhorizon scales, they are actually equivalent : $\mathcal{R} \simeq \zeta$.

We will no longer discuss properties of these variables as it is not necessary right now. We will see after why we do need them.

2.3 Perturbations

We are now ready to start the derivation of perturbations. We will closely follow the development made in [18]. What follows is a shortened version of the usual treatment presented in a multitude of books, lecture notes and articles. One in-depth development is made in [8].

2.3.1 Evolution of perturbations

The first step will be to consider the equation of motion at a classical level. In curved spacetime, the Klein-Gordon equation for a scalar field is written as :

$$\frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^{\mu}}\left(\sqrt{-g}g^{\mu\nu}\frac{\partial\phi}{\partial x^{\nu}}\right)+\frac{\partial V}{\partial\phi}=0. \tag{19}$$

The background metric for the universe is FLRW, but we are now dealing with perturbations ; therefore, the metric we will insert in 19 will be the perturbed metric. That gives the expression :

$$\delta\ddot{\phi}+3H\delta\dot{\phi}+\left(-\frac{1}{a^2}\nabla^2+\frac{d^2V}{d\phi^2}\right)\delta\phi=-2\Phi\frac{dV}{d\phi}+3\left(\dot{\Phi}+\dot{\Psi}\right)\dot{\phi}_0. \tag{20}$$

The next step is to perform a Fourier transform, and to choose a specific gauge to simplify the expression (20). It is possible to choose a gauge such that the metric perturbations make a negligible contribution to the equation of motion during inflation. We end up with

$$\delta\ddot{\phi}_{\boldsymbol{k}}+3H\delta\dot{\phi}_{\boldsymbol{k}}+\left[\left(\frac{k}{a}\right)^2+m^2\right]\delta\phi_{\boldsymbol{k}}=0. \tag{21}$$

We now consider H and m as being constant. Technically, they vary slowly , but the actual computation will only require their values at a given moment, so that we can consider them as constants. Moreover, since we are in slow-roll inflation, the inflaton has negligible mass $m^2 \ll H^2$. The solutions of (21) hence are :

$$\delta\phi_{\boldsymbol{k}}(t)=A_{\boldsymbol{k}}w_k(t)+B_{\boldsymbol{k}}w_k^*(t) \tag{22}$$

where

$$w_k(t)=\left(i+\frac{k}{aH}\right)\exp\left(\frac{ik}{aH}\right). \tag{23}$$

The expression 22 is a classical solution for the perturbation of the equation of motion in a FLRW background.

2.3.2 Vacuum perturbations in Minkowski space

Before going on with the computation of the power spectrum in the case of an inflating universe, we will consider the Minkowski case. It will gives us useful results that will be used in the next section.

The equation of motion for a free scalar particle in Minkowski space in Fourier space is

$$\ddot{\phi}_{\boldsymbol{k}} + E_k^2 \phi_{\boldsymbol{k}} = 0$$

where $E_k^2 = k^2 + m^2$. This is the equation for a harmonic oscillator. We will not go through the whole process of quantisation of the harmonic oscillator. We will simply review the main features of the quantisation.

There are creation $\hat{a}_{\boldsymbol{k}}$ and annihilation operators $\hat{a}_{\boldsymbol{k}}^{\dagger}$ for each Fourier mode $\boldsymbol{k}$. The groundstate $|0\rangle$ is defined as the one such that $\hat{a}_{\boldsymbol{k}}|0\rangle = 0$ for all $\boldsymbol{k}$, and the one particle states are $\hat{a}_{\boldsymbol{k}}^{\dagger}|0\rangle = |1_{\boldsymbol{k}}\rangle$. Those states are normalised, $\langle 1_{\boldsymbol{k}} \mid 1_{\boldsymbol{k}'}\rangle = \delta_{\boldsymbol{k}\boldsymbol{k}'}$. The commutation relations are

$$[\hat{a}_{\boldsymbol{k}}, \hat{a}_{\boldsymbol{k}'}] = \left[\hat{a}_{\boldsymbol{k}}^{\dagger}, \hat{a}_{\boldsymbol{k}'}^{\dagger}\right] = 0, \qquad \left[\hat{a}_{\boldsymbol{k}}, \hat{a}_{\boldsymbol{k}'}^{\dagger}\right] = \delta_{\boldsymbol{k}\boldsymbol{k}'}.$$

The field operator is

$$\hat{\phi} = \sum \underbrace{\left(w_k(t)\,\hat{a}_{\boldsymbol{k}} + w_k^*(t)\,\hat{a}_{\boldsymbol{k}}^{\dagger}\right)}_{\equiv \hat{\phi}_{\boldsymbol{k}}} e^{i\boldsymbol{k}\cdot\boldsymbol{x}},$$

where

$$w(t) = \frac{1}{\sqrt{2L^3 E_k}} e^{-iE_k t}. \tag{24}$$

The conjugate variable is

$$\dot{\hat{\phi}}_{\boldsymbol{k}} = -iE_k\left(w_k(t)\,\hat{a}_{\boldsymbol{k}} - w_k^* \hat{a}_{\boldsymbol{k}}^{\dagger}\right),$$

and the commutator is

$$\left[\hat{\phi}_{\boldsymbol{k}}, \dot{\hat{\phi}}_{\boldsymbol{k}"}\right] = iL^{-3}\delta_{\boldsymbol{k},-\boldsymbol{k}'}.$$

We are now able to compute the power spectrum

$$\Delta_\phi^2(k) = L^3 \frac{k^3}{2\pi^2} \langle|\phi_{\boldsymbol{k}}|\rangle^2.$$

