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# The MSSM, Beauty Decays and the Path Towards a Phenomenological Framework

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**Abstract**

Supersymmetry is the yet-unobserved symmetry between integer-spin-bosonic and half-integer-spin-fermionic states, which evades the limitations of the Coleman-Mandula theorem that restricts the symmetry groups of quantum field theories. The Minimally Supersymmetric Standard Model (MSSM) is the simplest theoretical realisation of supersymmetry that is compatible with the observed Standard Model (SM) of particle physics. In the contemporary landscape of beyond the SM (BSM) theory, beauty decays involving $b$-hadrons act as sensitive probes to assess the experimental validity of new models of fundamental interactions, such as the MSSM. Building from the foundations of supersymmetry and the construction of the MSSM, this report presents a review on the formulation of the framework used to study MSSM effects in beauty decays through the low-energy effective field theory (LEFT). Examples of matching calculations to obtain Wilson coefficients at LO and NLO are given, and the relevance of such an approach to constrain and challenge BSM models is emphasised. Recent applications of this approach on beauty decay observables are introduced to motivate the effectiveness of such an approach and provide a starting point for further developments.

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# Contents

## Lorentz Convention and Notation

- $\mu, \nu, \dots = 0, 1, 2, 3$ — indices for tensor representation of the Lorentz group
- $\alpha, \beta, \dots = (0,)1, 2(, 3)$ — indices for Weyl (Dirac) Lorentz spinor representation
- $\eta_{\mu\nu} = \text{diag}(-1, 1, 1, 1)$ — Minkowski metric in (- + + +) signature
- $\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}$ — Dirac matrix Clifford algebra
- $\theta_\alpha, \psi_\alpha, \chi_\alpha, \bar{\theta}_{\dot{\alpha}}, \bar{\psi}_{\dot{\alpha}}, \bar{\chi}_{\dot{\alpha}}$ — common two-component Weyl spinors
- $\Psi_\alpha$ — common four-component Dirac spinor
- $\epsilon^{\alpha\beta} = \epsilon^{\dot{\alpha}\dot{\beta}} = -\epsilon_{\alpha\beta} = -\epsilon_{\dot{\alpha}\dot{\beta}} = \epsilon^{ij} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ — two-index Levi-Civita symbols

The author would like to acknowledge that the notation in this document has been largely inspired by Prof. Kellogg S. Stelle's *The Standard Model and Beyond* and *Supersymmetry* courses, the text *Supersymmetry and Supergravity* (Wess & Bagger, 1992), and the review *The Standard Model effective field theory at work* (Isidori, Wilsch, & Wyler, 2023).

# 1 Introduction

The Standard Model (SM) of particle physics is arguably the most successful theory of fundamental interactions in our modern understanding of the universe. It is the quantum field theory (QFT) that codifies under the same picture three out of the four known interactions, namely the electric, weak (under the electroweak) and strong forces. When quantised, the SM gives rise to all the particles and force carriers that have been observed in experiments of particle physics for over 50 years. After the discovery of the Higgs boson at CERN in 2012 (Aad et al., 2012; Chatrchyan et al., 2012), every particle component of the SM has been experimentally observed. Furthermore, the measurements of their properties has yielded ever successful agreement with those predicted by the SM, with no experiment finding disagreement at the $5\sigma$ level commonly accepted as the discovery threshold. Nevertheless, lower-significance discrepancies and theoretical arguments have pushed us to believe that the SM cannot be the final picture.

Efforts to understand physics beyond the SM (BSM) can be said to begin from its most notable shortcoming: the absence of the interaction of gravity. Experimental hints such as the anomalous magnetic dipole of the muon in Muon $g-2$ (Abi et al., 2021), or deviations in the $W$ boson mass (CDF-Collaboration et al., 2022) provide contemporary questions addressed by BSM theories. Beauty decays (or the decays of $b$-hadrons) in particular act today as probes for new physics, being rare processes very sensitive to underlying theories. Most notably, their experimental involvement in branching fraction ratio, or angular distribution anomalies (Aaij et al., 2022), justify the interest of the physics community in their direction to explore BSM phenomena.

Supersymmetry is the mathematical construction that, on the most basic level, postulates a new symmetry between bosonic and fermionic states of a theory. It is the only symmetry that evades the Coleman-Mandula theorem, which restricts the form of the internal symmetry groups of QFTs (Coleman & Mandula, 1967; Haag, Łopuszański, & Sohnius, 1975). Moreover, it presents remarkable properties, such as the emergence of supergravity when it is locally gauged or the elimination of quadratic divergences emerging from bosonic loops. Supersymmetry is a theoretical concept, and as such must be realised if we are to observe it in nature. It is therefore most natural to extend the SM in this direction, and its most basic implementation yields the theory of the Minimally Supersymmetric Standard Model (MSSM). From the start, the MSSM presents a solution to the question of the Grand Unification of couplings when coupled with a SU(5) unified BSM theory (Dimopoulos & Georgi, 1981) and yields a resolution of the hierarchy, or weak-scale-instability, problem. Moreover, it provides a natural explanation for the emergence of electroweak symmetry breaking (EWSB) at the observed scale in the SM, and provides potential candidates for cosmological dark matter particles. As such, the MSSM (and its extensions), provide a fantastic variety of promising avenues for the exploration of BSM physics.

Despite its promising motivation, the experimental absence of superpartners imposes that supersymmetry, if it is in fact a symmetry of nature, must be broken and that such particles must be very heavy compared to experimental scales. Nevertheless, weak-scale processes can still be affected by supersymmetric effects from virtual superpartners at higher loop order. Wilsonian effective field theory (EFT) (Wilson, 1972) is a powerful framework under which these hypothetical effects can be studied at chosen energy scales. By viewing the SM under the modern light of an EFT, the MSSM can be matched onto experimentally relevant effective theories to both constrain its parameter space, and so its BSM effects, as well as to assess its phenomenological consistency with experiments prior to any direct detection.

On this ground, this dissertation aims to develop the theoretical and phenomenological framework commonly used to study MSSM effects in weak-scale physics, with a specific emphasis on beauty decays. The goals of this report therefore are to:

- Provide a comprehensive review of the mathematical formalism used to develop manifestly supersymmetric field theories (Section 3)
- Use the superspace and superfield formalism to introduce the MSSM and its flavour-relevant interactions (Sections 4 and 5)
- Present the concepts underpinning effective field theory under the specific light of the SMEFT and the LEFT (Section 6)
- Demonstrate computations of EFT Wilson coefficient matching at leading and non-leading order (LO+NLO), using heavy-SM and MSSM examples (Section 7)
- Give a high-level introduction of specific applications of the EFT formalism in studying MSSM effects in beauty processes (Section 8)

As such, this report presents a starting point for the more in-depth phenomenological assessment of the question of supersymmetry found in contemporary literature.

# 2 Standard Model and Beauty Decays

In this introductory section, we review the Standard Model and present beauty decays.

## 2.1 The Standard Model

The Standard Model (SM) is a field theory containing three generations of up-type and down-type quarks (giving six quark flavours in total), three generations of leptons and their corresponding neutrinos, 12 gauge vector bosons and a scalar boson (the Higgs). Behind its success lies its foundation upon symmetries and group-theoretical constructions. In fact, the SM is simply the most general renormalisable QFT, written in Lagrangian form (made explicit in section 2.1.1), which respects the postulate of special-relativistic invariance (invariance under the action of the Poincaré group) and gauges the direct-product internal symmetry group $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$. After the marvellous success of quantum field theory and renormalisation through quantum electrodynamics (QED), efforts were made to find the unification of its formulation with the discovered $W^{\pm}$ and $Z$ (discovered later) bosons mediating the weak interaction. This culminated in the Weinberg-Salam theory of electroweak unification (Weinberg, 1967; Salam & Ward, 1959), relying on Glashow's $\mathrm{SU}(2) \times \mathrm{U}(1)$ electroweak symmetry group construction (Glashow, 1959) with Yang-Mills (Yang & Mills, 1954) type gauge bosons acquiring mass through the Higgs mechanism (Higgs, 1964). Further development following Gell-Mann's and Ne'eman's eightfold way classification of mesons and baryons (Gell-Mann, 1961) led to the development of quantum chromodynamics (QCD), introducing the final SU(3) colour symmetry to the gauge group of the SM. Although extensions to the SM have been proposed, to account for neutrino masses through right-handed neutrinos in the seesaw mechanism (Yanagida, 1979) for example, its core form has remained unchanged since the 1970's. In its current form, it awaits significant tension with experiments to lead the exploration of extensions or alternative theories.

In this section, the minimal SM in its most commonly seen form is presented (in Section 2.1.1) as well as a discussion on the quark and lepton mass eigenstates (Section 2.1.2) giving rise also to the Cabibbo-Kobayashi-Maskawa (CKM) matrix. In particular, emphasis is placed on the misalignment of the quark mass and flavour bases inducing flavour changing charged currents (FCCCs), and neutral currents (FCNCs) at higher loops, through quark interaction with the $W$ boson.

### 2.1.1 Particle Content and Lagrangian

The SM is a field theory written in the Lagrangian formalism, with particles arising from the quantisation of each field, built upon the internal symmetries encoded by the direct-product Lie group $\mathrm{SU}(3)_{\mathrm{C}} \times \mathrm{SU}(2)_{\mathrm{L}} \times \mathrm{U}(1)_{\mathrm{Y}}$ (the subscripts $\mathrm{C}, \mathrm{L}$ and $\mathrm{Y}$ have been added to the groups to distinguish the kind of symmetry to which they are associated). The $\mathrm{SU}(3)_{\mathrm{C}}$ group encodes colour symmetry under which only quarks are charged, $\mathrm{SU}(2)_{\mathrm{L}}$

is the group associated to weak isospin and $\mathrm{U}(1)_\mathrm{Y}$ that associated to weak hypercharge. The SM internal symmetry group is not realised globally, rather it is gauged to a local symmetry by allowing the introduction of spacetime dependency in its transformations. Finally, the SM is also required to be invariant under special relativity, and the representation of the fields under the Poincaré group is associated to their quantum number of spin.

The SM then divides its fields, and therefore particles, into matter (fermionic, half integer spin, Grassmann-number-valued and anticommuting) and force (bosonic, whole integer spin) fields, each transforming under a different representation of the three symmetry groups. The spinor matter fields and Higgs-scalar field transform in either the trivial or fundamental representation for each group, whereas the vector gauge fields transform in an inhomogeneous modification of the adjoint representation for the symmetry they gauge. From the violation of parity symmetry (Wu, Ambler, Hayward, Hoppes, & Hudson, 1957), each matter Weyl spinor field has left and right versions, with the exceptions of neutrinos which are only left-handed in the minimal version of the SM considered in this analysis. Table 1 gives a detailed breakdown of each field along with its spin and representation with respect to the internal gauge group.

The $\mathrm{SU}(3)_\mathrm{C} \times \mathrm{SU}(2)_\mathrm{L} \times \mathrm{U}(1)_\mathrm{Y}$ gauge symmetry is built into the SM using a Yang-Mills construction. To the groups in the direct product are associated coupling constants $g_3, g_2, g_1$ in the same order. In the case of the $\mathrm{SU}(3)_\mathrm{C}, \mathrm{SU}(2)_\mathrm{L}$ non-Abelian groups, the respective Lie algebras $\mathfrak{su}(3)$ and $\mathfrak{su}(2)$ follow the generator commutation relations:

$$[T^\vartheta, T^\sigma] = i f^{\vartheta\sigma\tau} T^\tau \qquad \text{for} \quad \mathfrak{su}(3) \tag{2.1}$$

$$[t^i, t^j] = i\epsilon^{ijk} t^k \qquad \text{for} \quad \mathfrak{su}(2) \tag{2.2}$$

In the case of the $\mathrm{U}(1)_\mathrm{Y}$ Abelian group, the respective Lie algebra $\mathfrak{u}(1)$ is the trivial algebra, containing only one generator which we can denote $Y$, the weak hypercharge. To a choice of generators $T^\vartheta, t^i, Y$ respecting the Lie algebras there corresponds a choice of representation, and the gauge fields for each symmetry group live in the Lie algebra: $G_\mu = G_\mu^\vartheta T^\vartheta, W_\mu = W_\mu^I t^I, B_\mu \equiv B_\mu$. If we take the transformation in an arbitrary representation:

$$M_C := e^{i\xi^\vartheta(x)T^\vartheta}, \quad M_L := e^{i\theta^i(x)t^i}, \quad M_Y := e^{i\eta(x)Y} \tag{2.3}$$

$$M := M_C M_L M_Y = e^{i\xi^\vartheta(x)T^\vartheta + i\theta^i(x)t^i + i\eta(x)Y} \tag{2.4}$$

Then a gauge transformation on field $\phi(x)$ in an arbitrary representation, along with the associated gauge fields, takes the form:

$$\phi(x) \to M\phi(x) \tag{2.5}$$

$$G_\mu \to M_C G_\mu M_C^\dagger + \frac{i}{g_3}(\partial_\mu M_C) M_C^\dagger \tag{2.6}$$

$$W_\mu \to M_L W_\mu M_L^\dagger + \frac{i}{g_2}(\partial_\mu M_L)M_L^\dagger \tag{2.7}$$

$$B_\mu \to M_Y B_\mu M_Y^\dagger + \frac{i}{g_1}(\partial_\mu M_Y)M_Y^\dagger = B_\mu - \frac{Y}{g}\partial_\mu \eta(x) \tag{2.8}$$

Having introduced the transformation under a representation, the covariant derivative takes the (first for general gauge field $A_\mu = A_\mu^a T^a$, then SM specific) form:

$$\begin{aligned} D_\mu =& \partial_\mu + igA_\mu^a T^a & (2.9)\\ =& \partial_\mu + ig_3 G_\mu^\vartheta T^\vartheta + ig_2 W_\mu^i t^i + ig_1 Y B_\mu & (2.10)\end{aligned}$$

Noting that the generators for the fundamental representations of $\mathrm{SU(3)_C}, \mathrm{SU(2)_L}$ are $T^\vartheta = \frac{1}{2}\lambda^\vartheta, t^i = \frac{1}{2}\sigma^i$ respectively, we follow the representations in Table 1 to obtain the following expressions for the gauge-covariant derivatives for each field in the minimal SM:

$$D_\mu \phi = \partial_\mu \phi + \frac{i}{2} g_2 W_\mu^i \sigma^i \phi + \frac{i}{2} g_1 B_\mu \phi \tag{2.11}$$

$$D_\mu \ell_L^f = \partial_\mu \ell_L^f + \frac{i}{2} g_2 W_\mu^i \sigma^i \ell_L^f - \frac{i}{2} g_1 B_\mu \ell_L^f \tag{2.12}$$

$$D_\mu q_L^f = \partial_\mu q_L^f + \frac{i}{2} g_3 G_\mu^\vartheta \lambda^\vartheta q_L^f + \frac{i}{2} g_2 W_\mu^i \sigma^i q_L^f + \frac{i}{6} g_1 B_\mu q_L^f \tag{2.13}$$

$$D_\mu u_R^f = \partial_\mu u_R^f + \frac{i}{2} g_3 G_\mu^\vartheta \lambda^\vartheta u_R^f + \frac{2i}{3} g_1 B_\mu u_R^f \tag{2.14}$$

$$D_\mu d_R^f = \partial_\mu d_R^f + \frac{i}{2} g_3 G_\mu^\vartheta \lambda^\vartheta d_R^f - \frac{i}{3} g_1 B_\mu d_R^f \tag{2.15}$$

Finally, we define the Abelian and non-Abelian field strength tensors for Yang-Mills for each gauge symmetry as:

$$G_{\mu\nu}^\vartheta = \partial_\mu G_\nu^\vartheta - \partial_\nu G_\mu^\vartheta + g_3 f^{\theta\sigma\tau} G_\mu^\sigma G_\nu^\tau \tag{2.16}$$

$$W_{\mu\nu}^i = \partial_\mu W_\nu^i - \partial_\nu W_\mu^i + g_2 \epsilon^{ijk} W_\mu^j W_\nu^k \tag{2.17}$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu \tag{2.18}$$

We are now able to write the minimal SM Lagrangian. This contains a gauge-kinetic sector (2.19a), a matter-kinetic sector (2.19b), a Higgs scalar field sector (2.19c), a Yukawa sector (2.19d) and a a "theta" sector (2.19e). In the following, $\tilde{G}_{\mu\nu}^\vartheta = \frac{1}{2}\epsilon_{\mu\nu\alpha\beta} G^{\vartheta\alpha\beta}$, and $\tilde{\phi} = i\sigma^2\phi^*$.

$$\begin{aligned} \mathcal{L}_{\mathrm{SM}} =& -\frac{1}{4} G_{\mu\nu}^\vartheta G^{\vartheta\mu\nu} - \frac{1}{4} W_{\mu\nu}^i W^{i\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} & (2.19a)\\ & - \bar{\ell}_L^f i \not{D} \ell_L^f - \bar{e}_R^f i \not{D} e_R^f - \bar{q}_L^f i \not{D} q_L^f - \bar{u}_R^f i \not{D} u_R^f - \bar{d}_R^f i \not{D} d_R^f & (2.19b)\\ & - |D_\mu \phi|^2 + \mu^2 |\phi|^2 - \frac{1}{2}\lambda |\phi|^4 & (2.19c)\\ & - \Big( [y_e]_{fg} \bar{\ell}_L^f e_R^g \phi + [y_u]_{fg} \bar{q}_L^f u_R^g \tilde{\phi} + [y_d]_{fg} \bar{q}_L^f d_R^g \phi + \mathrm{h.c.} \Big) & (2.19d)\\ & - \frac{g_3^2 \theta_3}{32\pi^2} G_{\mu\nu}^\vartheta \tilde{G}^{\vartheta\mu\nu} - \frac{g_2^2 \theta_2}{32\pi^2} W_{\mu\nu}^i \tilde{W}^{i\mu\nu} - \frac{g_1^2 \theta_1}{32\pi^2} B_{\mu\nu} \tilde{B}^{\mu\nu} & (2.19e)\end{aligned}$$

| **Field Name** | **Field** | $\mathrm{SU(3)_C}$ | $\mathrm{SU(2)_L}$ | $\mathrm{U(1)}_Y$ |
|---|---|---|---|---|
| LH quark doublet | $q_L^f = \begin{pmatrix} u_L^f \\ d_L^f \end{pmatrix}$ | **3** | **2** | $+\frac{1}{6}$ |
| RH up-type quark singlet | $u_R^f$ | **3** | **1** | $+\frac{2}{3}$ |
| RH down-type quark singlet | $d_R^f$ | **3** | **1** | $-\frac{1}{3}$ |
| LH lepton doublet | $\ell_L^f = \begin{pmatrix} \nu_L^f \\ e_L^f \end{pmatrix}$ | **1** | **2** | $-\frac{1}{2}$ |
| RH electron singlet | $e_L^f$ | **1** | **1** | $-1$ |
| Higgs doublet | $\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$ | **1** | **2** | $+\frac{1}{2}$ |
| Gluon | $G_\mu^\vartheta$ | - | - | - |
| W | $W_\mu^i$ | - | - | - |
| B | $B_\mu$ | - | - | - |

Table 1: SM flavour-basis field breakdown with their corresponding gauge group representations. The doublet or singlet classification comes from the $\mathrm{SU(2)_L}$ representation and doublets are explicitly in column vector form. LH and RH stand for left- and right-handedness.

### 2.1.2 Mass Eigenstates and the CKM Matrix

The gauge bosons and the fermions acquire mass by interacting with the Higgs boson in the electroweak symmetry breaking (EWSB) phase: vector gauge bosons do this through the covariant derivative term of (2.19c) and fermions through Yukawa interactions in the (2.19d) sector. The Higgs potential as seen in (2.19c) is:

$$V(\phi) = -\mu^2|\phi|^2 + \frac{1}{2}\lambda|\phi|^4 \tag{2.20}$$

Therefore, if we take the Higgs complex doublet to have real vacuum expectation value (vev) $\phi = \frac{1}{\sqrt{2}}(0, v)$, minimising the potential in (2.20) yields:

$$v^2 = \frac{2\mu^2}{\lambda} \tag{2.21}$$

In this vacuum configuration, examination of the $\mathrm{SU(2)_L} \times \mathrm{U(1)_Y}$ Lie group generators shows that there is only one remaining unbroken generator, $Q := t^3 + Y$. Hence, the symmetry breaking pattern is the classical electroweak to electrodynamics pattern $\mathrm{SU(2)_L} \times \mathrm{U(1)_Y} \to \mathrm{U(1)_{EM}}$. Expanding the Higgs around the vacuum state and going into unitary gauge, we lose three of the four degrees of freedom of the complex doublet (corresponding to the three broken electroweak symmetry generators) and are left with the physical Higgs $h(x)$:

$$\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix} \tag{2.22}$$

Expanding $\mathcal{L}_{\text{SM}}$ in unitary gauge around the Higgs vacuum configuration, the gauge fields $W^i_\mu$ and $B_\mu$ develop the following tree-level masses in the following eigenstate arrangements:

$$m_\gamma = 0 \quad \text{for} \quad \mathcal{A}_\mu := \cos\theta_W B_\mu + \sin\theta_W W^3_\mu \tag{2.23}$$

$$m_W = \frac{vg_2}{2} \quad \text{for} \quad W^\pm_\mu := \frac{1}{\sqrt{2}}(W^1_\mu \mp iW^2_\mu) \tag{2.24}$$

$$m_Z = \frac{v}{2}\sqrt{g_1^2 + g_2^2} \quad \text{for} \quad Z := \cos\theta_W W^3_\mu - \sin\theta_W B_\mu \tag{2.25}$$

With the Weinberg mixing angle $\theta_W$ being defined through:

$$\sin\theta_W = \frac{g_1}{\sqrt{g_1^2 + g_2^2}}, \quad \cos\theta_W = \frac{g_2}{\sqrt{g_1^2 + g_2^2}} \tag{2.26}$$

For fermions (the quark and lepton sectors), masses are given once again by a small expansion around the Higgs vev. Taking (2.19d) and (2.22):

$$\mathcal{L}_{\text{SM}} = \dots - \frac{1}{\sqrt{2}}v\left([y_e]_{fg}\bar{e}^f_L e^g_R + [y_u]_{fg}\bar{u}^f_L u^g_R + [y_d]_{fg}\bar{d}^f_L d^g_R + \text{h.c.} + \text{h.o.t.}\right) \tag{2.27}$$

Therefore we can see that the mass eigenstates with definite mass eigenvalues are given by diagonalising the Yukawa mass matrices with biunitary transformations:

$$[y_e]_{fg} = U^{e_L\dagger}_{fm}[\hat{y}_e]_{mn}U^{e_R}_{ng} \tag{2.28}$$

$$[y_u]_{fg} = U^{u_L\dagger}_{fm}[\hat{y}_u]_{mn}U^{u_R}_{ng} \tag{2.29}$$

$$[y_d]_{fg} = U^{d_L\dagger}_{fm}[\hat{y}_d]_{mn}U^{u_R}_{ng} \tag{2.30}$$

The physical fermions (denoted with a hat) are now defined by absorbing the unitary transformations applied to the mass matrices, making the Yukawa sector diagonal:

$$\hat{e}^f_{L/R} = U^{e_{L/R}}_{fm}e^m_{L/R}, \quad \hat{u}^f_{L/R} = U^{u_{L/R}}_{fm}u^m_{L/R}, \quad \hat{d}^f_{L/R} = U^{d_{L/R}}_{fm}d^m_{L/R} \tag{2.31}$$

$$\implies \mathcal{L}_{\text{SM}} = \dots - \frac{v}{\sqrt{2}}\left(\sum_f [\hat{y}_e]_f \bar{\hat{e}}^f_L \hat{e}^f_R + \sum_f [\hat{y}_u]_f \bar{\hat{u}}^f_L \hat{u}^f_R + \sum_f [\hat{y}_d]_f \bar{\hat{d}}^f_L \hat{d}^f_R + \text{h.c.}\right) + \dots \tag{2.32}$$

Giving the fermion masses $m_{ef} = \frac{v}{\sqrt{2}}[\hat{y}_e]_f, m_{uf} = \frac{v}{\sqrt{2}}[\hat{y}_u]_f, m_{df} = \frac{v}{\sqrt{2}}[\hat{y}_d]_f$, where $f$ is any choice of generation. The diagonalisation of the Yukawa sector induces a mixing of quark generations in the matter kinetic sector (2.19b) through their interactions with $W^\pm$ bosons. This can be shown by expanding the charged current term in the covariant derivative for the left-handed quarks:

$$-\bar{q}^f_L i\not{D} q^f_L = \dots - \frac{ig_2}{\sqrt{2}}\left(\bar{u}^f_L \not{W}^+ d^f_L + \bar{d}^f_L \not{W}^- u^f_L\right) + \dots \tag{2.33}$$

$$= \dots - \frac{ig_2}{\sqrt{2}}\left(\bar{\hat{u}}^m_L (U^{u_L}_{mf}U^{d_L\dagger}_{fn})\not{W}^+ \hat{d}^n_L + \bar{\hat{d}}^n_L (U^{d_L}_{nf}U^{u_L\dagger}_{fm})\not{W}^- \hat{u}^m_L\right) + \dots \tag{2.34}$$

$$= \cdots - \frac{ig_2}{\sqrt{2}} \left( \bar{\hat{u}}_L^m V_{mn}^{\text{CKM}} \not{W}^+ \hat{d}_L^n + \bar{\hat{d}}_L^n V_{nm}^{\text{CKM}\dagger} \not{W}^- \hat{u}_L^m \right) + \ldots \tag{2.35}$$

To summarise, quark flavour changing charged currents (FCCCs) arise from the Cabibbo-Kobayashi-Maskawa (Cabibbo, 1963; Kobayashi & Maskawa, 1973) matrix $V^{\text{CKM}} = U^{u_L} U^{d_L \dagger}$, which in turn derives from the distinct biunitary transformations used to diagonalise the up-type and down-type quark masses in the Yukawa sector.

## 2.2 Beauty Decays and Phenomenological Relevance

Beauty decays are important flavour processes in experimental particle physics to probe potential candidate theories of new physics beyond the SM. By a beauty decay we intend a hadronic process in which a bottom $b$ quark (originally known as the "beauty" quark) changes its flavour into another generation of quark. Due to quark confinement, these processes involve $b$-hadrons, meaning hadrons which contain a $b$ quark as one of its constituents, which decay into a wide range of secondary products such as other hadrons or leptons. Example of such SM beauty decays are $\bar{B} \to \bar{K}^* \ell^+ \ell^-$ and $\bar{B}_s \to \mu^+ \mu^-$. Being very rare, these decays can place very stringent experimental bounds on what hypothesised higher-energy BSM theories can imply.

Beauty decays are currently used to probe two specific SM phenomena in particular: that of CP violation and of flavour-changing-neutral-currents (FCNCs). CP violation is strongly related to the complex phase of the CKM matrix. Nevertheless, the CKM mechanism alone is not sufficient to explain the observed baryon-antibaryon asymmetry in the universe (Aaij et al., 2017), suggesting that new physics might be involved. Beauty decays, especially $b \to s$, are very sensitive to such a mechanism and therefore can provide important experimental insight into BSM processes that can possibly account for this.

On the other hand, by FCNCs we intend processes through which quarks change their flavour but not their electric charge, such as the process $b \to s\gamma$. In the minimal SM, there is no tree-level vertex that can account for such a process, as the only flavour-violating vertex is the charged $W$-boson-quark interaction (Section 2.1.2 and 5.2). Hence these must happen at higher loop order. The strong suppression of FCNCs in the minimal SM resulting from the Glashow-Iliopoulos-Maiani (GIM) mechanism (Glashow, Iliopoulos, & Maiani, 1970) implies that new physics must also share the suppression of these processes (Misiak, Pokorski, & Rosiek, 1997). This provides important restrictions for BSM theories. In the case of the MSSM (Section 4), additional flavour changing vertices are generated, and FCNC results from beauty decays can be used to restrict hypothetical squark flavour violation (Behring, Gross, Hiller, & Schacht, 2012).

In contemporary analyses, beauty decays have gained substantial popularity for hinting at the violation of lepton flavour universality (LFU). By LFU we intend the condition that all leptons couple in exactly the same way to the gauge fields (except for mass difference contributions). Two common observables associated to this LFU discrepancy are the $R_K$ and $R_K^*$ branching fraction ratios, defined as:

$$R_K = \frac{\mathcal{B}(B^+ \to K^+\mu^+\mu^-)}{\mathcal{B}(B^+ \to K^+e^+e^-)} \simeq_{\text{SM}} 1 \qquad R_{K^*} = \frac{\mathcal{B}(B^+ \to K^*\mu^+\mu^-)}{\mathcal{B}(B^+ \to K^*e^+e^-)} \simeq_{\text{SM}} 1 \tag{2.36}$$

While recent analysis has lowered the previous $4\sigma$ deviation from SM predictions to $3.1\sigma$ (Aaij et al., 2022), this avenue further motivation for developing a framework in which to study candidate NP effects on beauty processes.

In the current experimental landscape, beauty decays occur at energy ranges around the electroweak scale despite the higher energies achieved in particle colliders. To explain this we consider, for example, beauty decays as observed from CERN's LHC proton-proton collisions at the current energy of 13.6 TeV. This is initially much higher than the electroweak scale. Nevertheless at bunch crossing, a proton-proton collision occurs at the primary vertex, which subsequently produces more quarks (and antiquarks) each sharing a part of that initial energy, of which only a fraction can bind into $b$-hadrons. The eventual $b$-hadrons therefore decay at secondary vertices with energies greatly reduced from the initial 13.6 TeV to energies to the $\sim \mathcal{O}(100)$ GeV range. We can use this consideration to frame the discussion of this report under the EWSB presented in Section 2.1.2.

# 3 Supersymmetric Field Theory

In the simplest terms, supersymmetry can be said to posit a symmetry of nature that intrinsically relates ”matter” and ”force” particles in theories of fundamental interactions. In this statement, we intend matter fields as fermions transforming as spinors under Lorentz transformations (half-integer spins) and force fields as bosons transforming as tensors (whole-integer spin), of which vector and scalar fields are a part of. Supersymmetry transformations more specifically involve particles and their respective superpartners, which are particles whose spin differs by a half compared to that of their base partner, paired in a specific manner. A Lagrangian that is invariant under supersymmetry transformations is then invariant under a ”rotation” of bosonic particles into their fermionic superpartners and vice-versa according to said pairing. This section attempts to present a foundational description of supersymmetry and the objects that transform under its representations, as to motivate and facilitate the supersymmetric field-theoretical development of the MSSM in the later Section 4.[1]

Before introducing supersymmetry formally, it is useful to consider the spinor representation in more detail. The Lorentz group, contained in the Poincaré group as the subgroup of special-relativistic transformations, is the non-compact and not simply-connected Lie group $\mathrm{SO}(3,1) \subset$ Poincaré. Its Lie algebra $\mathfrak{so}(3,1)$ is isomorphic to the Lie algebra $\mathfrak{spin}(3,1)$, which in turn is the Lie algebra for the group $\mathrm{Spin}(3,1) \simeq \mathrm{SL}(2,\mathbb{C})$, the simply-connected double-cover of $\mathrm{SO}(3,1)$. Weyl spinors are then defined as the two-component Grassmann (anticommuting) fields $\psi_\alpha$, $\bar{\psi}^{\dot{\alpha}}$ transforming in the two fundamental and conjugate representations of $\mathrm{SL}(2,\mathbb{C})$ respectively. Spinors transforming in the fundamental representation, denoted $(\frac{1}{2},0)$, are defined as left-handed, while those transforming in the conjugate representation $(0,\frac{1}{2})$ are defined as right-handed.

It follows that four component Dirac spinors transform as $(\frac{1}{2},0) \oplus (0,\frac{1}{2})$. This is made explicit by choosing the Weyl representation of four component spinors and the Clifford algebra, where (conventions as in (Wess & Bagger, 1992)):

$$\gamma_\mu = \begin{pmatrix} 0 & \sigma_\mu \\ \bar{\sigma}_\mu & 0 \end{pmatrix}, \quad \gamma_5 = \gamma_0\gamma_1\gamma_2\gamma_3 = \begin{pmatrix} -i\mathbb{1} & 0 \\ 0 & i\mathbb{1} \end{pmatrix} \tag{3.1}$$

$$\sigma_\mu = (\mathbb{1}, \sigma), \quad \bar{\sigma}_\mu = (\mathbb{1}, -\sigma) \tag{3.2}$$

Defining the usual projection operators $P_L = \frac{1}{2}(\mathbb{1} + i\gamma_5)$ and $P_R = \frac{1}{2}(\mathbb{1} - i\gamma_5)$, left and right handed Weyl spinors can be packaged into Weyl-representation Dirac spinors as:

$$\Psi = \begin{pmatrix} \psi_\alpha \\ \bar{\chi}^{\dot{\alpha}} \end{pmatrix}, \quad \Psi_L = P_L\Psi = \begin{pmatrix} \psi_\alpha \\ 0 \end{pmatrix}, \quad \Psi_R = P_R\Psi = \begin{pmatrix} 0 \\ \bar{\chi}^{\dot{\alpha}} \end{pmatrix} \tag{3.3}$$

To manipulate two-component Weyl spinor indices, we can either employ complex conjugation or use the $\mathrm{SL}(2,\mathbb{C})$ invariant $\epsilon$ tensors (as defined in Notation and Conventions)

[1]The experienced reader is authorised to skip this review and pass directly to Section 4!

as follows:

$$(\psi_\alpha)^* = \bar{\psi}_{\dot{\alpha}} \tag{3.4}$$

$$\epsilon^{\alpha\beta}\psi_\beta = \psi^\alpha, \quad \epsilon_{\dot{\alpha}\dot{\beta}}\bar{\psi}^{\dot{\beta}} = \bar{\psi}_{\dot{\alpha}}, \quad \epsilon^{\alpha\gamma}\epsilon_{\gamma\beta} = \delta^\alpha_\beta, \quad \epsilon^{\dot{\alpha}\dot{\gamma}}\epsilon_{\dot{\gamma}\dot{\beta}} = \delta^{\dot{\alpha}}_{\dot{\beta}} \tag{3.5}$$

Using these, we therefore have the Weyl spinor contractions $(\theta\theta) = \theta^\alpha\theta_\alpha = \epsilon^{\alpha\beta}\theta_\beta\theta_\alpha$ and $(\bar{\theta}\bar{\theta}) = \bar{\theta}_{\dot{\alpha}}\bar{\theta}^{\dot{\alpha}} = \epsilon^{\dot{\alpha}\dot{\beta}}\theta_{\dot{\alpha}}\theta_{\dot{\beta}}$ and we can therefore show:

$$(\theta\theta) = -\theta_\alpha\epsilon^{\alpha\beta}\theta_\beta = -2\theta_1\theta_2, \quad (\bar{\theta}\bar{\theta}) = 2\bar{\theta}_1\bar{\theta}_2 \tag{3.6}$$

Finally, it is also useful to note that vectors, such as $x^\mu$, transform in the $(\frac{1}{2}, \frac{1}{2})$ representation of $\mathrm{SL}(2, \mathbb{C})$.

## 3.1 Coleman-Mandula and the Supersymmetry Algebra

Supersymmetry is not merely speculative in its nature. Rather, it is a unique kind of symmetry which arises from the very nature of the mathematical description of symmetries in physical theories and its restrictions. Crucial in the historical development of physical symmetries, the ”no-go” theorem of Coleman and Mandula (Coleman & Mandula, 1967) is a fundamental result which, through S-matrix considerations, provides a proof for the only structure that continuous symmetries, that is symmetries described by Lie or infinite-parameter groups, of special-relativistic theories may exhibit. Presenting the theorem in a descriptive form, as almost exactly found in (Coleman & Mandula, 1967), the result states:

*Coleman-Mandula Theorem*
Let $\mathcal{G}$ be a connected symmetry group of the S-matrix, such that:

1. $\mathcal{G}$ contains a subgroup locally isomorphic to the Poincaré group
2. For any $M > 0$, there are only a finite number of one-particle states with mass less than $M$
3. Elastic scattering amplitudes are analytic functions of the $s$ and $t$ Mandelstam variables
4. The S-matrix is nontrivial such that any two one-particle momentum eigenstates scatter
5. The generators of $\mathcal{G}$, written as integral operators in momentum space, have distributions for their kernels

Then, $\mathcal{G}$ must be locally isomorphic to the direct product the Poincaré group $\mathcal{P}$ and an internal symmetry group $\mathcal{T}$ (defined as an arbitrary group commuting with $\mathcal{P}$), which can in turn be generated by a semi-simple Lie algebra and so be, for example, a direct product group.

Hence $\mathcal{G} = \mathcal{P} \times \mathcal{T}$, which we can interpret in the words of the authors as "the impossibility of combining space-time and internal symmetries in any but a trivial way". Supersymmetry bypasses the theorem by playing on its notion of a symmetry group. By introducing anticommutator relations to its Lie algebras and therefore allowing for spinor transformation generators, it was shown (Gol'fand & Likhtman, 1971) (and rediscovered in many contemporary publications (Volkov & Akulov, 1972; Volkov & Soroka, 1973; Wess & Zumino, 1974b)) that $\mathbb{Z}_2$-graded Lie algebras are obtained for which the Coleman-Mandula theorem does not apply in its strictest sense. Letting $M_{\mu\nu}$ be the generators of Lorentz transformations and $P_\mu$ the generators of spacetime translations of even $\mathbb{Z}_2$-grading, together generating the Poincaré algebra, while odd grading is applied to the new (Weyl two-component) spinorial supercharges $Q_\alpha$ and $\bar{Q}_{\dot{\alpha}}$, we obtain the following Super-Poincaré algebra $\mathcal{SP}$:

$$[M_{\mu\nu}, M_{\sigma\tau}] = - i\eta_{\nu\sigma}M_{\mu\tau} + i\eta_{\mu\sigma}M_{\nu\tau} + i\eta_{\nu\tau}M_{\mu\sigma} - i\eta_{\mu\tau}M_{\nu\sigma} \tag{3.7a}$$

$$[P_\mu, P_\nu] = 0 \tag{3.7b}$$

$$[M_{\mu\nu}, P_\sigma] = - i\eta_{\nu\sigma}P_\mu + i\eta_{\mu\sigma}P_\nu \tag{3.7c}$$

$$\{Q_\alpha, Q_\beta\} = \{\bar{Q}_{\dot{\alpha}}, \bar{Q}_{\dot{\beta}}\} = 0 \tag{3.7d}$$

$$\{Q_\alpha, \bar{Q}_{\dot{\beta}}\} = 2\sigma^\mu_{\alpha\dot{\beta}}P_\mu \tag{3.7e}$$

$$[Q_\alpha, P_\mu] = [\bar{Q}_{\dot{\alpha}}, P_\mu] = 0 \tag{3.7f}$$

It is worth mentioning that the above relations hold for $\mathcal{N} = 1$ supersymmetry, with $\mathcal{N}$ referring to the number of supercharges. For a different $\mathcal{N}$, the supercharges are supplemented with an index $i = 1, \ldots, \mathcal{N}$ and the fundamental relation (3.7e) is replaced with $\{Q^i_\alpha, \bar{Q}^j_{\dot{\beta}}\} = 2\sigma^\mu_{\alpha\dot{\beta}}\delta^{ij}P_\mu$, although this will not be relevant for the analysis presented in this report. With the graded Lie algebra modification, the symmetry group $\mathcal{G}$ examined in the Coleman-Mandula theorem was then shown (Haag et al., 1975) to necessarily only take the following modified form, which simply replaces the Poincaré algebra with the Super-Poincaré algebra and keeps the remaining structure identical:

$$\mathcal{G} = \mathcal{SP} \times \mathcal{T} \tag{3.8}$$

Finally, it is fundamental to highlight that supercharges satisfy the vanishing commutation relation (3.7f) and commute with internal symmetries (from the direct product in (3.8)). This directly implies that particles and superpartners have the same mass and quantum numbers with respect to the internal symmetries of a given theory.

## 3.2 Superspace and Superfields

To simplify the construction of supersymmetric theories, it is useful to briefly review the concepts of superspace and superfields defined on superspace. These enable the construction of Lagrangians that are manifestly supersymmetric invariant, useful to produce intelligible formulations of the MSSM, as well as offering a direct way of deriving supersymmetry transformations on components of the same supermultiplet.