For the vacuum state, the expectation value of $|\phi_{\boldsymbol{k}}|^2$ is

$$\begin{aligned}\langle 0 \mid \hat{\phi}_{\boldsymbol{k}}\hat{\phi}^{\dagger}_{\boldsymbol{k'}} \mid 0\rangle &= |w_k|^2 \langle 0 \mid \hat{a}_{\boldsymbol{k}}\hat{a}^{\dagger}_{\boldsymbol{k}} \mid 0\rangle + w_k^2\langle 0 \mid \hat{a}_{\boldsymbol{k}}\hat{a}_{-\boldsymbol{k}} \mid 0\rangle \\ &\quad + (w_k^*)^2 \langle 0 \mid \hat{a}^{\dagger}_{-\boldsymbol{k}}\hat{a}^{\dagger}_{\boldsymbol{k}} \mid 0\rangle + |w_k|^2 \langle 0 \mid \hat{a}^{\dagger}_{-\boldsymbol{k}}\hat{a}_{-\boldsymbol{k}} \mid 0\rangle \\ &= |w_k|^2 \langle 1_{\boldsymbol{k}} \mid 1_{\boldsymbol{k}}\rangle = |w_k|^2 .\end{aligned}$$

Using (24), we see that $|w_k|^2 = 1/\left(2L^3E_k\right)$, giving the final result for the power spectrum in Minkowski space :

$$\Delta^2_{\phi}(k) = \frac{k^3}{4\pi^2 E_k}.$$

2.3.3 Vacuum perturbations during inflation

We now turn to the inflation case. The background evolution and the perturbations will be treated differently ; we are going to quantise the perturbation keeping the background field at a classical level.

We have already seen that the solutions of the equation (21) are

$$w(t) = \frac{H}{\sqrt{2L^3k^3}}\left(i + \frac{k}{aH}\right)\exp\left(\frac{ik}{aH}\right)$$

where the prefactor is added for later convenience.

When the considered scale k is well inside the horizon, i.e. when $k \gg aH$, $\delta\phi_{\boldsymbol{k}}$ oscillates quickly with respect to H^{-1}. If we consider distances and times to be much smaller than Hubble scale, the curvature can be neglected and we can actually consider to be in a Minkowski space.

The operator for the perturbations is then

$$\delta\hat{\phi}_{\boldsymbol{k}}(t) = w_k(t)\,\hat{a}_{\boldsymbol{k}} + w_k^*\hat{a}^{\dagger}_{-\boldsymbol{k}},$$

and the power spectrum of inflation perturbations is then as in Minkowski space

$$\Delta^2_{\phi} = L^3\frac{k^3}{2\pi^2}|w_k|^2 .$$

Well before horizon exit, the field operator $\delta\hat{\phi}_{\boldsymbol{k}}$ and the Minkowski one are similar and one should expect similar fluctuations. However, when $k \ll aH$, the function $w_k(t)$ tends towards the constant

$$w_k(t) \to \frac{iH}{\sqrt{2L^3k^3}},$$

so after horizon exit, the fluctuations stop to evolve and the power spectrum is then

$$\Delta_\phi^2 = L^3 \frac{k^3}{2\pi^2} |w_k|^2 = \left(\frac{H}{2\pi}\right)^2 .$$

This result was obtained when H was considered constant, which is actually not the case. But, in the end, the quantity that will be of interest is the power spectrum of the gauge invariant variable $\mathcal{R}$. Therefore, as the value of $\mathcal{R}$ remains constant after horizon crossing, we only have to be able to compute its power spectrum at horizon crossing, when $k = aH$. This is the reason why we use the quantity $\mathcal{R}$, together with the fact that we do not need to worry about the validity of the development made in a peculiar gauge, since $\mathcal{R}$ is gauge-invariant.

Therefore, the power spectrum for the comoving curvature perturbation is

$$\Delta_\mathcal{R}^2(k) = \left(\frac{H}{\dot{\phi}_0}\right)^2 \Delta_\phi^2(k) = \left(\frac{H^2}{2\pi\dot{\phi}_0}\right)^2_{aH=k} .$$

That will be the main quantity used to study the scalar fluctuations in the following.

2.4 Power spectrum and spectral index

We have just derived the power spectrum for the scalar perturbations :

$$\Delta_s^2 = \frac{H_\star^4}{(2\pi)^2 \dot{\phi}_0^2},$$

where the star indicates that the expression is evaluated at horizon crossing $k = aH$. Using (9) and (10), we can recast it in the form

$$\Delta_s^2 = \frac{1}{8\pi^2} \frac{H_\star^2}{M_{\mathrm{Pl}}^2 \varepsilon_{H\star}}. \tag{25}$$

In terms of the potential slow-roll parameters, we have

$$\Delta_s^2 = \frac{1}{24\pi^2} \frac{V}{M_{\mathrm{Pl}}^4 \varepsilon_{V\star}}.$$

We are now ready to compute the spectral index n_s :

$$n_s - 1 = \frac{d\ln\Delta_s^2}{d\ln k} = \frac{d\ln\Delta_s^2}{d\ln N} \frac{d\ln N}{d\ln k}$$

where N is the number of e-folds. Using (16) and the following expression

$$\frac{d\ln\varepsilon_H}{dN} = 2\left(\varepsilon_H - \eta_H\right),$$

the first term becomes

$$\begin{aligned}\frac{d\ln\Delta_s^2}{d\ln N} &= \frac{d}{d\ln N}\left(\ln\left(\frac{1}{8\pi^2}\frac{H_\star^2}{M_{\mathrm{Pl}}^2\varepsilon_{H\star}}\right)\right)\\ &= 2\frac{d\ln H_\star}{dN} - \frac{d\ln\varepsilon_{H\star}}{dN}\\ &= -2\varepsilon_{H\star} - 2\left(\varepsilon_{H\star} - \eta_{H\star}\right)\\ &= 2\eta_{H\star} - 4\varepsilon_{H\star}.\end{aligned}$$