These concepts were first introduced by Salam and Strathdee (Salam & Strathdee, 1974b) to develop a method of deriving Wess-Zumino type supersymmetry transformations (Wess & Zumino, 1974b, 1974a), using group theoretic arguments. They rely on the coset space construction of manifolds on which the Poincaré and Super-Poincaré groups can act. The discussion in this report largely retraces the discussion by Tong (Tong, 2022) and uses results from- and convention close to that used in (Wess & Bagger, 1992).

### 3.2.1 Superspace

Poincaré transformations (i.e. Lorentz boosts combined with translations) act on Minkowski $\mathbb{R}^{1,3}$ space. In the coset space construction, Minkowski space can be seen as:

$$\mathbb{R}^{1,3} = \frac{\text{Poincaré}}{\text{Lorentz}} \tag{3.9}$$

Let $M^{\mu\nu}$ be the generators of Lorentz transformations and $P_\mu$ the generators of translations (i.e. momenta). Hence, for a Poincaré group element $G(\omega, x)$, we obtain the coset representatives $\tilde{G}(x)$ specified by the four-vector coordinate $x$ as follows (Tong, 2022):

$$G(\omega, x) = \exp\left[\frac{i}{2}\omega_{\mu\nu}M^{\mu\nu} - ix^\mu P_\mu\right] = \tilde{G}(x)\exp\left[\frac{i}{2}\omega_{\mu\nu}M^{\mu\nu}\right] \tag{3.10}$$

$$\implies \tilde{G}(x) = \exp\left[-ix^\mu P_\mu\right] \tag{3.11}$$

Supersymmetry transformations in $\mathcal{N} = 1$ are generated by the Weyl spinor supercharge $Q_\alpha$ and the conjugate supercharge $\bar{Q}_{\dot{\alpha}}$. It is noted that for a different number of supercharges, the following discussion can be extended by summing over the index $i = 1, \ldots, \mathcal{N}$ on products that include the supercharge $Q^i_\alpha$, although it will not be needed. To parametrise the translation part of the transformation we use the vector parameter $x^\mu$, transforming in $(\frac{1}{2}, \frac{1}{2})$ of $\mathrm{SL}(2, \mathbb{C})$, and to parametrise the supersymmetric translation we use the two-component Grassmann variables $\theta_\alpha$ in $(\frac{1}{2}, 0)$ and $\bar{\theta}_{\dot{\alpha}}$ in $(0, \frac{1}{2})$.

The way to construct a Super-Poincaré group element from supercharges then is:

$$G(\omega, x, \theta, \bar{\theta}) = \exp\left[\frac{i}{2}\omega_{\mu\nu}M^{\mu\nu} - ix^\mu P_\mu + i\theta^\alpha Q_\alpha + i\bar{\theta}_{\dot{\alpha}}\bar{Q}^{\dot{\alpha}}\right] \tag{3.12}$$

The coset space construction of superspace follows in exactly the same manner as Minkowski space, except for the replacement of the Poincaré group with the Super-Poincaré group:

$$\text{Superspace} = \frac{\text{Super-Poincaré}}{\text{Lorentz}} \tag{3.13}$$

As for Minkowski, we can separate the Lorentz component of the Super-Poincaré transformation to obtain a representative of the remaining coset:

$$G(\omega, x, \theta, \bar{\theta}) = \exp\left[-ix_\mu P^\mu + i\theta^\alpha Q_\alpha + i\bar{\theta}_{\dot{\alpha}}\bar{Q}^{\dot{\alpha}}\right] L(\omega) \tag{3.14}$$

$$= S(x, \theta, \bar{\theta}) L(\omega) \tag{3.15}$$

$$\implies S(x, \theta, \bar{\theta}) = \exp\left[-ix_\mu P^\mu + i\theta^\alpha Q_\alpha + i\bar{\theta}_{\dot{\alpha}}\bar{Q}^{\dot{\alpha}}\right] \tag{3.16}$$

With $L(\omega)$ being a Lorentz transformation, then $S(x, \theta, \bar{\theta})$ can be taken as such a representative of the coset specified by the three parameters $(x, \theta, \bar{\theta})$. We can now parametrise superspace as the eight-dimensional space specified by these three ”coordinates”.

### 3.2.2 Superspace Transformations

We have defined superspace as the coset space of the Super-Poincaré group with respect to the group of Lorentz transformations. Therefore, it is now possible to derive how the coordinate triple $(x, \theta, \bar{\theta})$ changes under the action of a superspace transformation.

We begin by considering whether a purely-spacetime transformation has the same effect as the typical translation operator. Using (3.16), we have that:

$$S(a, 0, 0) S(x, \theta, \bar{\theta}) = \exp\left[-ia^\mu P_\mu\right] \exp\left[-ix^\mu P_\mu + i\theta^\alpha Q_\alpha + i\bar{\theta}_{\dot{\alpha}}\bar{Q}^{\dot{\alpha}}\right] \tag{3.17}$$

$$= \exp\left[-i(x^\mu + a^\mu) P_\mu + i\theta^\alpha Q_\alpha + i\bar{\theta}_{\dot{\alpha}}\bar{Q}^{\dot{\alpha}}\right] \tag{3.18}$$

Where the fact that $\left[-ia^\mu P_\mu, -ix^\nu P_\nu + i\theta^\alpha Q_\alpha + i\bar{\theta}_{\dot{\alpha}}\bar{Q}^{\dot{\alpha}}\right] = 0$ (using the commutator identities in (3.7b) and (3.7f)) was used along with the Baker-Campbell-Hausdorff formula:

$$\exp[X] \exp[Y] = \exp\left[X + Y + \frac{1}{2}[X, Y] + \frac{1}{12}[X - Y, [X, Y]] + \dots\right] \tag{3.19}$$

Therefore, it is verified that the coordinate transformation induced by a purely spacetime group action has the expected non-supersymmetric effect, that is:

$$x \to x + a, \quad \theta \to \theta, \quad \bar{\theta} \to \bar{\theta} \tag{3.20}$$

Now, we examine in the same manner how a purely Grassmann superspace transformation affects the superspace coordinates, to understand how a supersymmetry transformation affects a superfield (introduced in the following section 3.2.3). Therefore we are interested in computing the explicit form of:

$$S(0, \zeta, \bar{\zeta}) S(x, \theta, \bar{\theta}) = \exp\left[i\zeta^\alpha Q_\alpha + i\bar{\zeta}_{\dot{\alpha}}\bar{Q}^{\dot{\alpha}}\right] \exp\left[-ix^\mu P_\mu + i\theta^\alpha Q_\alpha + i\bar{\theta}_{\dot{\alpha}}\bar{Q}^{\dot{\alpha}}\right] \tag{3.21}$$

We compute the commutator of the two exponentials:

$$\left[i\zeta^\alpha Q_\alpha + i\bar{\zeta}_{\dot{\alpha}}\bar{Q}^{\dot{\alpha}}, -ix^\mu P_\mu + i\theta^\alpha Q_\alpha + i\bar{\theta}_{\dot{\alpha}}\bar{Q}^{\dot{\alpha}}\right] \tag{3.22}$$

$$= -[\zeta^\alpha Q_\alpha, \theta^\beta Q_\beta] - [\bar{\zeta}_{\dot{\alpha}} \bar{Q}^{\dot{\alpha}}, \theta^\beta Q_\beta] - [\zeta^\alpha Q_\alpha, \bar{\theta}_{\dot{\alpha}} \bar{Q}^{\dot{\beta}}] - [\bar{\zeta}_{\dot{\alpha}} \bar{Q}^{\dot{\alpha}}, \bar{\theta}_{\dot{\alpha}} \bar{Q}^{\dot{\beta}}] \tag{3.23}$$

$$= \zeta^\alpha \theta^\beta \{Q_\alpha, Q_\beta\} - \bar{\zeta}^{\dot{\alpha}} \theta^\beta \{\bar{Q}_{\dot{\alpha}}, Q_\beta\} - \zeta^\alpha \bar{\theta}^{\dot{\beta}} \{Q_\alpha, \bar{Q}_{\dot{\beta}}\} + \bar{\zeta}^{\dot{\alpha}} \bar{\theta}^{\dot{\beta}} \{\bar{Q}_{\dot{\alpha}}, \bar{Q}_{\dot{\beta}}\} \tag{3.24}$$

$$= 2(\theta\sigma^\mu\bar{\zeta} - \zeta\sigma^\mu\bar{\theta})P_\mu \tag{3.25}$$

With the commutator being proportional to $P_\mu$, all higher commutators in the Baker-Campbell-Hausdorff formula vanish and we arrive to:

$$\begin{aligned} S(0,\zeta,\bar{\zeta})S(x,\theta,\bar{\theta}) &= \exp\left[-i(x^\mu + i\theta\sigma^\mu\bar{\zeta} - i\zeta\sigma^\mu\bar{\theta})P_\mu + i(\theta^\alpha + \zeta^\alpha)Q_\alpha + i(\bar{\theta}_{\dot{\alpha}} + \bar{\zeta}_{\dot{\alpha}})\bar{Q}^{\dot{\alpha}}\right] \\ &= S(x + i\theta\sigma\bar{\zeta} - i\zeta\sigma\bar{\theta}, \theta + \zeta, \bar{\theta} + \bar{\zeta}) \end{aligned} \tag{3.26}$$

Hence, we obtain that the transformation affects both superspace and spacetime coordinates and is given by:

$$x \to x + i\theta\sigma\bar{\zeta} - i\zeta\sigma\bar{\theta}, \quad \theta \to \theta + \zeta, \quad \bar{\theta} \to \bar{\theta} + \bar{\zeta} \tag{3.27}$$

### 3.2.3 Superfields and Superfield Types

A superfield can now be defined as a field, transforming in any representation of the Lorentz group (e.g. scalar, spinor or vector implemented by appending the correct index), which is valued on the superspace coordinate triple $(x, \theta, \bar{\theta})$, such as the scalar superfield $\Phi = \Phi(x, \theta, \bar{\theta})$, and transforms according to the transformation $S(x, \theta, \bar{\theta})$ of (3.16) (Salam & Strathdee, 1974b). Having derived the explicit transformation of the coordinates in (3.20), (3.27), superfields hence transform in the following two ways:

$$\Phi(x,\theta,\bar{\theta}) \to S(a,0,0)\Phi S(a,0,0)^\dagger = \Phi(x+a,\theta,\bar{\theta}) \tag{3.28}$$

$$\Phi(x,\theta,\bar{\theta}) \to S(0,\zeta,\bar{\zeta})\Phi S(0,\zeta,\bar{\zeta})^\dagger = \Phi(x + i\theta\sigma\bar{\zeta} - i\zeta\sigma\bar{\theta}, \theta + \zeta, \bar{\theta} + \bar{\zeta}) \tag{3.29}$$

Due to the Grassmann nature of the components of the Weyl spinors $\theta_\alpha$ and $\bar{\theta}_{\dot{\alpha}}$, the Taylor expansion of superfields with respect to the $\theta$ and $\bar{\theta}$ parameters truncates exactly after a finite number of terms. Consider the following, where we make use of (3.6) and $\eta^2 = 0$ for Grassmann $\eta$:

$$(\theta\theta)\theta_\alpha = 2\theta_2\theta_1\theta_\alpha = 2\theta_2(\theta_1\theta_\alpha) = -2(\theta_2\theta_\alpha)\theta_1 = 0 \tag{3.30}$$

Since either $\alpha = 1, 2$. A similar result follows for $\bar{\theta}\bar{\theta}$. Hence, the highest order we can achieve in a Taylor expansion of a superfield is a $\theta\theta\bar{\theta}\bar{\theta}$ term. If we take a scalar superfield, it can be written in terms of arbitrary ordinary spacetime fields (i.e. depending on $x$ only), the collection of which can be said to define a supermultiplet:

$$\begin{aligned} \Phi(x,\theta,\bar{\theta}) = \phi(x) &+ \theta\psi(x) + \bar{\theta}\bar{\chi}(x) + \theta\theta M(x) + \bar{\theta}\bar{\theta}N(x) \\ &+ \theta\sigma^\mu\bar{\theta}V_\mu(x) + \theta\theta\bar{\theta}\bar{\lambda}(x) + \bar{\theta}\bar{\theta}\theta\rho(x) + \theta\theta\bar{\theta}\bar{\theta}D(x) \end{aligned} \tag{3.31}$$

From (3.29), the transformation of the superfield implies the transformation of the individual components of the multiplet (Ferrara, Zumino, & Wess, 1974). Superfields with appended spinor or vector indices can also be obtained in the same manner, although they will not be required in the analysis considered in this report. The crucial observation is that, considering the spacetime-valued fields contained in the superfield, and therefore the components of the supermultiplet, there is an equivalent number of bosonic and fermionic (off-shell) degrees of freedom. For the bosonic contributions, we obtain two real d.o.f.s from each $\phi, M, N$ and $D$ scalar fields and eight real d.o.f.s from $V_\mu$ for a total of 16. This matches the four real d.o.f.s for each of the four two-component spinors $\psi, \chi, \lambda$ and $\rho$.

Given that superfields are representations of the Super-Poincaré group, we can ask whether or not these are reducible or whether they can be further broken down into further representations. Superfields are not in fact reducible, and the fermionic covariant derivatives can be defined to divide their space:

$$\mathcal{D}_\alpha = \partial_\alpha + i\sigma^\mu_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\partial_\mu \tag{3.32}$$

$$\bar{\mathcal{D}}_\alpha = -\bar{\partial}_{\dot{\alpha}} - i\theta^\alpha\sigma^\mu_{\alpha\dot{\alpha}}\partial_\mu \tag{3.33}$$

Where $\partial_\alpha = \frac{\partial}{\partial\theta^\alpha}$ and $\partial_{\dot{\alpha}} = \frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}}$ are partial derivatives with respect to the Grassmann parameters, satisfying for example $\partial_\beta\theta^\alpha = \delta^\alpha_\beta$. Now, chiral and anti chiral superfields can be defined using the following conditions, as first proposed by (Ferrara et al., 1974): a chiral field satisfies $\bar{\mathcal{D}}_{\dot{\alpha}}\Phi = 0$ while an antichiral field satisfies $\mathcal{D}_\alpha\Phi = 0$.

It follows that the conjugation of a superfield has the opposite chirality. From the vanishing anticommutation relations of the fermionic covariant derivatives and the supersymmetry transformation charges, the chiral and antichiral subspaces are invariant. Moreover, neither of the chirality conditions are satisfied only trivially, although they impose restrictions on the forms of the superfields. Using the fact that $\bar{\mathcal{D}}_{\dot{\alpha}}(y^\mu) := \bar{\mathcal{D}}_{\dot{\alpha}}(x^\mu + i\theta^\alpha\sigma^\mu_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}) = 0$, we have that $\Phi(x,\theta,\bar{\theta}) = \Phi(y,\theta)$ and hence we can perform the following simplifying Taylor expansion:

$$\begin{aligned}\Phi(x,\theta,\bar{\theta}) = \Phi(y,\theta) &= \phi(y) + \sqrt{2}\theta\psi(y) + \theta\theta F(y)\\ &=\phi(x) + i\theta\sigma^\mu\bar{\theta}\partial_\mu\phi(x) + \frac{1}{4}\theta\theta\bar{\theta}\bar{\theta}\Box\phi(x)\\ &\quad + \sqrt{2}\theta\psi(x) - \frac{i}{\sqrt{2}}\theta\theta\partial_\mu\psi(x)\sigma^\mu\bar{\theta} + \theta\theta F(x)\end{aligned} \tag{3.34}$$

A similar expansion derived with an equivalent observation holds for antichiral superfields too. Examining the supermultiplet components, we again note the equivalence between the number of bosonic and fermionic off-shell degrees of freedom. The complex scalar fields $\phi$ and $F$ each carry 2 real bosonic d.o.f.s for a total of 4, and the complex two-component spinor $\psi$ carries 4 real fermionic d.o.f.s.

Other than chiral and antichiral superfields, a further superfield that appears extensively in superspace actions, including that of the MSSM, is the real vector superfield, defined by the condition:

$$V(x,\theta,\bar{\theta}) = V^\dagger(x,\theta,\bar{\theta}) \tag{3.35}$$

From the reality condition, the real vector superfield cannot be chiral without being antichiral. As for a usual spacetime gauge theory with real gauge fields $A_\mu^i$, real superfields are the supersymmetric representations containing such gauge fields and their supersymmetric fermionic partners, the gauginos. Using the reality condition of (3.35), we obtain (Wess & Bagger, 1992) the expansion:

$$\begin{aligned}V(x,\theta,\bar{\theta}) =& C(x) + \Big[i\theta\chi(x) + \text{h.c.}\Big] \\ &+ \Big[\frac{i}{2}\theta\theta(M(x) + iN(x)) + \text{h.c.}\Big] \\ &- \theta\sigma^\mu\bar{\theta}V_\mu(x) + \Big[i\theta\theta\bar{\theta}\big(\bar{\lambda}(x) + \frac{i}{2}\sigma^\mu\partial_\mu\bar{\chi}(x)\big) + \text{h.c.}\Big] \\ &+ \frac{1}{2}\theta\theta\bar{\theta}\bar{\theta}\left(D(x) + \frac{1}{2}\Box C(x)\right)\end{aligned} \tag{3.36}$$

In the above, it is possible for the superfield $V$ to be matrix valued, as will be for non-Abelian supersymmetric gauge theory. Also, a d.o.f. count will show yet again the equality between the number of fermionic and bosonic off-shell d.o.f.s. Finally, it is important to note that chiral, antichiral and vector superfields form irreducible representations of the Super-Poincaré group.

### 3.3 Superspace Actions with Super Yang-Mills

The SM, as presented in Section 2.1, contains propagating fields of which the massive ones obtain their mass from Higgs field and Yukawa potential interactions, all transforming under specific representations of a local internal symmetry implemented through a Yang-Mills construction. It is written in the Lagrangian formulation, which uses an action that can be decomposed into sectors. Therefore, its supersymmetric extension requires at least the following types of action terms:

- Kinetic terms for the matter fields
- Kinetic (field strength) terms for the gauge fields
- Potential terms

Firstly, actions employing superfields must be integrals over superspace in an analog way to non-supersymmetric spacetime actions being integrals of Lagrangian densities over spacetime. Schematically, the correspondence is:

$$\mathcal{S} = \int \mathrm{d}^4x\mathcal{L}(x) = \int \mathrm{d}^4x\mathrm{d}^2\theta\mathrm{d}^2\bar{\theta}\ell(x,\theta,\bar{\theta}) \tag{3.37}$$

$$\text{with} \quad \int \mathrm{d}^2\theta = \frac{1}{2}\int \mathrm{d}\theta_1\mathrm{d}\theta_2, \int \mathrm{d}^2\bar{\theta} = -\frac{1}{2}\int \mathrm{d}\bar{\theta}_1\mathrm{d}\bar{\theta}_2 \implies \left[\int \mathrm{d}^2\theta\right]^\dagger = \int \mathrm{d}^2\bar{\theta} \tag{3.38}$$

Secondly, integrals of Grassmann quantities respect the Berezin integration correspondence between integrals and derivatives, with both integrals and derivatives also being anticommuting quantities:

$$\int \mathrm{d}\eta = \partial_\eta = \frac{\partial}{\partial \eta}, \quad \int \mathrm{d}\bar{\eta} = \partial_{\bar{\eta}} = \frac{\partial}{\partial \bar{\eta}} \tag{3.39}$$

$$\implies \int \mathrm{d}^2\theta\theta^2 = \int \mathrm{d}^2\bar{\theta}\bar{\theta}^2 = 1 \tag{3.40}$$

### 3.3.1 Basic Matter Superspace Actions

We can make some initial considerations on the simplest form of a superspace integral, that of a simple unrestricted superfield given by equation (3.31):

$$\mathcal{S} = \int \mathrm{d}^4x\mathrm{d}^2\theta\mathrm{d}^2\bar{\theta}\Phi(x, \theta, \bar{\theta}) \tag{3.41}$$

First of all, using Berezin integration from equation (3.40) and the fact that $\partial_\eta f = 0$ for $f$ independent of $\eta$, as well as (3.31):

$$\begin{aligned}\mathcal{S} &= \int \mathrm{d}^4x\mathrm{d}^2\theta\mathrm{d}^2\bar{\theta}\Phi(x, \theta, \bar{\theta}) = \int \mathrm{d}^4x\mathrm{d}^2\theta\mathrm{d}^2\bar{\theta}(\theta\theta)(\bar{\theta}\bar{\theta})D(x) \\ &= \int \mathrm{d}^4xD(x)\end{aligned} \tag{3.42}$$

Therefore a whole superspace integral on an arbitrary superfield only produces a common spacetime integral over the highest component $D$-term that it contains. Moreover, under a supersymmetry transformation $D$ changes by a total derivative term (Wess & Bagger, 1992), therefore the action remains invariant under a supersymmetry transformation. With this consideration, we deduce that integrals of unconstrained superfields (and therefore even products, sums, etc. of superfields) over the full superspace produce viable supersymmetry-invariant integrals.

Imposing Lagrangians to satisfy a reality condition, we can take $\Phi$ to be chiral and build the real action:

$$\begin{aligned}\mathcal{S}_{\text{chiral}} &= \int d^4x\mathrm{d}^2\theta\mathrm{d}^2\bar{\theta}\Phi^\dagger\Phi \\ &= \int d^4x\mathrm{d}^2\theta\mathrm{d}^2 \left[\cdots + \theta\theta\bar{\theta}\bar{\theta}[-\partial_\mu\phi^\dagger\partial^\mu\phi - i\bar{\psi}\bar{\sigma}^\mu\partial_\mu\psi + F^\dagger F]\right] \\ &= \int d^4x[-\partial_\mu\phi^\dagger\partial^\mu\phi - i\bar{\psi}\bar{\sigma}^\mu\partial_\mu\psi + F^\dagger F]\end{aligned} \tag{3.43}$$

Where the final expansion comes from taking the highest component of the superfield product using equation (3.34). This kinetic term, consisting of two (one propagating, one not) scalar fields with a propagating Weyl spinor field, is the well-known kinetic term for the free Wess-Zumino action (Wess & Zumino, 1974a). The auxiliary field F, whose role is to match the number of bosons to fermions, has an algebraic on-shell equation fixing it in terms of the remaining fields and discarding it as a field of physical value.

The next component that we require is that implementing a potential term in superspace. First, we observe that superspace integrals can also be taken for chiral fields (3.34) over half of superspace, and that the resulting Lagrangian density changes by a total derivative under supersymmetry transformations (Wess & Bagger, 1992):

$$\int \mathrm{d}^4x \mathrm{d}^2\theta \Phi = \int \mathrm{d}^4x F(x) \tag{3.44}$$

Hence, we can pick any functional $W[\Phi]$ required to be holomorphic in $\Phi$ which is chiral $\bar{\mathcal{D}}_{\dot{\alpha}} W[\Phi] = 0$ and define the superpotential term as below, including the Hermitian conjugation to ensure reality of the action term:

$$\mathcal{S}_{\text{superpotential}} = \int \mathrm{d}^2\theta W[\Phi] + \text{h.c.} \tag{3.45}$$

With this form of $\mathcal{S}_{\text{superpotential}}$, the only term contributing will be the $F$ -term of the holomorphic superpotential functional $W[\Phi]$. By picking a function to specify the scalar component of the $W$ superfield in terms of the scalar component of the $\Phi$ argument superfield (i.e. $W(\phi)$) we obtain the following form for the superpotential superfield, where the total derivative term will not contribute to the action and can be ignored:

$$W[\Phi] = W(\phi) + \frac{\partial W}{\partial \phi}\left[\sqrt{2}\theta\psi + \theta\theta F\right] - \frac{1}{2}\frac{\partial^2 W}{\partial \phi^2}\theta\theta\psi\psi + \partial_\mu U^\mu \tag{3.46}$$

The most general renormalisable superpotential term is a mixture of (not necessarily diagonal) "mass" bilinear coupling terms and "Yukawa" trilinear coupling terms:

$$W[\Phi] = \frac{1}{2}m_{ij}\Phi_i\Phi_j + \frac{1}{3}y_{ijk}\Phi_i\Phi_j\Phi_k \tag{3.47}$$

Matching a kinetic term with a superpotential term, we obtain the full Wess-Zumino action (Wess & Zumino, 1974a) in superfield formalism:

$$\mathcal{S}_{\text{WZ}} = \int d^4x \mathrm{d}^2\theta \mathrm{d}^2\bar{\theta}\Phi^\dagger\Phi + \left[\int \mathrm{d}^2\theta W[\Phi] + \text{h.c.}\right] \tag{3.48}$$

### 3.3.2 Super Yang-Mills Actions

From the direct product nature of the admissible symmetries of a supersymmetric special-relativistic theory (3.8), it has already been stated that the mass and quantum numbers of particle-superpartner pairs must be the same. More precisely, the Super-Poincaré transformations must commute with the internal symmetry group $\mathcal{T}$ of the theory. Let $\mathfrak{t} = \text{Lie}(\mathcal{T})$ be its Lie algebra, with generators satisfying the commutation relation:

$$[T^i, T^j] = if^{ijk}T^k, \{T^i\} \subset \mathfrak{t} \tag{3.49}$$

Then, we must have that $[S(x, \theta, \bar{\theta}), T^i] = 0$, which specifically and importantly implies that all components in a superfield must transform in the same representation of $\mathcal{T}$.

To implement Super Yang-Mills (SYM) constructions for gauge transformations, we pick the superfield type that can contain vector gauge fields, the real vector superfield $V(x,\theta,\bar{\theta})$. It is defined as a superfield, rather than a constant, reflecting the superspace dependence of local symmetries for a supersymmetric theory. Just as the gauge field for non-supersymmetric theory, we take the real vector superfield to live in the Lie algebra $\mathfrak{t}$, hence $V = V^a T^a$. Then a gauge transformation can be implemented by a chiral superfield $\Lambda(x,\theta,\bar{\theta})$, $\bar{\mathcal{D}}_{\dot{\alpha}}\Lambda = 0$, as follows:

$$e^V \to e^{V'} = e^{-i\Lambda^\dagger} e^V e^{i\Lambda} \implies V \to V' \tag{3.50}$$

With the explicit form of $V'$ not required. Then, a suitable choice of $\Lambda$ sets $V$ of (3.36) to the Wess-Zumino gauge (Wess & Bagger, 1992), for which only the physical fields (and one auxiliary field) of $V$ remain:

$$V(x,\theta,\bar{\theta}) \to V(x,\theta,\bar{\theta}) = -\theta\sigma^\mu\bar{\theta}V_\mu(x) + \big(i(\theta\theta\bar{\theta}\bar{\lambda}(x) + \text{h.c.}\big) + \frac{1}{2}\theta\theta\bar{\theta}\bar{\theta}D(x) \tag{3.51}$$

In this form, it is evident that the gauge field $V_\mu$ has a superpartner gaugino spinor field $\lambda_\alpha$. By combining a supersymmetry transformation with a subsequent Wess-Zumino gauge transformation, the field $V$ can be chosen to remain in Wess-Zumino gauge after a Super-Poincaré transformation. To introduce a kinetic term for the superfield containing the gauge and gaugino fields, we define the field strength superfield:

$$W_\alpha = -\frac{1}{4}\bar{\mathcal{D}}^2(e^{-V}\mathcal{D}_\alpha e^V) \tag{3.52}$$

Which undergoes the transformation:

$$\begin{aligned}
\implies W_\alpha \to &-\frac{1}{4}\bar{\mathcal{D}}^2(e^{-i\Lambda}e^{-V}e^{i\Lambda^\dagger}\mathcal{D}_\alpha(e^{-i\Lambda^\dagger}e^V e^{i\Lambda})) \\
=&-\frac{1}{4}\bar{\mathcal{D}}^2(e^{-i\Lambda}e^{-V}\mathcal{D}_\alpha(e^V e^{i\Lambda})) \\
=&-\frac{1}{4}\bar{\mathcal{D}}^2\left[e^{-i\Lambda}\mathcal{D}_\alpha e^{i\Lambda} + e^{-i\Lambda}e^{-V}\mathcal{D}_\alpha(e^V)e^{i\Lambda}\right] \\
=&-\frac{1}{4}e^{-i\Lambda}\bar{\mathcal{D}}^2(e^{-V}\mathcal{D}_\alpha e^V)e^{i\Lambda} = e^{-i\Lambda}W_\alpha e^{i\Lambda}
\end{aligned} \tag{3.53}$$

Given this transformation, we have the following invariant SYM action constructed in a very similar fashion to non-supersymmetric Yang-Mills, where the trace is taken over the gauge indices:

$$\begin{aligned}
\mathcal{S}_{\text{SYM}} &= \int \mathrm{d}^4x\mathrm{d}^2\theta\,\mathrm{Tr}(W^\alpha W_\alpha) + \text{h.c.} \\
&= \int \mathrm{d}^4x\Big[-\frac{1}{4}V^a_{\mu\nu}V^{a\mu\nu} - i\bar{\lambda}^a\bar{\sigma}^\mu D_\mu\lambda^a + \frac{1}{2}D^aD^a\Big]
\end{aligned} \tag{3.54}$$

In the above, $V^a_{\mu\nu} = \partial_\mu V^a_\nu - \partial_\nu V^a_\mu + gf^{bca}V^b_\mu V^c_\nu$ and the covariant derivative $D_\mu$ is as in 2.9 (noting that $D^a$ is distinguished by its index as an auxiliary field, not a covariant derivative). Finally, we define the transformations for chiral superfields in a given representation of the internal symmetry gauge group $\mathcal{T}$ as:

$$\Phi \to e^{-i\Lambda}\Phi = e^{-i\Lambda^a T^a}\Phi = [e^{-i\Lambda^a T^a}]_{ij}\Phi_j \tag{3.55}$$

$$\implies \Phi^\dagger \to \Phi^\dagger e^{i\Lambda^\dagger} \tag{3.56}$$

Now, the chiral superfield kinetic term of (3.43) breaks gauge invariance. To write a kinetic term coupling superfields to supergauge fields, we define the Kähler potential term:

$$\Phi^\dagger e^V \Phi = \Phi_i^\dagger [e^V]_{ij}\Phi_j \to \Phi^\dagger e^{i\Lambda^\dagger} e^{-i\Lambda^\dagger} e^V e^{i\Lambda} e^{-i\Lambda}\Phi = \Phi^\dagger e^V \Phi \tag{3.57}$$

The partial action term containing the Kähler potential can be expanded to obtain the following form, mostly recognisable from non-supersymmetric field theory:

$$\begin{aligned} \mathcal{S}_{\mathrm{M}} &= \int \mathrm{d}^4x \mathrm{d}^2\theta \mathrm{d}^2\bar{\theta} \Phi^\dagger e^V \Phi \\ &= \int \mathrm{d}^4x [-(D_\mu\phi_i)^\dagger (D^\mu \phi_i) - i\bar{\psi}_i \bar{\sigma}^\mu D_\mu \psi_i + F_i^\dagger F_i \\ &\quad + \sqrt{2} i [(\phi_i^* T^a_{ij}\psi_j)\lambda^a + \mathrm{h.c.}] + (\phi_i^* T^a_{ij}\phi_j) D^a] \end{aligned} \tag{3.58}$$

If instead we combine the kinetic Kähler potential term with the kinetic term for SYM, we obtain the general form of a SYM action with coupled superfields, fundamental in the formulation of the MSSM:

$$\mathcal{S}_{\mathrm{SYM+M}} = \left[\int \mathrm{d}^4x \mathrm{d}^2\theta \mathrm{Tr}(W^\alpha W_\alpha) + \mathrm{h.c.}\right] + \int \mathrm{d}^4x \mathrm{d}^2\theta \mathrm{d}^2\bar{\theta} \Phi^\dagger e^V \Phi \tag{3.59}$$

Insertion of a superpotential term of the type in (3.47) now must also simply follow the requirement of preserving gauge invariance when it is transformed under a local symmetry for the theory to be fully gauge invariant supersymetrically. As a final word on notation, the following section on the MSSM (Section 4) adopts the scaling $V \to 2gV$ to present the Lagrangian in canonical form.

# 4 The Minimally Supersymmetric Standard Model (MSSM)

Supersymmetry is a hypothesised symmetry of nature which has to be realised within the context of a theory. The most natural and simplest extension of the SM in this direction is the Minimally Supersymmetric Standard Model (MSSM). From the supersymmetric correspondence between fermionic and bosonic degrees of freedom, the MSSM supplements each SM field with a supersymmetric counterpart, a superpartner. Every integer spin boson in the SM is complemented with a half-integer spin bosino and every half-integer fermion is combined with an integer spin sfermion (scalar fermion). Using the framework developed in Section 3.2, we know that these are held in supermultiplets bound by supersymmetric transformations. While this simplicity is the starting point of this minimal extension to the SM, we soon see that further complications arise, such as the necessary emergence of a second Higgs doublet or the experimentally-motivated restrictions to interaction terms and mass-giving mechanisms.

Before delving into the technicalities of presenting a mathematical formulation of the MSSM, it is useful to briefly review the reasons why supersymmetric extensions of the SM are worth considering. The first is the resolution of the hierarchy problem (Dimopoulos & Georgi, 1981) regarding the quadratic divergences in the mass of the Higgs boson, which require a fine-tuning hypothesis to explain their surprising cancellation. By introducing fermionic superpartners, whose mass divergence is of logarithmic order, supersymmetry implying mass degeneracy between the bosonic and fermionic states exactly explains the absence of such divergences.

The second theoretical issue which supersymmetry in the SM solves has to do with the hypothesis that, just as weak-scale $\mathrm{SU}(3)_\mathrm{C} \times \mathrm{U}(1)_\mathrm{EM}$ gauge symmetry is the residual gauge group after breaking of $\mathrm{SU}(3)_\mathrm{C} \times \mathrm{SU}(2)_\mathrm{L} \times \mathrm{U}(1)_\mathrm{Y}$, that very same gauge group would be a residual of a larger group, such as $\mathrm{SU}(5)$. Simply stated, this conjectures that the gauge groups (and coupling constants) should unify at higher energy scales. Using renormalisation group evolution (see Section 6.2) of $g_1, g_2, g_3$ using just the SM does not cause their three trajectories to cross at a single point, while adding supersymmetry causes the meeting to be exact (Dimopoulos, Raby, & Wilczek, 1981).

Finally, there are indirect observations that are compatible with and support the argument that supersymmetry could be realised in nature. For example, considerations on grand unification led to prediction of a heavy top quark (Bagger, 1996), or the dark matter properties hint at compatibility with the MSSM light neutralino (see later) (Kane, 2002). It is also worth noting that all of these issues receive an explanation simultaneously by the incorporation of supersymmetry, which derived from disparate considerations (Section 3.1), into the SM.

Having discussed the theoretical motivations behind the MSSM, the aim of this section is to use the superspace and superfield formalism mainly developed in Sections 3.2 and 3.3 to formulate the MSSM in Lagrangian form. The details of the less-trivial terms present in the Lagrangian, notably the R-parity conserving superpotential and spontaneous supersymmetry breaking (SSB) from the soft-breaking term is discussed (Sections 4.2.3, 4.2.4). Finally, we obtain the mass-basis particles (in Section 4.3) that are present in the theory.

## 4.1 MSSM Particle Content

We begin the discussion of the MSSM formulation by considering its particle contents. Table 2 presents a breakdown of each MSSM field and its corresponding internal symmetry gauge group representations. Each supermultiplet containing matter particles (and the Higgs) is represented through chiral superfields, denoted $\Phi_X$, while gauge supermultiplets are represented through real vector superfields $V_Y$. Superpartners to minimal SM particles are denoted with a tilde, such as $\tilde{\ell}^f_L$. As seen for example in (3.43) or (3.54), each superfield also contains off-shell bosonic degrees of freedom which have no kinetic term in the action, i.e. $D$ and $F$ terms. These will yield algebraic equations of motion (see Section 4.2.6) which will lose on-shell degrees of freedom. Since these are not real particles and are only required to close supermultiplet transformations off-shell, they are not, and will not be considered as, physical fields. Also, in order to have right-handed quark and lepton fields represented as chiral superfields, we must intend $\Phi_u$, $\Phi_d$ and $\Phi_e$ to contain the charge-conjugated versions of the desired particles. The physical particles can be then re obtained by undoing the conjugation.

A notable difference between the SM and MSSM is the presence of two separate scalar Higgs doublets, namely $H_1$ and $H_2$ contained in $\Phi_{H_1}$ and $\Phi_{H_2}$, each with their corresponding Higgsinos. This derives from the form of superfield Yukawa couplings in the general superpotential term of (3.47). While the minimal SM (2.1.1) employs the conjugated Higgs doublet $\tilde{\phi}$ to produce Yukawa mass terms for up-type quarks, (3.47) presents no opportunity to include a conjugate chiral superfield. A second Higgs doublet chiral superfield, $H_2$ in this case, must then be used to produce Yuwaka-type terms for up-type quark superfields.

## 4.2 The MSSM Lagrangian

Using the superfields and representations discussed in Section 4.1, we can write down an explicit expression for the action of the MSSM. For convenience, we define the Lagrangian through $\mathcal{S}_{\text{MSSM}} = \int \mathrm{d}^4x \mathcal{L}_{\text{MSSM}}$, separating the superspace integrals from the spacetime integral, to obtain a Lagrangian not depending on superspace coordinates. $\mathcal{L}_{\text{MSSM}}$ is presented in equation (4.1). Its form mostly follows that presented by Kuroda in (Kuroda, 1999), with the important addition of three generations of (s)quark and (s)lepton superfields indexed by $f = 1, 2, 3$, which their original discussion does not consider but is given

| **Field Name** | **Field** | **Particle Contents** | $\mathrm{SU(3)_C}$ | $\mathrm{SU(2)_L}$ | $\mathrm{U(1)_Y}$ |
|---|---|---|---|---|---|
| LH (s)quark | $\Phi_{q^f}$ | $\tilde{q}_L^f = \begin{pmatrix} \tilde{u}_L^f \\ \tilde{d}_L^f \end{pmatrix}, \quad q_L^f = \begin{pmatrix} u_L^f \\ d_L^f \end{pmatrix}$ | $\mathbf{3}$ | $\mathbf{2}$ | $+\frac{1}{6}$ |
| RH up (s)quark | $\Phi_{u^f}$ | $\tilde{u}_R^{cf}, \quad u_R^{cf}$ | $\bar{\mathbf{3}}$ | $\bar{\mathbf{1}}$ | $-\frac{2}{3}$ |
| RH down (s)quark | $\Phi_{d^f}$ | $\tilde{d}_R^{cf}, \quad d_R^{cf}$ | $\bar{\mathbf{3}}$ | $\bar{\mathbf{1}}$ | $+\frac{1}{3}$ |
| LH (s)lepton | $\Phi_{\ell^f}$ | $\tilde{\ell}_L^f = \begin{pmatrix} \tilde{\nu}_L^f \\ \tilde{e}_L^f \end{pmatrix}, \quad \ell_L^f = \begin{pmatrix} \nu_L^f \\ e_L^f \end{pmatrix}$ | $\mathbf{1}$ | $\mathbf{2}$ | $-\frac{1}{2}$ |
| RH (s)electron | $\Phi_{e^f}$ | $\tilde{e}_R^{cf}, \quad e_R^{cf}$ | $\bar{\mathbf{1}}$ | $\bar{\mathbf{1}}$ | $+1$ |
| Higgs/Higgsino (1) | $\Phi_{H_1}$ | $H_1 = \begin{pmatrix} H_1^0 \\ H_1^- \end{pmatrix}, \quad \tilde{H}_1 = \begin{pmatrix} \tilde{H}_1^0 \\ \tilde{H}_1^- \end{pmatrix}$ | $\mathbf{1}$ | $\mathbf{2}$ | $-\frac{1}{2}$ |
| Higgs/Higgsino (2) | $\Phi_{H_2}$ | $H_2 = \begin{pmatrix} H_2^+ \\ H_2^0 \end{pmatrix}, \quad \tilde{H}_2 = \begin{pmatrix} \tilde{H}_2^+ \\ \tilde{H}_2^0 \end{pmatrix}$ | $\mathbf{1}$ | $\mathbf{2}$ | $+\frac{1}{2}$ |
| Gluino / gluon | $V_{\mathrm{C}}$ | $\tilde{G}^{\vartheta}, \quad G_\mu^{\vartheta}$ | - | - | - |
| Wino / W | $V_{\mathrm{L}}$ | $\tilde{W}^i, \quad W_\mu^i$ | - | - | - |
| Bino / B | $V_{\mathrm{Y}}$ | $\tilde{B}, \quad B_\mu$ | - | - | - |