For the second factor, we use the fact that we are at horizon crossing, so $k = aH$ and

$$\ln k = \ln\left(aH\right) = \ln a + \ln H = N + \ln H.$$

Therefore, using again (16) and since ε_H is small,

$$\frac{d\ln k}{dN} = 1 - \varepsilon_H \qquad \Rightarrow \qquad \frac{dN}{d\ln k} \simeq 1 + \varepsilon_H.$$

This gives the final expression at first order in potential slow-roll parameters for the spectral index :

$$n_s - 1 = \left(2\eta_{H\star} - 4\varepsilon_{H\star}\right)\left(1 + \varepsilon_{H\star}\right) \simeq 2\eta_{V\star} - 6\varepsilon_{V\star}. \tag{26}$$

3 Effective Action for Inflation

We are now turning to a completely different approach, developing an effective field theory for inflation. Many successful effective field theories have already been used in different areas of physics, as in particle and nuclear physics, or in condensed matter. The value of the effective theories has been proven at many occasions.

The existence of unknown physics at Planckian energies and the will to still be able to describe the dynamics of inflation naturally lead to the use of an effective theory for inflation at lower energies.

The advantage is that we can describe in the most general theory the perturbations around a quasi de Sitter background, using the lowest dimension operators compatible with the symmetries [4, 19]. There is a clean separation between the contributions of high energy physics and what comes from the symmetries of the considered theory. We have one unified theory, able to describe the whole physics of inflation – eventually through the inclusion of higher order terms in the action – that allows us to derive in a one-go computation the spectrum of primordial perturbations, even if we have few indications on the actual physical mechanism driving inflation.

As a first step, we will focus on the derivation of the effective action for inflation, closely following the development made in [4]. Afterwards, we will consider a few limits and results.

3.1 The unitary gauge

We now turn to the construction of an effective action, in the case of a single scalar inflation model. Such an action can be derived from general symmetries that constrain the terms appearing in the Lagrangian. Let us introduce the final result, from [4, 19, 20] :

$$
\begin{aligned}
S = \int d^4x\sqrt{-g}\Bigg[&\frac{1}{2}M_{\mathrm{Pl}}^2 R + M_{\mathrm{Pl}}^2 \dot{H} g^{00} - M_{\mathrm{Pl}}^2\left(3H^2+\dot{H}\right) \\
&+ \frac{M_2(t)^4}{2!}\left(g^{00}+1\right)^2 + \frac{M_3(t)^4}{3!}\left(g^{00}+1\right)^3 + \dots \\
&- \frac{\overline{M}_1(t)^3}{2}\left(g^{00}+1\right)\delta K^\mu{}_\mu - \frac{\overline{M}_2(t)^2}{2}\delta K^\mu{}_\mu{}^2 - \frac{\overline{M}_3(t)^2}{2}\delta K^\mu{}_\nu \delta K^\mu{}_\nu + \dots\Bigg],
\end{aligned}
\tag{27}
$$

where $c(t)$ and $\Lambda(t)$ are some functions of time, $M_{1,2,3}$, $\overline{M}_{1,2,3}$ are time-dependent mass scales, and $\delta K_{\mu\nu} = K_{\mu\nu} - a^2 H h_{\mu\nu}$ is the variation of the extrinsic curvature of constant time surfaces with respect to the unperturbed FLRW. The first term in (27) is the well-known Einstein-Hilbert action.

To begin, we will consider only a single scalar field that drives inflation, but the result we obtain is actually more general.

Such scalar field provides a natural clock based on the proper time of the field. The existence of that clock breaks the symmetry under time diffeomorphisms, and the symmetry breaking will lead to the apparition of a Goldstone boson.

Furthermore, to be more accurate, the scalar field ϕ is invariant under all (space and time) diffeomorphisms, but it is the perturbation $\delta\phi$ that is *not.*

It is only invariant under spatial diffeomorphisms, while it transforms as :

$$t \to t + \xi^0 (t, \boldsymbol{x}) \qquad \delta\phi \to \delta\phi + \dot{\phi}_0 (t) \xi^0$$

under time diffeomorphisms. This induces a peculiar choice of gauge, the so-called unitary gauge.

The unitary gauge is such that the scalar degree of freedom, $\delta\phi$, is contained in the metric rather than in the scalar field. In other words, $\delta\phi = 0$. It means that the metric has now three degrees of freedom ; the two helicities and the scalar mode. Later, in order to restore the gauge invariance and to write the same theory in different gauges, we will use a trick called the Stückelberg mechanism. The Goldstone boson π will become apparent and will take the place of the scalar degree of freedom $\delta\phi$.

3.2 The operators

The game is now to write down all the possible operators for the action, given a set of rules – as usual when one develops an effective field theory. Those operators need to be function of the metric fluctuations, and to be invariant under the time dependent spatial diffeomorphisms $x^i \to x^i + \xi^i (t)$. Also, in a theory which is only invariant under spatial diffeomorphisms and not under time diffeomorphisms, there is a preferred slicing of the spacetime given by $\tilde{t}(x)$ – surfaces of constant $\tilde{t}$ are surfaces of constant value for the scalar field. The unitary gauge is actually chosen such that the time coordinate corresponds to the slicing $\tilde{t}$.

We are now ready to review what terms are allowed in the effective action for inflation, following [4].

First of all, expressions that are invariant under all diffeomorphisms can be used. They are polynomials of the Riemann tensor $R_{\mu\nu\rho\sigma}$ and of its covariant derivatives, contracted with the metric or the antisymmetric tensor to give a scalar. At first order, we will have the Ricci scalar R in the Einstein-Hilbert term.

Also, any generic function of t can be used in front of any term of the Lagrangian.

The gradient $\partial_\mu \tilde{t}$ becomes ${\delta^0}_\mu$ in unitary gauge (since t and $\tilde{t}$ coincide). Therefore, this allows us to leave a free upper zero index ; one example is g^{00}, which will often be used.