Table 2: MSSM flavour basis breakdown of superfields and their physical particle supermultiplet contents. The additional $c$ superscript, such as in $u^c$, denotes a requirement to intend the field as the conjugate of the physical one. Vectors denote $\mathrm{SU(2)_L}$ doublets. LH and RH stand for left- and right-handedness.

explicitly in this report. The soft supersymmetry breaking term $\mathcal{L}_{\text{soft}}$ is given explicitly in equation (4.14), as it is not strictly part of the main structure of the MSSM but rather can change depending on the model.

$$
\begin{aligned}
\mathcal{L}_{\text{MSSM}} = &\int \mathrm{d}\theta^2 \left[\frac{1}{4}W_{\text{Y}}W_{\text{Y}} + \frac{1}{2}\text{Tr}(W_{\text{L}}W_{\text{L}}) + \frac{1}{2}\text{Tr}(W_{\text{C}}W_{\text{C}})\right] + \text{h.c.} && \text{(4.1a)}\\
&+ \sum_{f=1}^{3}\int \mathrm{d}\theta^2\mathrm{d}\bar{\theta}^2 \Phi^{\dagger}_{\ell f} e^{2\left(g_2 \frac{1}{2}\sigma^i V^i_{\text{L}} + g_1 \frac{1}{2} Y_\ell V_{\text{Y}}\right)} \Phi_{\ell f} && \text{(4.1b)}\\
&+ \sum_{f=1}^{3}\int \mathrm{d}\theta^2\mathrm{d}\bar{\theta}^2 \Phi^{\dagger}_{e f} e^{2\left(g_1 \frac{1}{2} Y_e V_{\text{Y}}\right)} \Phi_{e f} && \text{(4.1c)}\\
&+ \sum_{f=1}^{3}\int \mathrm{d}\theta^2\mathrm{d}\bar{\theta}^2 \Phi^{\dagger}_{q f} e^{2\left(g_3 \frac{1}{2}\lambda^\vartheta V^\vartheta_{\text{C}} + g_2 \frac{1}{2}\sigma^i V^i_{\text{L}} + g_1 \frac{1}{2} Y_q V_{\text{Y}}\right)} \Phi_{q f} && \text{(4.1d)}\\
&+ \sum_{f=1}^{3}\int \mathrm{d}\theta^2\mathrm{d}\bar{\theta}^2 \Phi^{\dagger}_{u f} e^{2\left(-g_3 \frac{1}{2}\lambda^{*\vartheta} V^\vartheta_{\text{C}} + g_1 \frac{1}{2} Y_u V_{\text{Y}}\right)} \Phi_{u f} && \text{(4.1e)}\\
&+ \sum_{f=1}^{3}\int \mathrm{d}\theta^2\mathrm{d}\bar{\theta}^2 \Phi^{\dagger}_{d f} e^{2\left(-g_3 \frac{1}{2}\lambda^{*\vartheta} V^\vartheta_{\text{C}} + g_1 \frac{1}{2} Y_d V_{\text{Y}}\right)} \Phi_{d f} && \text{(4.1f)}\\
&+ \int \mathrm{d}\theta^2\mathrm{d}\bar{\theta}^2 \Phi^{\dagger}_{H_1} e^{2\left(g_2 \frac{1}{2}\sigma^i V^i_{\text{L}} + g_1 \frac{1}{2} Y_{H_1} V_{\text{Y}}\right)} \Phi_{H_1} && \text{(4.1g)}\\
&+ \int \mathrm{d}\theta^2\mathrm{d}\bar{\theta}^2 \Phi^{\dagger}_{H_2} e^{2\left(g_2 \frac{1}{2}\sigma^i V^i_{\text{L}} + g_1 \frac{1}{2} Y_{H_2} V_{\text{Y}}\right)} \Phi_{H_2} && \text{(4.1h)}\\
&+ \sum_{f,g=1}^{3}\left[[y_e]_{fg}\int \mathrm{d}\theta^2(\Phi_{H_1}\cdot\Phi_{lf})\Phi_{e^g} + \text{h.c.}\right] && \text{(4.1i)}\\
&- \sum_{f,g=1}^{3}\left[[y_u]_{fg}\int \mathrm{d}\theta^2(\Phi_{H_2}\cdot\Phi_{q^f})\Phi_{u^g} + \text{h.c.}\right] && \text{(4.1j)}\\
&+ \sum_{f,g=1}^{3}\left[[y_d]_{fg}\int \mathrm{d}\theta^2(\Phi_{H_1}\cdot\Phi_{q^f})\Phi_{d^g} + \text{h.c.}\right] && \text{(4.1k)}\\
&- \mu\int \mathrm{d}\theta^2(\Phi_{H_1}\cdot\Phi_{H_2}) + \text{h.c.} && \text{(4.1l)}\\
&+ \mathcal{L}_{\text{soft}} && \text{(4.1m)}\\
&+ \mathcal{L}_{\text{fixing}} + \mathcal{L}_{\text{ghost}} && \text{(4.1n)}
\end{aligned}
$$

### 4.2.1 The Gauge and Matter Kinetic Sectors

Sectors (4.1a)-(4.1h) contain the kinetic terms for both matter and gauge fields. Sector (4.1a) specifically contains the three field strength terms for the colour, isospin and hypercharge gauge symmetries in the direct product gauge group $\text{SU}(3)_\text{C} \times \text{SU}(2)_\text{L} \times \text{U}(1)_\text{Y}$, arranged in three action terms of the type in (3.54). Sectors (4.1b)-(4.1h) employ the Kähler potential of (3.58) to couple each of the superfields to the gauge superfields in the correct representations according to Table 2. Considering three generations of quarks and leptons, there are three copies of each kinetic term, one for each generation.

It is worth noting that the "right-handed" quark chiral superfields, transform under the conjugate representations of $\mathrm{SU}(3)_\mathrm{C}$ and therefore according to the generators $-\lambda^{*\vartheta}$, with $\lambda^\vartheta$ being the usual Gell-Mann matrices. Also, their weak hypercharge is negative with respect to the one expected in the minimal SM.

### 4.2.2 The Superpotential Sector

Sectors (4.1i)-(4.1l) contain the superpotential terms for the MSSM interactions, incorporated as in (3.45). Collecting the terms written separately in the Lagrangian and arranged in a way that can be confronted with the general renormalisable superpotential of (3.47), the MSSM superpotential is:

$$\begin{aligned} W_\mathrm{MSSM} = -\mu\Phi_{H_1}\cdot\Phi_{H_2} + &\sum_{f,g=1}^{3}[y_e]_{fg}(\Phi_{H_1}\cdot\Phi_{l^f})\Phi_{e^g} \\ -&\sum_{f,g=1}^{3}[y_u]_{fg}(\Phi_{H_2}\cdot\Phi_{q^f})\Phi_{u^g} \\ +&\sum_{f,g=1}^{3}[y_d]_{fg}(\Phi_{H_1}\cdot\Phi_{q^f})\Phi_{d^g} \end{aligned} \tag{4.2}$$

In the above, the dot is defined as $\Phi\cdot\Psi = \epsilon^{ij}\Phi_i\Psi_j$ for $\mathrm{SU}(2)_\mathrm{L}$ doublets. It is clear that we can associate $\mu$ with a mass-contributing term for the Higgs superfields. The remaining terms instead provide Yukawa couplings that will contribute to the development of mass for the matter superfields after electroweak symmetry breaking, analysed in detail in Section 4.3. From term (4.1j), it is also seen that the Higgs superfield doublet $H_2$ acts as the $\tilde{\phi}$ field for the up-type quark superfield.

All terms of $W_\mathrm{MSSM}$ respect gauge invariance, as required. For example, we consider the Yukawa term for the up-type quark superfield. First, summing the weak hypercharges for the term yields $\frac{1}{2}+\frac{1}{6}-\frac{2}{3}=0$. Secondly, an $\mathrm{SU}(2)_\mathrm{L}$ transformation gives:

$$\begin{aligned} (\Phi_{H_2}\cdot\Phi_{q^f})\Phi_{u^g} \to &\epsilon^{ij}[e^{-2g_2 i\Lambda_\mathrm{L}}]_{ik}(\Phi_{H_2})_k[e^{-2g_2 i\Lambda_\mathrm{L}}]_{jl}(\Phi_{q^f})_l\Phi_{u^g} \\ &= ([e^{-2g_2 i\Lambda_\mathrm{L}}]_{ik}[e^{-2g_2 i\Lambda_\mathrm{L}}]_{jl}\epsilon^{ij})(\Phi_{H_2})_k(\Phi_{q^f})_l\Phi_{u^g} \\ &= (\det[e^{-2g_2 i\Lambda_\mathrm{L}}])\epsilon^{kl}(\Phi_{H_2})_k(\Phi_{q^f})_l\Phi_{u^g} \\ &= \epsilon^{kl}(\Phi_{H_2})_k(\Phi_{q^f})_l\Phi_{u^g} = (\Phi_{H_2}\cdot\Phi_{q^f})\Phi_{u^g} \end{aligned} \tag{4.3}$$

Where the unit determinant follows from:

$$\det[e^{-2g_2 i\Lambda_\mathrm{L}}] = e^{-2g_2 i\Lambda^a_\mathrm{L}\mathrm{tr}[T^a]} = e^0 = 1 \tag{4.4}$$

Finally, we consider an $\mathrm{SU}(3)_\mathrm{C}$ transformation where, in the following, $\sigma, \tau$ and $\rho$ are taken to be explicit $\mathrm{SU}(3)_\mathrm{C}$ indices:

$$(\Phi_{H_2}\cdot\Phi_{q^f})\Phi_{u^g} \to (\Phi_{H_2}\cdot(e^{-2g_3 i\Lambda^\vartheta T^\vartheta})_{\sigma\tau}(\Phi_{q^f})^\tau)(e^{2g_3 i\Lambda^\vartheta T^{*\vartheta}})_{\sigma\rho}\Phi^\rho_{u^g}$$

$$= (\Phi_{H_2} \cdot (\Phi_{q^f})^\tau)(e^{-2g_3 i\Lambda^\vartheta T^\vartheta})_{\sigma\tau}(e^{2g_3 i\Lambda^\vartheta T^{\dagger\vartheta}})_{\rho\sigma}\Phi^\rho_{u^g}$$
$$= (\Phi_{H_2} \cdot (\Phi_{q^f})^\tau)(e^{-2g_3 i\Lambda^\vartheta T^\vartheta})_{\sigma\tau}(e^{-2g_3 i\Lambda^\vartheta T^\vartheta})^\dagger_{\rho\sigma}\Phi^\rho_{u^g}$$
$$= (\Phi_{H_2} \cdot (\Phi_{q^f})^\tau)\delta_{\rho\tau}\Phi^\rho_{u^g} = (\Phi_{H_2} \cdot \Phi_{q^f})\Phi_{u^g} \tag{4.5}$$

The other terms follow similar transformations that guarantee their gauge invariance.

It is worth mentioning that the $-\mu\Phi_{H_1} \cdot \Phi_{H_2}$ term is usually included within the soft breaking component of the Lagrangian, $\mathcal{L}_{\text{soft}}$, as in (Misiak et al., 1997). Nevertheless, the term is supersymmetric invariant, and therefore can be considered as part of $W_{\text{MSSM}}$ superpotential.

### 4.2.3 R-Parity

While the superpotential $W_{\text{MSSM}}$ respects supersymmetry, the gauge symmetries of the theory, renormalisability and is holomorphic in the superfields, there are some additional terms that have been omitted. This is because they produce phenomenology inconsistent with experiments that can be fully removed by adding the symmetry of R-parity, or matter parity, to the MSSM.

Under R-parity (Fayet & Iliopoulos, 1974; Farrar & Fayet, 1978), each field in a supermultiplet transforms by an overall integer phase $\phi(x) \to \eta_R\phi(x)$ with $\eta_R = \pi_R(\phi)$ given by the parity function. More specifically, minimal SM fields can be assigned the R-parity $\eta_R = +1$ simultaneously with superpartners obtaining the phase $\eta_R = -1$. Simultaneous transformation of the superspace coordinates by $\theta \to -\theta$, $\bar{\theta} \to -\bar{\theta}$ ensures that each supermultiplet transforms by a single overall phase. As shown in (Farrar & Weinberg, 1983), this formulation is equivalent to defining the parity function from the quantum number of the MSSM fields:

$$\pi_R(\phi) = (-1)^{2S_\phi}(-1)^{3(B_\phi - L_\phi)} \tag{4.6}$$

$W_{\text{MSSM}}$ is symmetric under R-parity transformations. As an example, we consider the following:

$$\pi_R(H_1) = (-1)^{2\cdot 0}(-1)^0 = +1 \tag{4.7}$$
$$\pi_R(\tilde{\ell}^f_L) = (-1)^{2\cdot 0}(-1)^{3(0-1)} = -1 \tag{4.8}$$
$$\pi_R(\tilde{e}^f_L) = (-1)^{2\cdot 0}(-1)^{3(0-1)} = -1 \tag{4.9}$$
$$\implies (\Phi_{H_1} \cdot \Phi_{\ell^f})\Phi_{e^g} \to (+1)(-1)^2(\Phi_{H_1} \cdot \Phi_{\ell^f}) = (\Phi_{H_1} \cdot \Phi_{\ell^f}) \tag{4.10}$$

Invariance of all other terms follows similarly. We can attempt extending $W_{\text{MSSM}}$ with parity violating terms. Examples of terms breaking R-parity invariance that can be added to (4.2) are the following (Martin, 1998):

$$W_{\Delta L} = \frac{1}{2}y_{fgh}(\Phi_{\ell^f} \cdot \Phi_{\ell^g})\Phi_{e^h} + y'_{fgh}(\Phi_{\ell^f} \cdot \Phi_{q^g})\Phi_{d^h} + \mu'_f(\Phi_{\ell^f} \cdot \Phi_{H_1}) \tag{4.11}$$

$$W_{\Delta B} = \frac{1}{2} y''_{fgh} \Phi_{u^f} \Phi_{d^g} \Phi_{d^h} \tag{4.12}$$

Using the results in (4.8) and (4.9), we can for example show that the first term of $W_{\Delta L}$ is odd under R-parity:

$$(\Phi_{\ell^f} \cdot \Phi_{\ell^g}) \Phi_{e^h} \to (-1)^3 (\Phi_{\ell^f} \cdot \Phi_{\ell^g}) = -(\Phi_{\ell^f} \cdot \Phi_{\ell^g}) \tag{4.13}$$

Besides R-parity, both $W_{\Delta L}$ and $W_{\Delta B}$ respect all of the usual requirements for being part of $W_{\text{MSSM}}$. Taking these additions integrated over superspace to define interaction vertices, we can see that (4.11) contains an overall lepton number increase of 1, while (4.12) contains an overall baryon number increase of 1. Hence, these terms violate either lepton or baryon universality.

The most striking result of including these terms in the superpotential is the decay of the proton, which can for example happen in the process $p^+ \to \pi^0 e^+$ mediated by bottom squarks. With results from Super-Kamiokande observing its longest lifetime limit as $1.67 \times 10^{34}$ years (Ikeda, 2015; Bajc, Hisano, Kuwahara, & Omura, 2016), R-parity is justified as an additional symmetry restricting the form of the MSSM superpotential. It is worth noting that R-parity violation can be included in supersymmetric extensions to the SM beyond the MSSM (Barbier et al., 2005), although it will not be a topic of discussion beyond simple observations in this report.

#### 4.2.4 Soft-Breaking Terms

The result that particles sharing a supermultiplet must be degenerate in mass (and share the same quantum numbers) implies experimentally that superpartners should be observed, for example, at the same energies as observed particles in particle colliders. If we examine the experimentally observed SM particles in Table 1, no two boson-fermion pairs can be seen to have the same gauge group quantum numbers. Therefore, it is concluded that no superpartners have been observed in nature. To maintain supersymmetry as a valid hypothesised symmetry of observable nature, it is possible to arrange for its breaking such that additional mass is developed for the unobserved particles. In the generality of supersymmetric theories, the first two fundamental models of supersymmetry breaking (SSB) were proposed by Fayet and Iliopoulos and subsequently by O'Raifertaigh.

In the Fayet-Iliopoulos mechanism of SSB (Fayet & Iliopoulos, 1974), it is required that two chiral superfields interact with a vector superfield, such as in supersymmetric QED (SQED). The $D$-term of the vector superfield then can develop a non-zero vev and cause the scalar potential, produced by solving its associated algebraic equation of motion, to give different masses to bosons and fermions coupling to it. Similarly to broken internal symmetry giving rise to a massless Goldstone boson, SSB via the Fayet-Iliopoulos mechanism gives rise to a massless fermion (Iliopoulos & Zumino, 1974; Salam & Strathdee, 1974a) (due to the spinorial nature of the conserved charges and currents of supersymmetry), which is typically called a Goldstino.

In O'Raifertaigh's mechanism of (O'Raifeartaigh, 1975), SSB can be achieved with three or more chiral superfields and the requirement of using a vector superfield is dropped. By including a linear term in the superpotential, $F$-term integration again produces mass differences within the same supermultiplet, and a Goldstino arises.

Unfortunately, neither mechanism is acceptable when supplementing theory with the experimental success of the observed SM (Chung et al., 2005). The Fayet-Iliopoulos mechanism fails by requiring extensions of the MSSM respecting further U(1) symmetries, which predicts particles that are too light for experiment. O'Raifertaigh's mechanism fails when considering the mass implications from the vanishing supertrace relation, which are again too light.

To avoid the experimental limitations imposed by describing an explicit mechanism for supersymmetry breaking, we include a $\mathcal{L}_{\text{soft}}$ term (4.1m) which is to be intended as an effective contribution to the MSSM Lagrangian (similar to those discussed in Section 6). Its effective nature can be justified as supersymmetry being spontaneously broken at high scales, with the effects being mediated to lower scales by messenger fields, such as as a consequences of coupling to supergravity or anomalies. Nevertheless, it is a soft-breaking term in the sense that all of its terms are renormalisable (notably different from the higher-dimensional effective terms later considered in Section 6), other than being gauge invariant, and do not give rise to quadratic divergences, although clearly explicitly breaking supersymmetry. Its form is given by:

$$
\begin{aligned}
\mathcal{L}_{\text{soft}} = &- \frac{1}{2}[m_{\tilde{G}} \tilde{G}^{\vartheta} \tilde{G}^{\vartheta} + m_{\tilde{W}} \tilde{W}^{i} \tilde{W}^{i} + m_{\tilde{B}} \tilde{B} \tilde{B} + \text{h.c.}] && (4.14a)\\
&- m^2_{\tilde{H}_1} H_1^{\dagger} H_1 - m^2_{\tilde{H}_2} H_2^{\dagger} H_2 + \left[b(H_1 \cdot H_2) + \text{h.c.}\right] && (4.14b)\\
&- \sum_{f,g=1}^{3} \left[[M^2_{\tilde{\ell}}]_{fg} \tilde{\ell}_L^{\dagger f} \tilde{\ell}_L^{g} + [M^2_{\tilde{e}}]_{fg} \tilde{e}_R^{c*f} \tilde{e}_R^{cg}\right. && (4.14c)\\
&\qquad \left. + [M^2_{\tilde{q}}]_{fg} \tilde{q}_L^{\dagger f} \tilde{q}_L^{g} + [M^2_{\tilde{u}}]_{fg} \tilde{u}_R^{c*f} \tilde{u}_R^{g} + [M^2_{\tilde{d}}]_{fg} \tilde{d}_R^{c*f} \tilde{d}_R^{g}\right] && (4.14d)\\
&- \sum_{f,g=1}^{3} \left[[A_{\tilde{e}}]_{fg} (\tilde{H}_1 \cdot \tilde{\ell}_L^{f}) \tilde{e}_R^{cg} - [A_{\tilde{u}}]_{fg} (\tilde{H}_2 \cdot \tilde{q}_L^{f}) \tilde{u}_R^{cg} + [A_{\tilde{d}}]_{fg} (\tilde{H}_1 \cdot \tilde{q}_L^{f}) \tilde{d}_R^{cg} + \text{h.c.}\right] && (4.14e)
\end{aligned}
$$

Where, in the above, Hermitian conjugation also acts to transpose $\text{SU}(2)_{\text{L}}$ doublets. (4.14a) provides the mass-adjusting terms for gluinos, (4.14b) terms contributing to Higgs masses and electroweak symmetry breaking, and (4.14c),(4.14d) provides mass contributions for sfermions (squarks and sleptons). Finally (4.14e) contains additional trilinear scalar interactions weighted by the $[A_{\tilde{X}}]$ mixing angle matrices.