We will now introduce the unit vector perpendicular to surfaces of constant $\tilde{t}$:

$$n_\mu = \frac{\partial_\mu \tilde{t}}{\sqrt{-g^{\mu\nu}\partial_\mu \tilde{t}\partial_\nu \tilde{t}}}.$$

We can define the induced spatial metric on surfaces of constant $\tilde{t}$:

$$h_{\mu\nu} = g_{\mu\nu} + n_\mu n_\nu.$$

It will allow us to project any tensor on the constant $\tilde{t}$ surfaces. In particular, the Riemann tensor of the induced 3d metric ${}^{(3)}R_{\mu\nu\rho\sigma}$ and covariant derivatives with respect to the metric $h_{\mu\nu}$ can be used.

We can also consider the covariant derivative of $\partial_\mu \tilde{t}$, or – equivalently – to covariant derivative to n_μ. Indeed, the derivative acting on the normalisation factor gives $\partial_\mu g^{00}$ which are also covariant and can be used in the unitary gauge Lagrangian. If we project the covariant derivative of n_μ onto the surface of constant $\tilde{t}$, we obtain the extrinsic curvature of these surfaces

$$K_{\mu\nu} = h^\sigma{}_\mu \nabla_\sigma n_\nu.$$

Since $n^\nu \nabla_\sigma n_\nu = \frac{1}{2}\nabla_\sigma \left(n^\nu n_\nu\right) = 0$, we see that the ν index is already projected on the surface. The covariant derivative of n_ν perpendicular to the surface is

$$n^\sigma \nabla_\sigma n_\nu = -\frac{1}{2}\left(-g^{00}\right)^{-1} h^\mu{}_\nu \partial_\mu \left(-g^{00}\right)$$

and so we see that there are no other terms appearing here than the one we already know of. Therefore, the covariant derivatives of n_μ can always be arranged using the extrinsic curvature $K_{\mu\nu}$, g^{00} and covariant derivatives of those terms.

That is pretty much all the terms we can construct given the symmetries of the problem. However, there is just one last subtlety ; the 3d induced Riemann tensor can actually be written as a combination of the extrinsic curvature perturbation and the induced metric :

$${}^{(3)}R_{\alpha\beta\gamma\delta} = h^\mu{}_\alpha h^\nu{}_\beta h^\rho{}_\gamma h^\sigma{}_\delta R_{\mu\nu\rho\sigma} - K_{\alpha\gamma}K_{\beta\delta} + K_{\beta\gamma}K_{\alpha\delta}.$$

Therefore, it is redundant and there is no need to include it in the Lagrangian. Similarly, we can avoid the explicit use of $h_{\mu\nu}$ as it can be expressed through the 4d metric $g_{\mu\nu}$ and n_μ. Also, as we can obtain the 3d covariant derivative of a projected tensor with the projection of the 4d covariant derivative, the use of the covariant derivatives with respect to the induced 3d metric can be left out.

In conclusion, we see that the most general action that can be written in unitary gauge is of the form

$$S = \int d^4x\sqrt{-g}F\left(R_{\mu\nu\rho\sigma}, g^{00}, K_{\mu\nu}, \nabla_\mu, t\right)$$

and the only free indices in the action must be upper 0's.

3.3 Derivation of the action

In order to show the power of the unitary gauge, let us write the action for the inflaton field in the following way :

$$S = \int d^4x\sqrt{-g}\left[\frac{M_{\rm Pl}^2}{2}R - c\left(t\right)g^{00} - \Lambda\left(t\right)\right] + S^{(2)} \tag{28}$$

where R is the Ricci scalar, g^{00} is the upper time-time component of the metric, $c\left(t\right)$ and $\Lambda\left(t\right)$ are two functions of t, and $S^{(2)}$ contains terms of at least quadratic order in fluctuations [21]. The usefulness of this expression is that it encompasses many different models, provided that you carefully choose $c\left(t\right)$ and $\Lambda\left(t\right)$. For instance, for the Lagrangian of a simple scalar field with a slow-roll potential,

$$\mathcal{L} = \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu - V\left(\phi\right),$$

the kinetic term can be written as

$$\frac{1}{2}g^{\mu\nu}\partial_\nu\phi\partial_\mu\phi = c\left(t\right)g^{00} \qquad \text{where } c\left(t\right) = \frac{\dot{\phi}_0^2}{2}.$$

The reason for withdrawing the $\partial_i\phi\partial_j\phi$'s is because $c\left(t\right)$ is multiplied by $g^{\mu\nu}$, and g^{00} is the only component of the metric to satisfy the condition of invariance under spatial diffeomorphisms. ϕ_0 is the only part of ϕ to appear since we are in the unitary gauge.

We already know that the three terms in (28) comply to the conditions cited above. Indeed, R is invariant under all diffeomorphisms, so is allowed, while any generic function of time t is also permitted in front of any term of the Lagrangian ; we therefore have $\Lambda\left(t\right)$ and another function $c\left(t\right)$ multiplied by g^{00}.

Let us now consider what terms can appear in the $S^{(2)}$ action. As already stated, they have to be at least of quadratic order, and to obey the constraints developed in section 3.2. We therefore get the following intermediate expression :

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}M_{\mathrm{Pl}}^2 R - c\,(t)\,g^{00} - \Lambda\,(t) + \frac{M_2\,(t)^4}{2!}\left(g^{00}+1\right)^2 \right. \tag{29}$$
$$\left. + \frac{M_3\,(t)^4}{3!}\left(g^{00}+1\right)^3 + \dots - \frac{\overline{M}_2\,(t)^2}{2}\delta K^{\mu\ 2}_{\ \mu} + \dots\right]$$

were $\delta K_{\mu\nu}$ is the variation of the extrinsic curvature $K_{\mu\nu}$:

$$\delta K_{\mu\nu} = K_{\mu\nu} - a^2 H h_{\mu\nu}.$$

We are almost done ; there are just two terms that need to be computed. Let us concentrate on those terms. They actually describe the background evolution of the scalar field and will be the only ones to contribute to the stress-energy tensor [4] :

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta S_{\mathrm{matter}}}{\delta g^{\mu\nu}}.$$

The components of the tensors are :

$$\begin{aligned}
T_{00} &= -\frac{2}{\sqrt{-g}}\frac{\delta\sqrt{-g}}{\delta g^{00}}\left(-g^{00}c\,(t) - \Lambda\,(t)\right) - \frac{2}{\sqrt{-g}}\sqrt{-g}\frac{\delta}{\delta g^{00}}\left(-g^{00}c\,(t) - \Lambda\,(t)\right)\\
&= -g_{00}g^{00}c\,(t) - \Lambda\,(t)\,g_{00} + 2c\,(t)\\
&= c\,(t) + \Lambda\,(t)\,,
\end{aligned}$$

and

$$\begin{aligned}
T_{ij} &= -\frac{2}{\sqrt{-g}}\frac{\delta\sqrt{-g}}{\delta g^{ij}}\left(-g^{00}c\,(t) - \Lambda\,(t)\right) - \frac{2}{\sqrt{-g}}\sqrt{-g}\frac{\delta}{\delta g^{ij}}\left(-g^{00}c\,(t) - \Lambda\,(t)\right)\\
&= -\frac{2}{\sqrt{-g}}\left(-\frac{1}{2}g_{ij}\right)\sqrt{-g}\left(-2g^{00}c\,(t) - \Lambda\,(t)\right)\\
&= a^2\,(t)\,\delta_{ij}\left(2c\,(t) - \Lambda\,(t)\right),
\end{aligned}$$

where we used :

$$\frac{\delta\sqrt{-g}}{\delta g^{\mu\nu}} = -\frac{1}{2}\sqrt{-g}\ g_{\mu\nu}.$$

We just need to use the Einstein equations (2) and $8\pi G = M_{\rm Pl}^{-2}$ to find the Friedmann ones :

$$H^2 = \frac{1}{3M_{\rm Pl}^2}\left[c\left(t\right) + \Lambda\left(t\right)\right],$$
$$\frac{\ddot{a}}{a} = \dot{H} + H^2 = -\frac{1}{3M_{\rm Pl}^2}\left[2c\left(t\right) - \Lambda\left(t\right)\right].$$

Solving to get the expressions for $c\left(t\right)$ and $\Lambda\left(t\right)$, we obtain :

$$c\left(t\right) = -\dot{H}M_{\rm Pl}^2$$
$$\Lambda\left(t\right) = M_{\rm Pl}^2\left(3H^2 + \dot{H}\right)$$

which gives immediately the final result (27).

This expression is very useful. It encompasses many different theories in one action, and the usual models for inflation can be derived from there again. For instance, the vanilla model of slow-roll inflation is

$$\dot{\phi}_0^2\left(t\right) = -2M_{\rm Pl}^2\dot{H}, \qquad V\left(\phi_0\left(t\right)\right) = M_{\rm Pl}^2\left(3H^2 + \dot{H}\right),$$

with all the $M_{2,3,\ldots}$ and $M_{1,2,\ldots}$ set to zero, but this is the simplest example. Many other models can be expressed choosing specific values for those parameters, and quantum corrections can be implemented. This is one of the main advantages of the treatment of effective field theories applied to inflation. It encompasses a whole set of different theories in one action, and allows the modelling of unknown higher energy physics.

3.4 Stückelberg mechanism

In order to get the result (27), we had to pick up a specific gauge. In this so called unitary gauge, the scalar degree of freedom has been eaten by the metric. We will now use a trick that will restore the gauge invariance of the action and let the scalar mode explicitly appear.

The idea is to perform a “broken time diffeomorphism to reintroduce the Goldstone boson which non-linearly realises this symmetry” [4, 19]. Let us follow the development made in [4]. They first concentrate on the operators $c\left(t\right)g^{00}$ and $\Lambda\left(t\right)$, then extend the approach to other terms. Hence, let us focus on the following action :

$$S = \int d^4x\sqrt{-g}\left[-c\left(t\right)g^{00} - \Lambda\left(t\right)\right]. \tag{30}$$

The broken time diffeomorphism is :

$$\begin{aligned} t \to \tilde{t} &= t + \xi^0(x), \\ x^i \to \tilde{x}^i &= x^i. \end{aligned}$$

The metric in the new coordinates is

$$g^{00}(x) = \frac{\partial x^0}{\partial \tilde{x}^\mu}\frac{\partial x^0}{\partial \tilde{x}^\nu}\tilde{g}^{\mu\nu}(\tilde{x}(x)),$$

and replacing it in (30) gives

$$\begin{aligned} S &= \int d^4x \sqrt{-\det\left(\frac{\partial x^\rho}{\partial \tilde{x}^\mu}\frac{\partial x^\sigma}{\partial \tilde{x}^\nu}\tilde{g}^{\mu\nu}(\tilde{x}(x))\right)}\left[-c(t)\frac{\partial x^0}{\partial \tilde{x}^\mu}\frac{\partial x^0}{\partial \tilde{x}^\nu}\tilde{g}^{\mu\nu}(\tilde{x}(x)) - \Lambda(t)\right] \\ &= \int d^4x \sqrt{-\tilde{g}(\tilde{x}(x))}\left|\frac{\partial x}{\partial \tilde{x}}\right|\left[-c(t)\frac{\partial x^0}{\partial \tilde{x}^\mu}\frac{\partial x^0}{\partial \tilde{x}^\nu}\tilde{g}^{\mu\nu}(\tilde{x}(x)) - \Lambda(t)\right]. \end{aligned}$$