### 4.2.5 Comment on Gauge Fixing and Ghost Fields

The terms (4.1a)-(4.1l) in $\mathcal{L}_{\text{MSSM}}$ can be considered to be part of the formulation of the MSSM at the classical level. In order to quantise the theory, we use a path integral approach. The correlators are obtained by the functional derivatives of the sourced generating functional $\mathcal{Z}[J, J_\alpha, J_\mu] = e^{i\mathcal{W}[J,J_\alpha,J_\mu]}$, including a factorisation of $\mathcal{Z}$ by the volume of gauge orbits. This sourced generating functional is supposed to be intended schematically, with the sources $J, J_\alpha$ and $J_\mu$ standing for the scalar, spinor and vector sources needed for the different Lorentz representations of fields present in the MSSM.

The two terms in (4.1n), namely $\mathcal{L}_{\text{fixing}}$ and $\mathcal{L}_{\text{ghost}}$, then provide the gauge-fixing and Faddeev-Popov ghost terms to account for the BRST quantisation of the MSSM theory at the path integral level. The explicit form of these terms is very extensive and not required by this analysis, but can be found in (Kuroda, 1999) or (Rosiek, 1990) (and its erratum (Rosiek, 2002)).

### 4.2.6 The Scalar Potential from Auxiliary Fields

In Section 3.31, we have seen that chiral and vector superfields both contain components, namely the $F^X$ and $D^\vartheta, D^i, D$ fields (where $X$ is a given supermultiplet label, coming from the chiral and vector superfields respectively), that do not contain on-shell d.o.f.s. This is because their equations of motion do not contain kinetic terms and are purely algebraic. When their solution is substituted back into the action, they give rise to additional interactions between the physical fields as additions to the potential chosen for the theory. This contribution is called the scalar potential, and in the case of the MSSM it holds part of the contributions for the mass-giving terms for its particles. Using the superpotential to gather terms contributing to particle masses will be crucial in Section 4.3 to rotate field multiplets into the physical mass basis of the MSSM.

To derive the form of the scalar potential, we remain in the generality of a SYM theory coupled to matter chiral superfields and a superpotential from Section 3.3, in particular combining equation (3.59) with a gauge invariant (but general) superpotential (3.46). Collecting the actions integrated over superspace coordinates, we obtain the general form (Ewerth, 2004a):

$$\begin{aligned}\mathcal{L} = &-\frac{1}{4}V^a_{\mu\nu}V^{a\mu\nu} - i\bar{\lambda}^a\bar{\sigma}^\mu D_\mu\lambda^a + \frac{1}{2}D^aD^a - (D_\mu\phi)^*_i(D^\mu\phi)_i - i\bar{\psi}_i\bar{\sigma}^\mu D_\mu\psi_i \\ &+ F^*_iF_i + \sqrt{2}i(\phi^*_iT^a_{ij}\psi_j\lambda^a + \text{h.c.}) + gD^a\phi^*_iT^a_{ij}\phi_j + \left(\frac{\partial^2 W}{\partial\phi_i}F_i - \frac{1}{2}\frac{\partial^2 W}{\partial\phi_i\partial\phi_j} + \text{h.c.}\right)\end{aligned} \tag{4.15}$$

Applying the Euler-Lagrange condition with respect to the real fields $D^a$ and $F^*_i$, we obtain the algebraic equations of motion for the auxiliary fields:

$$\frac{\partial\mathcal{L}}{\partial D^a} - \partial_\mu\frac{\partial\mathcal{L}}{\partial\partial_\mu D^a} = \frac{\partial\mathcal{L}}{\partial D^a} = D^a + g\phi^*_iT^a_{ij}\phi_j = 0 \implies D^a = -g\phi^*_iT^a\phi_j \tag{4.16}$$

$$\frac{\partial \mathcal{L}}{\partial F_i^*} - \partial_\mu \frac{\partial \mathcal{L}}{\partial \partial_\mu F_i^*} = \frac{\partial \mathcal{L}}{\partial F_i^*} = F_i + \left(\frac{\partial W}{\partial \phi_i}\right)^* = 0 \implies F_i = -\left(\frac{\partial W}{\partial \phi_i}\right)^* \tag{4.17}$$

By performing these substitutions in the inverse fashion, we can rewrite the Lagrangian as:

$$\begin{aligned}\mathcal{L} = &-\frac{1}{4}V^a_{\mu\nu}V^{a\mu\nu} - i\bar{\lambda}^a\bar{\sigma}^\mu D_\mu \lambda^a - (D_\mu\phi)^*_i(D^\mu\phi)_i - i\bar{\psi}_i\bar{\sigma}^\mu D_\mu\psi_i \\ &+ i\sqrt{2}(\phi_i^* T^a_{ij}\psi_j\lambda^a + \text{h.c.}) + \left(-\frac{1}{2}\frac{\partial^2 W}{\partial\phi_i\partial\phi_j} + \text{h.c.}\right) - F_i^*F_i - \frac{1}{2}D^aD^a\end{aligned} \tag{4.18}$$

Observing the final two terms, we define the scalar potential as:

$$\begin{aligned}V_{\text{aux}} &= F_i^*F_i + \frac{1}{2}D^aD^a \\ &= \left(\frac{\partial W}{\partial \phi_i}\right)^* \left(\frac{\partial W}{\partial \phi_i}\right) + \frac{1}{2}g^2(\phi_i^*T^a_{ij}\phi_j)^2\end{aligned} \tag{4.19}$$

From this expression, we can note that the scalar potential, as the name suggests, arises from the scalar components $\phi_i$ of the superfield coupled to the gauge symmetry. There is no contraction of higher spinor or vector terms in the supermultiplet.

Clearly, the MSSM contains a more complex gauge symmetry group than the single non-Abelian considered, but the scalar potential terms can still be collected easily. Each vector superfield with gauge index $a$ contributes a single term of the type $\frac{1}{2}D^aD^a$, while each chiral superfield labelled by $X$ contributes $(F^X)^\dagger F^X$. We take $\vartheta$ as a $\text{SU}(3)_\text{C}$ index and $i$ as a $\text{SU}(2)_\text{L}$ index, as well as labelling $\phi^X$ the scalar component for the $X$ multiplet. The total $F$-term contributions are given schematically by:

$$\sum_X (F^X)^\dagger F^X = \sum_X \sum_{\vartheta,i} \left|\frac{\partial W_{\text{MSSM}}}{\partial(\phi^X)^{\vartheta i}}\right|^2 \tag{4.20}$$

The $D$-term contributions for a given gauge group come from products of the scalar fields in non-trivial representations of the group weighted by the generators of the corresponding representation, as:

$$D^a = -g_G \sum_T \sum_X (\phi^X)^\dagger T^a \phi^X \tag{4.21}$$

Combining $F$ and $D$ term contributions of (4.20) and (4.21), the scalar potential of the MSSM can be descriptively given as:

$$V_{\text{scalar,MSSM}} = -\sum_X \sum_{\vartheta,i} \left|\frac{\partial W_{\text{MSSM}}}{\partial(\phi^X)^{\vartheta i}}\right|^2 - \frac{1}{2}\sum_{G\in\mathcal{T}} g_G^2 \left(\sum_T \sum_X (\phi^X)^\dagger T^a \phi^X\right)^2 \tag{4.22}$$

With $\mathcal{T} = \text{SU}(3)_\text{C} \times \text{SU}(2)_\text{L} \times \text{U}(1)_\text{Y}$ and $G$ running over the groups in the direct product and $g_G$ the $g_1, g_2$ and $g_3$ coupling constants.

## 4.3 MSSM Mass-Basis Particles

### 4.3.1 Electroweak Symmetry Breaking in the MSSM and $\tan\beta$

As in the minimal SM, the mechanism that most fundamentally contributes to the masses of MSSM particles is the development of the vevs for the two Higgs fields we have introduced. As in Section 2.1.2, the Higgs configuration that performs the $\mathrm{SU}(3)_\mathrm{C} \times \mathrm{SU}(2)_\mathrm{L} \times \mathrm{U}(1)_\mathrm{Y} \to \mathrm{SU}(3)_\mathrm{C} \times \mathrm{U}(1)_\mathrm{EM}$ EWSB is that minimising its self-interaction potential.

A full derivation of the two Higgs vevs is covered in Appendix A, based on the discussion present in (Kuroda, 1999). In short, the terms of the Higgs self-interaction potential can be recovered in the scalar potential (4.22) and the soft breaking terms (4.14). With a choice of gauge, the two Higgs doublets assume the configuration:

$$H_1 = \frac{1}{\sqrt{2}}\begin{pmatrix} v_1 \\ 0 \end{pmatrix}, \quad H_2 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_2 \end{pmatrix} \tag{4.23}$$

Using the vevs of the two doublets, we can also define the following $\beta$ angle:

$$\tan\beta = \frac{v_2}{v_1} \implies v^2 = v_1^2 + v_2^2, v_1 = v\sin\beta, v_2 = v\cos\beta \tag{4.24}$$

### 4.3.2 Mass Matrices, Physical Particles and the Super-CKM Basis

Having discussed EWSB using the two Higgs doublets required by the MSSM, it is now possible to observe how the superpartners to the minimal SM fields obtain their masses in the broken phase. The diagonalisation of superpartner mass matrices is important in identifying the mass basis fields which are commonly used to study the physical interactions in the MSSM. These appear both in the interaction vertices of Section 5 and the Feynman diagrams of Section 7. The explicit derivations for all of these mass matrices has been performed and verified with the literature ((Cho, Misiak, & Wyler, 1996) and (Kuroda, 1999) for generation-independent matrices) by collecting contributions from integrated out $F$ and $D$ fields (the scalar potential of (4.22)), from the soft supersymmetry breaking term (4.14), from the Higgs-superfield interactions and from the Yukawa terms. Two fully explicit examples of calculations can be found in Appendix B.

First, it is useful to re-express the $W$ and $Z$ boson masses in terms of the combined vevs of the two Higgs doublets (Kuroda, 1999) (which compared to the minimal SM adds the square of the vevs):

$$m_W^2 = \frac{1}{4}g_2^2 v^2 \tag{4.25}$$

$$m_Z^2 = \frac{1}{4}(g_2^2 + g_1^2)v^2 \tag{4.26}$$

$$v^2 = v_1^2 + v_2^2 \tag{4.27}$$

The first derived mass term is that for neutral Higgs particles, coming from the small field expansion around the vev of the two Higgs doublets. The four real fields obtained in this way contain one scalar, one pseudoscalar and two Goldstone fields. The two matrices collecting these physical states are:

$$\mathcal{M}_{\varphi^0} = \begin{pmatrix} m_A^2 \sin^2\beta + m_Z^2 \cos^2\beta & -(m_A^2 + m_Z^2)\cos\beta\sin\beta \\ -(m_A^2 + m_Z^2)\cos\beta\sin\beta & m_A^2\cos^2\beta + m_Z^2\sin^2\beta \end{pmatrix} \tag{4.28}$$

$$\mathcal{M}_{\chi^0} = (m_{H_1}^2 + m_{H_2}^2)\begin{pmatrix} \sin^2\beta & -\sin\beta\cos\beta \\ -\sin\beta\cos\beta & \cos^2\beta \end{pmatrix} \tag{4.29}$$

Where:

$$m_{H_1}^2 = m_{\tilde{H}_1}^2 + |\mu|^2 \tag{4.30}$$

$$m_{H_2}^2 = m_{\tilde{H}_2}^2 + |\mu|^2 \tag{4.31}$$

$$m_A^2 = m_{H_1}^2 + m_{H_2}^2 \tag{4.32}$$

This is followed by the two charged Higgs fields obtained by diagonalising the following matrix:

$$\mathcal{M}_{\phi^\pm} = (m_{H_1}^2 + m_{H_2}^2 + \frac{g_2^2}{4}v^2)\begin{pmatrix} \sin^2\beta & -\cos\beta\sin\beta \\ -\cos\beta\sin\beta & \cos^2\beta \end{pmatrix} \tag{4.33}$$

The following superfield types come from a mixing of Higgsino and $\mathrm{SU}(2)_\mathrm{L} \times \mathrm{U}(1)_\mathrm{Y}$ gaugino (wino and bino, or equivalently wino, zino and photino) terms after symmetry breaking. These linear combinations of fields can be both charged or neutral under the residual $\mathrm{U}(1)_\mathrm{EM}$ gauge symmetry, giving rise to charginos and neutralinos respectively. The chargino mass matrix, giving two fields, is given by:

$$\mathcal{M}_{\tilde{\chi}^\pm}^2 = \begin{pmatrix} m_{\tilde{W}} & \sqrt{2}m_W\cos\beta \\ \sqrt{2}m_W\sin\beta & \mu \end{pmatrix} \tag{4.34}$$

The neutralino mass matrix instead gives four fields and is given by:

$$\mathcal{M}_{\tilde{\chi}^0}^2 = \begin{pmatrix} m_{\tilde{B}} & 0 & -m_Z s_W\cos\beta & m_Z s_W\sin\beta \\ 0 & m_{\tilde{W}} & m_Z c_W\cos\beta & -m_Z c_W\sin\beta \\ -m_Z s_W\cos\beta & m_Z c_W\cos\beta & 0 & -\mu \\ m_Z s_W\sin\beta & -m_Z c_W\sin\beta & -\mu & 0 \end{pmatrix} \tag{4.35}$$

The following mass matrices all involve the superpartners of SM matter fields, namely squarks and sleptons. In Section 2.1.2 it was discussed how rotation of the flavour basis quarks and leptons into the mass basis is done through biunitary transformations, also giving rise to the CKM matrix. In preparing the mass matrix for squarks, it is convenient to apply the same flavour-to-mass-basis transformations (Ewerth, 2004a) for the quarks, obtaining the super-CKM basis (Dugan, Grinstein, & Hall, 1985) for squarks. This is convenient to use because we only deal with virtual squark effects (Hall, Kostelecky, &

Raby, 1986) and so do not have incoming or outgoing mass-basis squarks in interactions. The squark and slepton transformations are:

$$\begin{aligned} \nu_L^f &\to U^{\nu_L}_{fg}\nu_L^g \\ e_L^f &\to U^{e_L}_{fg}e_L^{*g} \quad e_R^{c*f} \to U^{e_R}_{fg}e_R^{c*g} \\ u_L^f &\to U^{u_L}_{fg}u_L^g \quad u_R^{c*f} \to U^{u_R}_{fg}u_R^{c*g} \\ d_L^f &\to U^{d_L}_{fg}d_L^g \quad d_R^{c*f} \to U^{d_R}_{fg}d_R^{c*g} \end{aligned} \tag{4.36}$$

We also define the following combinations of matrices:

$$\begin{aligned} M^2_{\tilde{\ell}} &\to U^{e_L\dagger}M^2_{\tilde{\ell}}U^{e_L} =: (M^2_{\tilde{e}})_{LL} \quad M^2_{\tilde{e}} \to U^{e_R\dagger}M^2_{\tilde{e}}U^{e_R} =: (M^2_{\tilde{e}})_{RR} \\ M^2_{\tilde{q}} &\to U^{u_L\dagger}M^2_{\tilde{q}}U^{u_L} =: (M^2_{\tilde{u}})_{LL} \quad M^2_{\tilde{u}} \to U^{u_R\dagger}M^2_{\tilde{u}}U^{u_R} =: (M^2_{\tilde{u}})_{RR} \\ M^2_{\tilde{q}} &\to U^{d_L\dagger}M^2_{\tilde{q}}U^{d_L} =: (M^2_{\tilde{d}})_{LL} \quad M^2_{\tilde{d}} \to U^{d_R\dagger}M^2_{\tilde{d}}U^{d_R} =: (M^2_{\tilde{d}})_{RR} \end{aligned} \tag{4.37}$$

The mixing angle matrices transform as:

$$\begin{aligned} A_{\tilde{e}} &\to \tfrac{1}{\sqrt{2}}v_1U^{e_L\dagger}A_{\tilde{e}}U^{e_R} =: (M^2_{\tilde{e}})_{LR} \\ A_{\tilde{u}} &\to -\tfrac{1}{\sqrt{2}}v_2U^{u_L\dagger}A_{\tilde{u}}U^{u_R} =: -(M^2_{\tilde{u}})_{LR} \\ A_{\tilde{d}} &\to \tfrac{1}{\sqrt{2}}v_1U^{d_L\dagger}A_{\tilde{d}}U^{d_R} =: (M^2_{\tilde{d}})_{LR} \end{aligned} \tag{4.38}$$

And the Yukawa matrices transformations:

$$\begin{aligned} y_e &\to \tfrac{1}{\sqrt{2}}v_1U^{e_R\dagger}y_eU^{e_L} =: M_e \\ y_u &\to -\tfrac{1}{\sqrt{2}}v_2U^{u_R\dagger}y_uU^{u_L} =: M_u \\ y_d &\to \tfrac{1}{\sqrt{2}}v_1U^{d_R\dagger}y_dU^{d_L} =: M_d \end{aligned} \tag{4.39}$$

The first mass matrices that we give are those for selectrons (and smuons, staus) and sneutrinos. There are six selectrons, matching the left and right-handed degrees of freedom of the three fermionic generations, and three sneutrinos from the single left-handed neutrinos. The selectron and sneutrino mass matrices are:

$$\mathcal{M}^2_{\tilde{e}} = \begin{pmatrix} (M^2_{\tilde{e}})_{LL} + M_e^2 + \frac{1}{2}(m_Z^2 - 2m_W^2)\cos 2\beta\mathbb{1} & (M^2_{\tilde{e}})_{LR} - M_e\mu\tan\beta \\ (M^2_{\tilde{e}})^\dagger_{LR} - M_e\mu\tan\beta & (M^2_{\tilde{e}})_{RR} + M_e^2 - m_Z^2 s_W^2\cos 2\beta\mathbb{1} \end{pmatrix} \tag{4.40}$$

$$\mathcal{M}^2_{\tilde{\nu}} = (M^2_{\tilde{e}})_{LL} + \frac{1}{2}m_Z^2\cos 2\beta\mathbb{1} \tag{4.41}$$

Now, we give two separate matrices for the up and down squarks of the three generations and their "left" and "right" handed components (each matrix is $6 \times 6$):

$$\mathcal{M}^2_{\tilde{u}} = \begin{pmatrix} (M^2_{\tilde{u}})_{LL} + M_u^2 + \frac{1}{6}(4m_Z^2 - m_Z^2)\cos 2\beta\mathbb{1} & (M^2_{\tilde{u}})_{LR} - M_u\mu\cot\beta \\ (M^2_{\tilde{u}})^\dagger_{LR} - M_u\mu^*\cot\beta & (M^2_{\tilde{u}})_{RR} + M_u^2 + \frac{2}{3}m_Z^2 s_W^2\cos 2\beta\mathbb{1} \end{pmatrix} \tag{4.42}$$

$$\mathcal{M}^2_{\tilde{d}} = \begin{pmatrix} (M^2_{\tilde{d}})_{LL} + M_d^2 - \frac{1}{6}(m_Z^2 + 2m_W^2)\cos 2\beta\mathbb{1} & (M^2_{\tilde{d}})_{LR} - M_d\mu\tan\beta \\ (M^2_{\tilde{d}})^\dagger_{LR} - M_d\mu^*\tan\beta & (M^2_{\tilde{d}})_{RR} + M_d^2 - \frac{1}{3}m_Z^2 s_W^2\cos 2\beta \end{pmatrix} \tag{4.43}$$

With the squark matrices given, all the mass-basis particles can now be obtained in terms of these mass matrices, and hence we can proceed to the quantisation of the superpartner fields.

# 5 MSSM Quantisation

This section presents an overview of the quantisation of the MSSM theory, necessary for the analysis presented in Section 7. The superpartner propagators are given (in 5.1), together with a reduced subset of the vast amount of vertices that involve the new particles (and the two Higgs doublets, in 5.2). Particular emphasis is placed on the vertices that play a part in quark and squark flavour-changing interactions, contrasting the only $W$ boson vertex in the minimal SM producing generational mixing.

As discussed in Section 2.2 and will be elaborated in Section 6.1 (when talking about the LEFT), our interest lies in the MSSM contributions at the energy scale around EWSB. For this reason, the propagators and vertices will refer to the particles in their mass-basis form, derived in the diagonalisation of the MSSM mass matrices in Section 4.3.2. The contributions presented in this section is a subset of those (with a slightly different notation) given in (Rosiek, 1990) (and its erratum (Rosiek, 2002)).

## 5.1 Particle and Superpartner Propagators

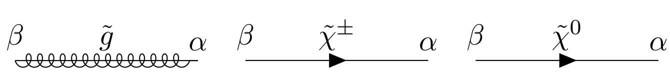
We begin by giving the diagrammatic form of the propagators for the particles and their superpartners, together with the contribution they represent in the computation of the expressions from Feynman diagrams.

The gauge boson propagators for photons, $Z$ and $W^{\pm}$ bosons and gluons, as also found in the minimal SM are the following:


Their momentum space contribution is:

$$D_{(G)\mu\nu}(p) = \frac{i\eta_{\mu\nu}}{p^2 + m^2 - i\epsilon} \tag{5.1}$$

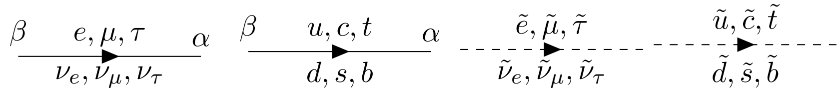
The fermionic gaugino (mixed with Higgsino) propagators, for the gluino, chargino and neutralino respectively, are the following:




Their momentum space contribution to Feynman diagrams is:

$$D_{(\tilde{G})\alpha\beta}(p) = \frac{-i(\gamma \cdot k + m)_{ab}}{p^2 + m^2 - i\epsilon} \tag{5.2}$$

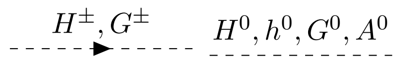
The matter particles and superpartner propagators are given below for leptons, quarks, sleptons and squarks respectively.




They are either fermions or scalars and therefore contribute:

$$D_{(M)\alpha\beta}(p) = \frac{-i(\gamma \cdot k + m)_{ab}}{p^2 + m^2 - i\epsilon} \qquad D_{(\tilde{M})}(p) = \frac{-i}{p^2 + m^2 - i\epsilon} \tag{5.3}$$

Finally, the charged and neutral Higgs propagators, both scalars, are represented diagrammatically as:

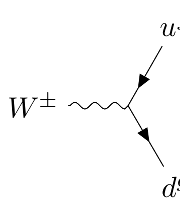



Their scalar contribution is:

$$D_{(\tilde{H})}(p) = \frac{-i}{p^2 + m^2 - i\epsilon} \tag{5.4}$$

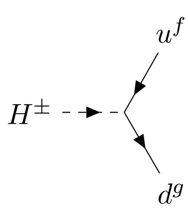
## 5.2 Relevant Vertices

In this section, we concentrate on giving the subset of vertices that are directly relevant for tree-level quark flavour changing charged interactions and for loop level FCNCs (Misiak et al., 1997). To avoid unnecessary complication for the MSSM-exclusive vertices, the reader is invited to consult (Rosiek, 1990) for their precise numerical mathematical contribution in diagrams. The first vertex that we consider is the SM $W$ boson and quark flavour changing interaction, the only flavour changing interaction in the minimal SM. Its diagrammatic form and contribution are given by:

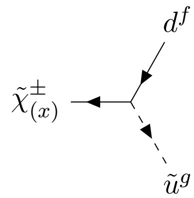



$$= \Big( - \frac{ie}{\sqrt{2} s_W} \Big) V^{\text{CKM}*}_{fg} \gamma^\mu P_L \tag{5.5}$$

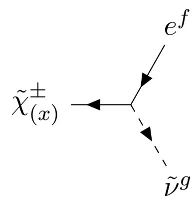
In the MSSM, a second tree-level flavour changing charged current is present, that given by the charged Higgs scalars. This introduces the following diagrammatic vertex



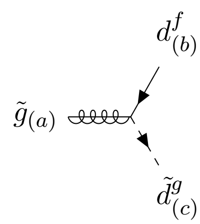

At one-loop level, there are many more interaction vertices that can appear to contribute to flavour-changing currents. To simplify matters, we only give those that will be used in the later Section 7. Commencing with the down-quark up-squark chargino vertex, we have the vertex:



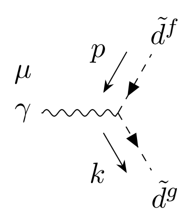

Similarly, charginos can interact with leptons and sneutrinos, through the vertex:




We now concentrate on the down-quark down-squark gluino interaction:




Finally, we give a vertex that is not flavour-changing but will be required later on, that is the squark to photon interaction that will be present in a penguin contribution:




As can be seen, the MSSM contains many more flavour-changing interactions than the minimal SM, suggesting the potential of MSSM effects to affect flavour-changing beauty decays growing exponentially at each higher loop order.

# 6 Effective Field Theory, SMEFT and LEFT

## 6.1 Effective Field Theory

When considering the additional free parameters, that the MSSM presents (such as soft-breaking masses or the Higgs vev $\beta$ angle) all, except for the minimal SM parameters, lack explicit experimental determination. Moreover, acknowledging the vast amount of possible interactions that the Lagrangian (4.1) contains, as seen in Section 5, a method of projecting the approximate overall effects of supersymmetry onto weak energy-scale processes must be developed if we wish to study its contributions specifically to beauty decays. Not only is it computationally infeasible to consider every possible contribution to a particular process from all the coupled degrees of freedom, but making exact predictions requires us to trust a model at arbitrary energy scales and fully, rather than partially, determine its constants and couplings from experiment. In the specific case of the MSSM, where superpartners have not been observed in experiments, this is especially troublesome.

The theory of developing actions to describe physics in specific ”regions” of energy scales is known as effective field theory (EFT). Theoretically, this modifies the viewpoint that a single theory must be valid at all scales. Theories can now be the effective manifestations of higher-energy, more fundamental constructions. Experimentally, the relevance of EFT is tied to most human observations happening at specific scales, at which higher-energy components can be integrated (or averaged) out, such as very massive d.o.f.s. For example, the LHC at CERN is operating its Run 3 data-taking at 13.6 TeV (or $1.36 \times 10^4$ GeV) for proton-proton collisions. This is many orders of magnitude below the expected GUT or Planck scales of $10^{16}$ and $10^{19}$ GeV where we expect new physics. Under this light, the SM is now understood to be the leading term of a ”low-energy” EFT (Isidori et al., 2023) of a model which contains BSM physics. The EFT approach of projecting the physics contained in a high-scale theory down to lower-scale observable phenomena, described exclusively by the low-scale fields, is called the ”top-down” approach to EFT.

While the following approach will be only briefly discussed, it is also possible to use the formalisms of EFT to place model independent constraints on new theories. This is known in the literature as the ”bottom-up” approach. By assuming that experimental tensions with theory derive from effective contributions to a low-scale action, such as the SM intended as such, it is possible to impose a-priori what the low-scale appearance of new physics with unknown UV completions should be.

### 6.1.1 The Wilsionian Effective Lagrangian

The foundational work on EFT was laid by Wilson (Wilson, 1972), initially inspired the study of Landau theory and later extending to the development of the renormalisation group (RG) and its equations (Landau, 1937; Wilson, 1983). As mentioned previously, the success of EFT lies in allowing theories typically not regarded as renormalisable through higher dimensional effective operators. The following discussion is based on that in (Isidori et al., 2023) and (Georgi, 1993).

A core requirement of quantum field theories is that they must be renormalisable in the standard procedures with the introduction of a finite number of counterterms. At the Lagrangian level for a $d = 4$ dimensional theory, this implies that all field-composite terms must be of dimension $\leq 4$ excluding the dimension brought by coefficient couplings, with the Lagrangian itself being of mass dimension $[\mathcal{L}] = 4$. From the Klein-Gordon, Dirac and Yang-Mills kinetic terms for scalar, spinor and vector fields respectively, we extract their mass dimension:

$$[\mathcal{L}] = 4 = [\partial_\mu \phi \partial^\mu \phi] = 2[\partial] + 2[\phi] = 2 + 2[\phi] \implies [\phi] = 1 \tag{6.1}$$

$$[\mathcal{L}] = 4 = [i\bar{\psi}\gamma^\mu \partial_\mu \psi] = [\partial] + 2[\psi] = 1 + 2[\psi] \implies [\psi] = \frac{3}{2} \tag{6.2}$$

$$[\mathcal{L}] = 4 = [\partial_\mu A_\nu \partial^\mu A^\nu] = 2[\partial] + 2[A] = 2 + 2[A] \implies [A] = 1 \tag{6.3}$$

Hence, a term like the Yukawa $\phi\bar{\psi}\psi$ is renormalisable from its dimensionality ($[\phi]+2[\psi] = 4$) and can be included in a Lagrangian (provided that it also respects the local gauge symmetries of the theory), while a four-fermion term $(\bar{\psi}\psi)(\bar{\psi}\psi)$ of dimension 6 typically cannot.

Terms such as the four-fermion operator can nevertheless appear in effective actions. The derivation of a Wilsonian effective Lagrangian can be understood at the level of path integrals. Let $\Lambda$ be a given energy scale at which we want to derive an effective theory. Schematically, we let $\phi_L$ and $\phi_H$ represent light and heavy fields with characteristic masses $m$ and $M$ respectively such that $m < \Lambda < M$ (these can be taken to be the maximum and minimum masses for the two groups of fields). By comparison with $\Lambda$, these set the low and high energy scales for effective analysis of the theory. Then the sourced generating functional for the theory $\mathcal{L}[\phi_L, \phi_H]$ is given by the following path integral over light and heavy field configurations:

$$\begin{aligned} \mathcal{Z}[J_L, J_H] &= \int \mathcal{D}\phi_L \int \mathcal{D}\phi_H \exp\left[i\mathcal{S}[\phi_L, \phi_H] + i\int \mathrm{d}^4x(\phi_L J_L + \phi_H J_H)\right] \\ &= \int \mathcal{D}\phi_L \int \mathcal{D}\phi_H \exp\left[i\int (\mathcal{L}[\phi_L, \phi_H] + \phi_L J_L + \phi_H J_H)\right] \end{aligned} \tag{6.4}$$

The effective action is then defined as the remnant Lagrangian after performing path-integration over the heavy fields $\phi_H$ (Isidori et al., 2023):

$$\begin{aligned} \mathcal{Z}[\phi_L] &= \int \mathcal{D}\phi_L \int \mathcal{D}\phi_H \exp\left[i\int (\mathcal{L}[\phi_L, \phi_H] + \phi_L J_L)\right] \\ &= \int \mathcal{D}\phi_L \exp\left[i\int (\mathcal{L}_{\mathrm{EFT}}[\phi_L] + \phi_L J_L)\right] \end{aligned} \tag{6.5}$$

The remaining effective Lagrangian $\mathcal{L}_{\text{EFT}}$ is independent of the $\phi_H$ heavy fields and, being the path integral a measure over all possible field configurations, contains their averaged contributions at the level of $\Lambda$. Formally, such a decoupling of light and heavy fields is encoded in the Appelquist-Carazzone theorem (Appelquist & Carazzone, 1975). Although the effective Lagrangian is fully independent of heavy fields, it is precisely the action of integrating over them that gives rise to non-renormalisable vertices composed of the remaining light fields, such as the four-fermion operator previously considered when we consider the spinors to be light.

### 6.1.2 The Operator Product Expansion and Effective Hamiltonians

The higher-dimensional operators produced follow two conditions (Georgi, 1993) which guarantee that the EFT requires only a finite number of parameters to perform calculations at any arbitrary, but finite, precision. The first condition is that there must be a finite number of operators for every specific dimension $d > 4$ (even if $d$ itself is not limited). The second is that the coefficients of these operators must be bounded above from $\mathcal{O}(1/\Lambda^{d-4})$, with $\Lambda$ being the previously chosen scale. Considering these conditions, the form of the effective Lagrangian is given by the following operator product expansion (OPE) (Isidori et al., 2023):

$$\mathcal{L}_{\text{EFT}} = \mathcal{L}_L + \sum_{d=5}^{\infty} \sum_i \frac{1}{\Lambda^{d-4}} \mathcal{C}_i^{(d)} \mathcal{O}_i^{(d)} \tag{6.6}$$

In the above, $\mathcal{L}_L$ is the remaining part of the original Lagrangian $\mathcal{L}$ which includes only renormalisable terms of dimension $\leq 4$. The $\mathcal{O}_i^{(d)}$ is a $d$-dimensional operator, indexed by $i$ taking only a finite number of values. Its coefficient $\mathcal{C}_i^{(d)}$ is called its Wilson coefficient and represents the coupling strength of the effective interaction. The infinite sum represents the contributions given by all the possible higher-dimensional operators composed of light fields, respecting the previously mentioned conditions. For a precision of choice, the infinite sum can be chosen to truncate at a specific dimension, introducing further approximation. A discussion on how to obtain the effective operators for a given theory from symmetry arguments is postponed to Section 6.3 under the example of the SM EFT.

For the purposes of the analysis presented in Section 7, it is useful to introduce the effective Hamiltonian. This is obtained through the Legendre transform of the Lagrangian, noting that there is no modification to the conjugate-pair-product part as no fields are added in the construction of the effective Lagrangian. Again, this expression can be truncated at a chosen finite dimension.

$$\mathcal{H}_{\text{EFT}} = \cdots - \sum_{d=5}^{\infty} \sum_i \frac{1}{\Lambda^{d-4}} \mathcal{C}_i^{(d)} \mathcal{O}_i^{(d)} \tag{6.7}$$

## 6.2 Renormalisation Group Evolution

We present here a summary on the renormalisation group (RG) and its application to EFT. The renormalisation group equations (RGEs) are the set of differential equations relating the values of couplings at different energy scales, under the fundamental assumption that physical observables are invariant under the arbitrary nature of such a scale. First approached in (Stueckelberg & Petermann, 1951) and subsequently formalised in (Gell-Mann & Low, 1954), they play the central role in Wilsonian EFT of evolving effective couplings between scales where fields are integrated out.

Let $\mu$ be a sliding scale appearing, for example, from using a $\overline{\text{MS}}$ renormalisation scheme to evaluate divergent loop integrals from a BSM theory. Let $g_i$ be an arbitrary coupling for a chosen theory, then its scale evolution (dependence on $\mu$) is given by the RGE (Weinberg, 1996):

$$\mu \frac{\mathrm{d}g_i}{\mathrm{d}\mu} = \beta_i(\dots, g_j(\mu), \dots) \tag{6.8}$$

Each coupling "constant" has an associated function $\beta_i$ which governs its evolution, that might depend on all other scale varying couplings of the theory. This differential equation can be very complex to solve based on the theory in question. As seen in Section 6.3 discussing the SMEFT and LEFT, there is usually theory specific effort in understanding RGE solutions.

We now consider an EFT which, alongside coupling constants, contains Wilson coefficients. When inserting effective operators in interactions, there are additional divergences that cannot be absorbed through field renormalisation and require effective operator renormalisation (Buchalla, Buras, & Lautenbacher, 1996). Those divergences can be absorbed by renormalisation inside the bare Wilson coefficients:

$$\mathcal{C}_i^{\langle 0\rangle} = Z_{ij}^{\mathcal{C}}\mathcal{C}_j = \mathcal{C}_i + (Z_{ij}^{\mathcal{C}} - \delta_{ij})\mathcal{C}_j \tag{6.9}$$

Where the second separation has been made in the direction of the addition to the counter term Lagrangian. By demanding the bare coefficients to be independent of scale, Wilson coefficients also absorb a dependence on $\mu$. The RG equation for the Wilson coefficients of a specific theory is given by (Buras, 2020):

$$\mu \frac{\mathrm{d}\mathcal{C}_i}{\mathrm{d}\mu} = \frac{1}{(4\pi)^2}\gamma_{ij}(\mu)\mathcal{C}_j(\mu) \tag{6.10}$$

The matrix $\gamma_{ij}$ is called the anomalous dimension matrix and encodes the mixing of dependence between the considered Wilson coefficients (or, alternatively, operators). The entries of $\gamma$ can contain the couplings of the underlying theory, and run with them. With its specific form once again depending on the EFT in question, these are the RGEs used to run the couplings away from the matching scale. Finally, the higher-dimensional terms in the renormalised Lagrangian take the form:

$$\mathcal{L}_{\text{EFT}} = \dots + \sum_{d=5}^{\infty}\sum_i \frac{1}{\Lambda^{d-4}}\mathcal{C}_i^{(d)}(\mu)\mathcal{O}_i^{(d)}(\mu) \tag{6.11}$$

The dependence of the operators on the sliding scale $\mathcal{O}(\mu)$ is to be noted, as they can contain further constants that run with scale such as gluon couplings.

## 6.3 Standard Model Effective Field Theories

### 6.3.1 SMEFT

It has already been stated that the modern viewpoint on the SM is to regard it as the renormalisable sector of an EFT of a more fundamental theory. If we take the SM to be valid up to an energy scale $\Lambda_{\text{BSM}}$ after which we expect new-physics contributions to become relevant, we can introduce the larger theory of the SMEFT using (6.6) and (6.11):

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \sum_{d=5}^{\infty} \sum_{i} \frac{1}{\Lambda_{\text{BSM}}^{d-4}} \mathcal{C}_i^{(d)}(\mu) \mathcal{O}_i^{(d)}(\mu) \tag{6.12}$$

We can choose $\Lambda_{\text{BSM}}$ as a scale that allows us to integrate out MSSM effects and project them onto the SMEFT, although the form of the effective operators $\mathcal{O}_i^{(d)}$ in (6.12) remains to be found. While this report does not discuss the model-independent constraining of BSM contributions, the bottom-up approach introduced in Section 6.1 does play a part. Although their dimensionality clearly is not restricted, the effective operators should obey the same symmetries as the renormalisable theory. Hence, enumerating all the possible terms that are Poincaré invariant and respect the gauge group $\text{SU}(3)_{\text{C}} \times \text{SU}(2)_{\text{L}} \times \text{U}(1)_{\text{Y}}$ makes the bottom-up approach reproduce all of the possible effective operators of the SMEFT. It is worth noting that the SMEFT is formulated prior to EWSB, which is why the LEFT is introduced in Section 6.3.2.

Infinite enumeration of all $\mathcal{O}_i^{(d)}$ is not possible, but results for specific dimensions $d$ are well known in the literature, in particular those for dimensions $d = 5$ and $d = 6$ which are used in this report. The first complete enumeration of $d = 5, 6$ operators for the SMEFT is due to (Buchmüller & Wyler, 1986), in which all the 81 possible terms can be found (up to Hermitian conjugation and combination of different generations). As an example, we reproduce the single $d = 5$ Weinberg term (Weinberg, 1979) and the $d = 6$ four-left-lepton operator in the SM notation of Table 1:

$$\mathcal{O}_{\phi\phi\ell\ell}^{(5)fg} = (\epsilon^{ij} \bar{\ell}_L^{cf} \phi_j)(\epsilon^{kl} \ell_L^g \phi_l) \tag{6.13}$$

$$\mathcal{O}_{\ell\ell\ell\ell}^{(6)fghl} = (\bar{\ell}_L^f \gamma_\mu \ell_L^g)(\bar{\ell}_L^h \gamma^\mu \ell_L^l) \tag{6.14}$$

Nevertheless, the approach provided by (Buchmüller & Wyler, 1986) in the derivation of the effective operators produces a redundant basis. In such a basis, some operators can be re-expressed in terms of others when applying techniques such as field redefinitions and integration by parts under the action integral, or the use of equations of motion for the theory. The accepted non-redundant basis for the SMEFT, referred to as the Warsaw basis, was derived first in (Iskrzynski, 2010) (in Polish) and later collected in (Grzadkowski, Iskrzyński, Misiak, & Rosiek, 2010). The total number of dimension 5 and 6 terms in the Warsaw basis is 60, 21 less than the previously considered redundant basis. Both the approach of a redundant and non-redundant basis has been introduced here because they can be applicable to two types of matching, namely off-shell and on-shell discussed in Section 6.4.