Then, performing the change of variable $x \to \tilde{x}$ for the integration,

$$\begin{aligned} S = \int d^4\tilde{x}\sqrt{-\tilde{g}(\tilde{x})}\Bigg[&- c\left(\tilde{t} - \xi^0(x(\tilde{x}))\right)\frac{\partial\left(\tilde{t} - \xi^0\right)}{\partial \tilde{x}^\mu}\frac{\partial\left(\tilde{t} - \xi^0\right)}{\partial \tilde{x}^\nu}\tilde{g}^{\mu\nu}(\tilde{x}) \\ &- \Lambda\left(\tilde{t} - \xi^0\right)\Bigg]. \end{aligned}$$

Now, here is the trick. We promote $\xi^0(x)$ to be a Goldstone field and we do the substitution

$$\xi^0(x(\tilde{x})) \to -\tilde{\pi}(\tilde{x})$$

which yields

$$S = \int d^4\tilde{x}\sqrt{-\tilde{g}(\tilde{x})}\Bigg[- c\left(\tilde{t} + \tilde{\pi}(\tilde{x})\right)\frac{\partial\left(\tilde{t} + \tilde{\pi}(\tilde{x})\right)}{\partial \tilde{x}^\mu}\frac{\partial\left(\tilde{t} + \tilde{\pi}(\tilde{x})\right)}{\partial \tilde{x}^\nu}\tilde{g}^{\mu\nu}(\tilde{x}) \tag{31}$$

$$- \Lambda\left(\tilde{t} + \tilde{\pi}(\tilde{x})\right)\Bigg]. \tag{32}$$

We can now compute (dropping the tilde in order to simplify the notation)

$$\begin{aligned} \frac{\partial(t+\pi)}{\partial x^\mu}\frac{\partial(t+\pi)}{\partial x^\mu}g^{\mu\nu} &= \frac{\partial(t)}{\partial x^\mu}\frac{\partial(t)}{\partial x^\mu}g^{\mu\nu} + 2\frac{\partial(t)}{\partial x^\mu}\frac{\partial(\pi)}{\partial x^\mu}g^{\mu\nu} + \frac{\partial(\pi)}{\partial x^\mu}\frac{\partial(\pi)}{\partial x^\mu}g^{\mu\nu} \qquad (33) \\ &= g^{00} + 2\partial_\mu\pi g^{0\mu} + g^{\mu\nu}\partial_\mu\pi\partial_\nu\pi \\ &= g^{00} + 2\dot{\pi}g^{00} + 2\partial_i\pi g^{0i} + \dot{\pi}^2 g^{00} + 2\partial_i\pi\dot{\pi}g^{0i} + g^{ij}\partial_i\pi\partial_j\pi \\ &= g^{00}(1+\dot{\pi})^2 + 2g^{0i}\partial_i\pi(1+\dot{\pi}) + g^{ij}\partial_i\pi\partial_j\pi, \end{aligned}$$

so we get a final form for (31). Replacing that directly in the expression (27), we have :

$$\begin{aligned} S = \int d^4x\sqrt{-g}\Bigg[&\frac{1}{2}M_{\rm Pl}^2 R - M_{\rm Pl}^2\left(3H^2\left(t+\pi\right)+\dot{H}\left(t+\pi\right)\right) \\ &+ M_{\rm Pl}^2\dot{H}\left(t+\pi\right)\left(g^{00}\left(1+\dot{\pi}\right)^2+2g^{0i}\partial_i\pi\left(1+\dot{\pi}\right)+g^{ij}\partial_i\pi\partial_j\pi\right) \\ &+ \frac{M_2\left(t+\pi\right)^4}{2}\left(g^{00}\left(1+\dot{\pi}\right)^2+2g^{0i}\partial_i\pi\left(1+\dot{\pi}\right)+g^{ij}\partial_i\pi\partial_j\pi+1\right)^2 \\ &+\frac{M_3\left(t+\pi\right)^4}{2}\left(g^{00}\left(1+\dot{\pi}\right)^2+2g^{0i}\partial_i\pi\left(1+\dot{\pi}\right)+g^{ij}\partial_i\pi\partial_j\pi+1\right)^3+\dots\Bigg] \end{aligned} \tag{34}$$

The scalar field that was previously eaten by the metric is now explicitly present ; the perturbations $\delta\phi$ of the scalar field are now represented by the Goldstone boson π. The terms involving extrinsic curvature $K^{\mu}{}_{\nu}$ have been omitted. The advantage here is that at high enough energy, decoupling happens, and the coupling with gravity can be neglected.

However, the two descriptions are not equivalent ; actually, the theory with the Goldstone boson is more general than the one with the single-clock inflaton we started from, and doesn't even assume the existence of a scalar field [22].

4 Single Field Inflation and Other Limits in EFT

We just developed an effective action to describe inflation of the univese. We can now check the consistency of this procedure with the results from section 2. The easiest comparison to do is the slow-roll inflation model, but others theories can also be studied.

4.1 Standard slow-roll inflation

Let us now consider a few examples of theories encompassed in the effective action (27). The simplest one would be to keep only the first three terms that correspond to the background evolution of the scalar field ϕ. When we fix all the parameters $M_{2,3}$ and $\overline{M}_{1,2,3}$ to zero, as already stated, we recover the case of slow-roll single field inflation with the correspondence

$$\dot{\phi}_0^2\left(t\right) = -2M_{\rm Pl}^2\dot{H}, \qquad V\left(\phi_0\left(t\right)\right) = M_{\rm Pl}^2\left(3H^2+\dot{H}\right).$$

If we then consider the Goldstone action (34) in this context :

$$S_{\text{slow roll}} = \int d^4x\sqrt{-g}\Big[\frac{1}{2}M_{\text{Pl}}^2 R - M_{\text{Pl}}^2\left(3H^2(t+\pi) + \dot{H}(t+\pi)\right)$$
$$+M_{\text{Pl}}^2\dot{H}(t+\pi)\left(g^{00}(1+\dot{\pi})^2 + 2g^{0i}\partial_i\pi(1+\dot{\pi}) + g^{ij}\partial_i\pi\partial_j\pi\right)\Big],$$

we can bring out some range of energy where mixing terms with gravity are significant or not. Indeed, at high enough energies, we can leave out the metric perturbations when working on the physics of the Goldstone boson. In order to determine the typical energy where the mixing terms can be neglected, let us have a look to the leading term of gravity and Goldstone boson mixing. It takes the form

$$\sim M_{\text{Pl}}^2\dot{H}\dot{\pi}\delta g^{00}.$$

Then, if we normalise the fields to

$$\begin{aligned}\pi_c &\sim M_{\text{Pl}}\dot{H}^{1/2}\pi,\\ \delta g_c^{00} &\sim M_{\text{Pl}}\delta g^{00},\end{aligned} \tag{35}$$

we have

$$\sim \sqrt{\dot{H}}\,\dot{\pi}_c\,\delta g_c^{00} = \sqrt{\varepsilon}H\pi_c\delta g_c^{00}.$$

where $\varepsilon = -\dot{H}/H^2$ is the slow-roll parameter. The mixing terms can be neglected for energies larger than the scale [4]

$$E_{\text{mix}} \sim \sqrt{\varepsilon}H.$$

This scale will be of importance, as it actually determine the range of energy that actually corresponds to the slow-roll inflation limit.

4.2 Other limits and links with inflation models

The above limit can be established for any model ; for instance, in the case where M_2 is taken to be large, the energy scale would be $E_{\text{mix}} \sim M_2^2/M_{\text{Pl}}^4$ [4]. This allows us to consider models with sound speed smaller than unity, a feature often linked to high values for non-Gaussianities.

Furthermore, if we allow non vanishing $\overline{M}_{2,3}$, we end up with the action corresponding to Ghost inflation, while if all the $\overline{M}$'s are set to zero but allowing higher order M's, there is a link with DBI inflation. Others theories of inflation, as for instance K-inflation, can be recovered through a careful choice of parameters in the effective action [23].

4.3 High energy regime

Let us go back to the general case. The aim here will be to see the behaviour of (34) in the high energy limit. Provided that inflation occurs in early enough times, i.e. at energies $E \gg E_{\mathrm{mix}}$ above a mixing scale that can be established, we are in a regime where the physics of the Goldstone boson decouples from the metric perturbations. Those can then be neglected. We therefore just have to consider a FLRW background,

$$ds^2 = -dt^2 + a^2(t)\, d\vec{x}^2,$$

and we can compute

$$g^{00}(1+\dot{\pi})^2 + 2g^{0i}\partial_i\pi(1+\dot{\pi}) + g^{ij}\partial_i\pi\partial_j\pi \quad \to \quad -1-\dot{\pi}^2-2\dot{\pi}+\frac{(\partial_i\pi)^2}{a^2}.$$

But the term

$$\sim -2M_{\mathrm{Pl}}^2\dot{\pi}\dot{H}$$

is no longer relevant, as it is only significant at energies $E \sim E_{\mathrm{mix}}$. We also neglect the terms

$$-3M_{\mathrm{Pl}}^2H^2 \quad \text{and} \quad -2M_{\mathrm{Pl}}^2\dot{H},$$

since we are ignoring the back-reaction of the perturbations on the metric.

Moreover, we see that, if we keep the terms up to the third order,

$$\frac{M_2^4}{2!}\left(-1-\dot{\pi}^2-2\dot{\pi}+\frac{(\partial_i\pi)^2}{a^2}+1\right)^2 \to 2M_2^4\left(\dot{\pi}^2+\dot{\pi}^3-\dot{\pi}\frac{(\partial_i\pi)^2}{a^2}\right)+\mathcal{O}^4$$
$$\frac{M_3^4}{3!}\left(-1-\dot{\pi}^2-2\dot{\pi}+\frac{(\partial_i\pi)^2}{a^2}+1\right)^3 \to \frac{4}{3}M_3^4(t)\,\dot{\pi}^3+\mathcal{O}^4$$

we end up with the following result [4] :

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}M_{\mathrm{Pl}}^2R - M_{\mathrm{Pl}}^2\dot{H}\left(\dot{\pi}^2-\frac{(\partial_i\pi)^2}{a^2}\right)\right. \qquad (36)$$
$$\left. +2M_2^4\left(\dot{\pi}^2+\dot{\pi}^3-\dot{\pi}\frac{(\partial_i\pi)^2}{a^2}\right)-\frac{4}{3}M_3^4\dot{\pi}^3+\dots\right].$$

The terms including the extrinsic curvature $K_{\mu\nu}$ have been withdrawn again, as they are significant only in a regime where the space is very close to a de-Sitter space [19].

4.3.1 Slow-roll inflation

We can now use the expression (36) in the case of slow-roll inflation to see if there is consistency with the canonical derivation of quantities, as the spectral index for instance.

As we are in slow-roll inflation, but in a regime of energy $E \gg E_{\text{mix}}$, we only need to consider the following terms in the action :

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}M_{\text{Pl}}^2\,R - M_{\text{Pl}}^2\dot{H}\left(\dot{\pi}^2 - \frac{(\partial_i\pi)^2}{a^2}\right)\right].$$

Let us now consider the curvature perturbation ξ[2] defined at linear order, in the unitary gauge when $\pi = 0$, as

$$g_{ij} = a^2\,(t)\,[1 + 2\xi\delta_{ij} + \gamma_{ij}]$$

where γ is traceless, transverse, and describe the two tensor degrees of freedom.

There is a simple relationship between ξ and the Goldstone boson π :

$$\xi = -H\pi = -\frac{H}{M_{\text{Pl}}\sqrt{\dot{H}}}\pi_c, \tag{37}$$

where we used (35) for the normalisation of the Goldstone boson.

This can be understood in the following way. As we are in a single field inflation, there is only one single classical trajectory, and a π fluctuation corresponds to a time delay $\pi \sim \delta t$ on this trajectory. But a curvature fluctuation corresponds to the amount of expansion of the universe by the end of inflation $\xi \sim H\delta t$, on surfaces where the physical clock is uniform [19]. So we get the result.

The variable ξ stays constant after horizon crossing ; therefore we will mainly be interested in computing the correlation function of ξ (that will gives power spectrum and spectral index) at horizon crossing, $k \sim aH$. The fact that we are in a slow-roll single scalar field in a quasi de Sitter background allows us to actually use the result :

$$\langle\pi_c\,(\boldsymbol{k}_1)\,\pi_c\,(\boldsymbol{k}_2)\rangle = (2\pi)^3\,\delta\,(\boldsymbol{k}_1 + \boldsymbol{k}_2)\,\frac{H_\star^2}{2k_1^3},$$

[2] In the article [4], they use ζ to refer to this quantity, possibly leading to some confusion. However, it is not exactly the same definition as the one we introduced in 18, even if they do coincide on superhorizon scales. Also, it is not a gauge invariant variable.

where as before the $\star$ indicates that we evaluate the quantity at horizon crossing. Therefore, using 37, the correlation function for the curvature perturbation is simply

$$\langle \xi(\boldsymbol{k}_1)\, \xi(\boldsymbol{k}_2) \rangle = (2\pi)^3\, \delta(\boldsymbol{k}_1 + \boldsymbol{k}_2)\, \frac{H_\star^4}{2M_{\rm Pl}^2 \left| \dot{H}_\star \right| k_1^3}.$$

This gives the expression for the power spectrum

$$\Delta_s^2 = \frac{H_\star^4}{2M_{\rm Pl}^2 \left| \dot{H}_\star \right|},$$

that matches (25). We can then compute the spectral index

$$n_s - 1 = \frac{d \ln \Delta_s^2}{d \ln k} = \frac{d}{d \ln k} \ln \frac{H_\star^4}{\left| \dot{H}_\star \right|} = \frac{1}{H_\star} \frac{d}{dt_\star} \ln \frac{H_\star^4}{\left| \dot{H}_\star \right|} = 4\frac{\dot{H}_\star}{H_\star^2} - \frac{\ddot{H}_\star}{H_\star \dot{H}_\star},$$

where $\frac{d}{d\ln k} = \frac{1}{H_\star}\frac{d}{dt_\star}$ because

$$\frac{d \ln k}{dt} = \frac{d \ln aH}{dt} = \frac{\dot{a}}{a} + \frac{\dot{H}}{H} = (1 - \varepsilon_H)\, H$$

and therefore

$$\frac{d}{d \ln k} = \frac{dt}{d \ln k} \frac{d}{dt} = \frac{1}{1-\varepsilon_H} \frac{1}{H} \frac{d}{dt} \simeq \frac{1}{H} \frac{d}{dt}.$$

Using the expressions for the Hubble slow-roll parameters (14), we obtain

$$n_s - 1 = 2\eta_{H\star} - 4\varepsilon_{H\star}$$

which is the same result as (26).

In the case where $M_2^4 \gg M_{\rm Pl}^2 \dot{H}$, we can also compute the power spectrum of curvature perturbation, which is given by [24] :

$$\langle \xi_{\boldsymbol{k}} \xi_{\boldsymbol{k}'} \rangle = (2\pi)^3\, \delta(\boldsymbol{k} + \boldsymbol{k}')\, \frac{1}{2k^3} \left. \frac{H^4}{M_{\rm Pl}^2 \left| \dot{H} \right| c_s} \right|_{-c_s k \eta = 1}$$

where η is the conformal time and

$$c_s^{-2} = 1 - \frac{2M_2^4}{M_{\rm Pl}^2 \dot{H}}$$

is the sound speed of fluctuations.

As we can see, the effective approach leads to the same results as the canonical approach. The consistency of the effective field theory is clear and links with different inflation models can be made.

Conclusion

The derivation of the power spectrum for scalar – and tensor – perturbations is of primary interest. It gives us tools to understand the evolution of the content of the universe, from primordial fluctuations to large scale structures. Since we cannot observe directly the inflation stage, we can only probe the model through its relics and implications.

Therefore, a serious number of models for inflation have been developed, each one leading to the prediction of a power spectrum with possibly different features. However, the derivation of the power spectrum – even in relatively simple cases such as single scalar field inflation – is intricate.

Consequently, the idea of an effective field for inflation seems to be attractive ; the derivation of the interesting quantities needs to be done only once, and afterwards, for *any* model of inflation, we would be able to directly write the corresponding power spectrum or spectral index, saving ourselves from long and fastidious computations. That would allow us, for instance, to quickly compare several models and point out differences between them, which is usually not straightforward.

Nevertheless, the situation is not as nice as we could first think. The action involves complicated terms, the connection between the effective side and a given model of inflation is not always obvious, and – at least for simple models – the computation of the power spectrum does not appear to be easier than in the canonical case.

But the application of an effective field theory to inflation is quite recent and, if up to now the comparison between it and the usual approach results more or less in a tie, the effective approach could soon appear to be more powerful in the long run, since the handling of higher order terms is simpler. Moreover, the application of effective field theory to the field of cosmology in general has been very active these last years (see [19, 20, 21, 23, 24, 25, 26]) and seems to be very promising.

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