In the wider scope of this exploration on MSSM effects on beauty decays, it is worth mentioning that effective operators for the SMEFT need not conserve neither baryon or lepton number. These arise in the minimal SM as accidental symmetries deriving from available global U(1) rotations and hence effective operators can violate these. In fact, baryon or lepton number is violated for odd $d$ (Helset & Kobach, 2020). We can either justify suppression of such terms based on the BSM theory to be matched or directly impose B and L conservation as additional global symmetries for the theory.

### 6.3.2 LEFT

While the SMEFT can be seen as a frontier of our understanding of the SM, the analysis presented in this report targets energy ranges around the EWSB scale. The appropriate EFT derived from the SM for this scale is the Low Energy EFT (LEFT), used in Section 7. When SSB reduces the gauge symmetry of the SM in the pattern $\mathrm{SU(3)_C} \times \mathrm{SU(2)_L} \times \mathrm{U(1)_L} \rightarrow \mathrm{SU(3)_C} \times \mathrm{U(1)_{EM}}$, the physical Higgs field $h$ (the remaining one after imposing unitary gauge), $W$ and $Z$ bosons in particular develop mass alongside the matter fields from the Yukawa interactions. In this broken phase, the LEFT is obtained by integrating out $h$, $W^{\pm}$, $Z$ and the heaviest top quark $t$. Hence, the remaining fields beside the photon and gluons $(\gamma, g)$ are the 5 quarks $(u, d, c, s, b)$ and the leptons $(\ell, \mu, \tau, \nu_\ell, \nu_\mu, \nu_\tau)$. Therefore, the LEFT resembles a modern perspective take on the Fermi theory of weak interactions (Fermi, 1934). Similarly to how effective operators are derived in the SMEFT case, (Jenkins, Manohar, & Stoffer, 2018b) presents a derivation and a full enumeration of all operators for the chosen dimensions $d = 5, 6$. These are not worth repeating explicitly as the matching of MSSM effects onto LEFT operators involves only a certain subset of these when specifying to beauty decays and are included in Section 7. The LEFT Lagrangian is:

$$\mathcal{L}_{\mathrm{LEFT}} = \mathcal{L}_{\mathrm{QCD}\times\mathrm{QED}} + \sum_{d=5}^{\infty} \sum_{i} \frac{1}{\Lambda_{\mathrm{EWSB}}^{d-4}} \mathcal{C}_i^{(d)}(\mu) \mathcal{O}_i^{(d)}(\mu) \tag{6.15}$$

Matching MSSM effects to the LEFT can be done in either one of two ways, of differing asymptotic precision. We describe the first. As an extension to the minimal SM, it is natural to calculate effective contributions from the MSSM at an energy scale $\Lambda_{\mathrm{BSM}} = \Lambda_{\mathrm{MSSM}}$ above the top quark mass, but below the mass of the lightest superpartner. This requires calculating the Wilson coefficients for SMEFT effective operators, then running these down to the EWSB scale $\mu_W \sim \mathcal{O}(m_W)$ using the SMEFT RG equations of the form (6.10) (Jenkins, Manohar, & Trott, 2013, 2014; Alonso, Jenkins, Manohar, & Trott, 2014). Then EWSB is performed by expanding around the Higgs vev, and integrating out the $h, W^{\pm}, Z$ fields to obtain the LEFT result. Finally, the LEFT RG equations can be used to evolve the operators and coefficients down to the mass-scale of the remaining heaviest field, the bottom quark.

While this approach performed in multiple steps is favourable to produce a better description of the low-energy physics (Isidori et al., 2023), it goes beyond the level of complexity of this report. The second, easier approach is that to directly perform the matching of MSSM effects onto the LEFT, integrating heavy fields from the MSSM and SM at and above the top quark mass-scale simultaneously. Once the Wilson coefficients have been calculated, they can be run down with the LEFT RG equations (Aebischer, Fael, Greub, & Virto, 2017; Jenkins, Manohar, & Stoffer, 2018a) to the scale of the bottom quark mass, once again.

### 6.3.3 Comment on Evanescent Operators

In reality, the operators in (6.15) have to be supplemented with one additional class of effective operators, that of evanescent operators. These appear in dimensional regularisation when we deform away from a four-dimensional spacetime integration to evaluate divergent integrals.

While these are fundamental for the correct calculation of $\overline{\text{MS}}$ renormalised Wilson coefficients in the MSSM matching, the technicalities of the procedure go beyond the level of this report.

## 6.4 Diagrammatic Matching

In this section, an outline for the method of computation of Wilson coefficients of MSSM effects in the LEFT is given. The $\mathcal{C}_i^{(d)}$ can be computed using Feynman diagrams. The fundamental idea is to require an equivalence of the UV-complete and effective theories at the chosen matching energy scale $\Lambda$. This can be achieved in two ways (Isidori et al., 2023).

The first, known as off-shell matching, involves requiring the low-energy and UV-complete quantum effective actions (not Wilsonian) to agree to the chosen order in $\Lambda$. The definition of irreducibility in 1PI to be taken is the light-line irreducibility (1LPI), where we say that a diagram is irreducible if it remains connected when a light particle line is cut. The contributing Feynman diagrams are then imposed to agree and the Wilson coefficients for a redundant basis of operators (not simplified using the equations of motion) are solved for. This is the method used in Section 7.

The second, on-shell matching, requires that for any given scattering process of light particles, the amplitudes computed using the effective or UV-complete theory agree. In this case, a non-redundant basis can be used although all diagrams must be computed for this approach.

Schematically, the off-shell and on-shell matching conditions, in the case of matching the MSSM to the LEFT, are respectively given by:

$$\Gamma_{\text{LEFT}}[\phi_L] = \Gamma_{\text{MSSM}}[\phi_L] \qquad \langle\beta|S_{\text{LEFT}}|\alpha\rangle = \langle\beta|S_{\text{MSSM}}|\alpha\rangle \tag{6.16}$$

Where $\alpha, \beta$ are initial and final states of light fields $\phi_L$ only. Off-shell matching in a redundant basis can be reduced to a non-redundant basis by subsequent use of the equations of motion. Instead of guaranteeing full equivalence in both approaches, we can introduce a further approximation, that of choosing the specific processes to match. Here, we require matching 1LPI or full diagrams contributing to specific scattering processes only. In this case, only a subset of effective operators is generated and the running given by RGEs may gain imprecision. Once again, this is a further approximation used in Section 7.

# 7 Beauty Decays in the MSSM

In this section of this report, we present the phenomenologically-relevant framework to study the effect of hypothetical MSSM fields on beauty decays, which were initially presented in Section 2.2. Using the EFT formalism developed in Section 6, the commonly used $-\Delta B = \Delta S = 1$ effective Hamiltonian will be presented in 7.1. Part of the MSSM formulated in Section 4, with its particles in the mass basis and their interactions of Section 5, will be projected onto the LEFT by employing off-shell diagrammatic matching up to non-leading (one-loop) order (NLO). We first calculate a simple leading (tree-level) order (LO) SM contribution to a tree-level Wilson coefficient in Section 7.2.2 as an introductory calculation for diagrammating matching. We then fully calculate a flavour-changing one-loop process (again integrating out only heavy SM fields), to demonstrate an NLO calculation of which the generalisation is applicable to MSSM loops. Following the lines of the previous box-diagram calculation, the closely related chargino-squark box-diagram will be presented, together with the Feynman diagram contribution that allows the (very involved) computation of its Wilson coefficient contribution (in Section 7.2.4). Finally, a gluino-squark contribution to a photon penguin diagram will be discussed (in Section 7.2.5), to introduce part of the further complications to be considered when calculating MSSM diagrams, and to conclude the NLO discussion.

## 7.1 B Decays Effective Hamiltonian

Based on the discussion in Section 2.2, we wish to perform the EFT matching of MSSM effects onto the LEFT, as to explore its potential effects between the energy scales of EWSB and the mass of the $b$ quark, typically explored in contemporary particle colliders. As discussed in Section 6.3.2, the light degrees of freedom that are part of the LEFT are the 5 light active flavours of quarks $(u, d, c, s, b)$, the six leptons $(e, \mu, \tau, \nu_e, \nu_\mu, \nu_\tau)$ and the two gauge bosons (collecting the 8 gluons) $(g, \gamma)$. These interact in the SM broken phase of QCD $\times$ QED, or $\mathrm{SU}(3)_\mathrm{C} \times \mathrm{U}(1)_\mathrm{EM}$. The analysis is chosen to involve effective operators of dimension at most $d = 6$. We also choose the additional imposition of baryon and lepton number conservation as a forced symmetry of operators, differently from their accidental nature in the MSSM (excluding the enforcement of R-parity).

With this background, we are able to introduce the effective Hamiltonian for beauty decays $-\Delta B = \Delta S = 1$ (Ali, Lunghi, Greub, & Hiller, 2002a; Cho et al., 1996; Ewerth, 2004a):

$$\mathcal{H}_\mathrm{EFT} = \cdots - \frac{4G_F}{\sqrt{2}} \left[ \sum_{q=u,c} \sum_{i=1}^{2} V_{qb}^\mathrm{CKM} V_{qs}^{\mathrm{CKM}*} \mathcal{C}_i^q \mathcal{O}_i^q + \sum_{q=u,c,t} \sum_{i=3}^{10} V_{qb}^\mathrm{CKM} V_{qs}^{\mathrm{CKM}*} \mathcal{C}_i^q \mathcal{O}_i \right] \quad (7.1)$$

Some common constants, such as CKM matrix $V^\mathrm{CKM}$ elements, have been factorised out from Wilson coefficients for convenience. The new constant appearing is the Fermi constant, defined as:

$$G_F = \frac{\sqrt{2} g_2^2}{8 m_W^2} \tag{7.2}$$

The effective operators that enter (7.1) are the following (where $T^\vartheta$ are the generators of $\mathrm{SU}(3)_\mathrm{C}$, $G^\vartheta_{\mu\nu}$ its field strength tensor and $F_{\mu\nu}$ the field strength for the EM gauge symmetry):

$$\mathcal{O}_1^q = (\bar{s}_L \gamma_\mu T^\vartheta q_L)(\bar{q}_L \gamma^\mu T^\vartheta T^a b_L) \tag{7.3a}$$

$$\mathcal{O}_2^q = (\bar{s}_L \gamma_\mu q_L)(\bar{q}_L \gamma^\mu b_L) \tag{7.3b}$$

$$\mathcal{O}_3 = (\bar{s}_L \gamma_\mu b_L) \sum_q (\bar{q}_L \gamma^\mu q_L) \tag{7.3c}$$

$$\mathcal{O}_4 = (\bar{s}_L \gamma_\mu T^\vartheta b_L) \sum_q (\bar{q}_L \gamma^\mu T^\vartheta q_L) \tag{7.3d}$$

$$\mathcal{O}_5 = (\bar{s}_L \gamma_{\mu_1} \gamma_{\mu_2} \gamma_{\mu_3} b_L) \sum_q (\bar{q}_L \gamma^{\mu_1} \gamma^{\mu_2} \gamma^{\mu_3} q_L) \tag{7.3e}$$

$$\mathcal{O}_6 = (\bar{s}_L \gamma_{\mu_1} \gamma_{\mu_2} \gamma_{\mu_3} T^\vartheta b_L) \sum_q (\bar{q}_L \gamma^{\mu_1} \gamma^{\mu_2} \gamma^{\mu_3} T^\vartheta q_L) \tag{7.3f}$$

$$\mathcal{O}_7 = \frac{e}{g_3^2} m_b (\bar{s}_L \sigma^{\mu\nu} b_R) F_{\mu\nu} \qquad (\text{where } \sigma^{\mu\nu} = \frac{1}{2}[\gamma^\mu, \gamma^\nu]) \tag{7.3g}$$

$$\mathcal{O}_8 = \frac{1}{g_3} m_b (\bar{s}_L \sigma^{\mu\nu} T^\vartheta b_R) G^\vartheta_{\mu\nu} \tag{7.3h}$$

$$\mathcal{O}_9 = \frac{e^2}{g_3^2} (\bar{s}_L \gamma_\mu b_L)(\bar{\ell}_L \gamma^\mu \ell_L) \tag{7.3i}$$

$$\mathcal{O}_{10} = -\frac{e^2}{g_3^2} (\bar{s}_L \gamma_\mu b_L)(\ell_L \gamma^\mu i \gamma_5 \ell_L) \tag{7.3j}$$

Where in (7.3j), an additional factor of $-i$ is included for matching conventions with the literature. Of the above, the operators $i = 1$ and 2 are known as the current-current operators, 3-6 as the QCD penguin operators, 7 and 8 the electromagnetic and chromomagnetic dipoles, and finally 9 and 10 the semileptonic operators. With $b$ mass of dimension $[m_b] = 1$, the operators in (7.3g),(7.3h) are of dimension $d = 5$. All other operators are of dimension $d = 6$, as limited by the chosen matching. While the LEFT contains many more operators, this subset is widely accepted to produce the Wilson coefficients with the greatest contributions to $b$ decay observables (Bobeth, Misiak, & Urban, 2000; Lunghi, Masiero, Scimemi, & Silvestrini, 2000) and furthermore allows closed matching of the subprocesses discussed in Section 7.2. A last comment to be made is the inclusion of $e$ and $g_3$ coupling constants in some operators, needed to define a loop expansion of the Wilson coefficients in terms of the strong coupling as discussed in Section 7.5.

## 7.2 MSSM-LEFT Diagrammatic Matching

The diagrammatic matching that we will perform on MSSM contributions to beauty decays is of the off-shell type at the scale $\Lambda = \mu_W \sim \mathcal{O}(m_W)$. As discussed in Section 6.4, this involves matching 1LPI MSSM diagrams to NLO to 1PI diagrams of the effective theory at the chosen energy scale, and we will restrict to matching certain subprocesses that participate to the decay $\bar{B} \to \bar{K}^{(*)}\ell^+\ell^-$. As discussed in (Bobeth, Buras, & Ewerth, 2005; Bobeth et al., 2000), these subprocesses are: $b \to s\ell^-\ell^+$, $b \to s\gamma$ $b \to sZ$, $b \to sq\bar{q}$ where $q = u, c$, and $b \to sg$.

It is to be noted that all the fields participating in the chosen subprocesses involve the light fields of the LEFT only, especially $b \to sq\bar{q}$ excludes the possibility of a heavy top quark process. Furthermore, a number of these subprocesses contribute to each other. For example $b \to s\gamma$ can be directly involved subprocess of $b \to s\ell^-\ell^+$, or $b \to sg$ can be involved in $b \to sq\bar{q}$.

As previously mentioned, for the purposes of this report only a very limited exemplary portion of (7.3) are examined. These specifically belong to the processes $b \to s\ell^-\ell^+$, $b \to s\gamma$ and $b \to sq\bar{q}$. For these processes, the discussion was loosely based on (Bobeth et al., 2000; Ewerth, 2004a).

### 7.2.1 Wilson Coefficient Loop Expansion

For convenience in matching at separate loop orders, we define the loop expansion of the Wilson coefficients by exploiting a rescaling of the $\mathrm{SU}(3)_\mathrm{C}$ QCD constant including its RG dependence:

$$\alpha_s(\mu) = \frac{g_3^2(\mu)}{4\pi} \tag{7.4}$$

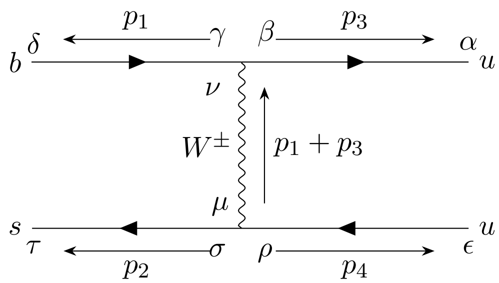
The loop expansion at any scale $\mu$ is then given by (Bobeth et al., 2005):

$$\mathcal{C}_i^q(\mu) = \mathcal{C}_i^{q(0)}(\mu) + \frac{\alpha_s(\mu)}{4\pi}\mathcal{C}_i^{q(1)}(\mu) + \mathcal{O}(\alpha_s(\mu)^2) \tag{7.5}$$

In this expansion, the coefficient $\mathcal{C}_i^{q(n)}$ represents from matching the $n$-loop MSSM contributions. We stop at the order $\mathcal{O}(\alpha_s)$ as we consider only NLO effects. This can be defined because each additional virtual gluon line closing a new loop contributing to the process will have to produce two coupling constants from its two internal vertices. The coefficients in (7.3) are normalised in $g_3$ such that (7.5) always holds.

### 7.2.2 A Tree Contribution to $b \to sq\bar{q}$

As a model LO contribution, the matching of the decay $b \to su\bar{u}$ mediated by a single $W$ boson is considered. This decay appears as part of the $b \to sq\bar{q}$ at tree level and involves only SM fields and so is a simple and instructive example.



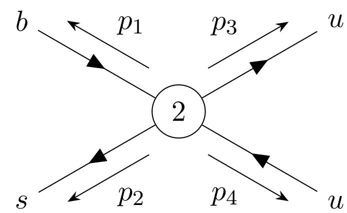




Figure 1: Tree contribution via $W$ boson exchange contributing to $b \to su\bar{u}$ subprocess matching

First, we note that the scattering involves four light (compared to the matching scale $\mu_W$) quark fields as initial and final particles. Therefore we expect the operators $\mathcal{O}_1^u, \mathcal{O}_2^u, \ldots$ to $\mathcal{O}_6$ to be generated by the UV theory matching. To simplify the discussion, we anticipate that only the $\mathcal{O}_2^u$ operator is in fact generated (Bobeth et al., 2000). Secondly, we note that this decay is a tree level decay, therefore the contribution that it generates will only be included in $\mathcal{C}_2^{u(0)}$. With these observations, Figure 1 shows the 1LPI diagram and its corresponding LEFT vertex. In this section, all references to the Wilson coefficient are supposed to be intended as $\mathcal{C}_2^u = \mathcal{C}_2^u(\mu_W)$.

First we calculate the LEFT contribution. Using $\mathcal{O}_2^u = (\bar{s}_L \gamma_\mu u_L)(\bar{u}_L \gamma^\mu b_L)$ and (7.1), the interaction part of the effective Hamiltonian and the Feynman rules give:

$$\begin{aligned} V^{\text{LEFT}}_{\alpha\delta\epsilon\tau}(p_1, p_2, p_3, p_4) = &\left( i \frac{4G_F}{\sqrt{2}} V^{\text{CKM}}_{ub} V^{\text{CKM}*}_{us} \mathcal{C}_2^u \right) \\ &\times \left( \frac{-i(\gamma \cdot p_2 + m_s)}{p_2^2 + m_s^2} \right)_{\alpha\beta} (\gamma_\mu P_L)_{\beta\gamma} \left( \frac{-i(-\gamma \cdot p_4 + m_u)}{p_4^2 + m_u^2} \right)_{\gamma\delta} \\ &\times \left( \frac{-i(\gamma \cdot p_3 + m_u)}{p_3^2 + m_u^2} \right)_{\epsilon\rho} (\gamma^\mu P_L)_{\rho\sigma} \left( \frac{-i(-\gamma \cdot p_1 + m_b)}{p_1^2 + m_b^2} \right)_{\sigma\tau} \end{aligned} \tag{7.6}$$

Where the indices $\alpha, \beta, \gamma, \delta, \epsilon, \rho, \sigma, \tau$ are Dirac spinor indices which are summed over (can be summed over horizontally without a metric). Amputating the external propagators (and renaming letters), we remain with the contribution of the effective vertex only:

$$V^{\text{LEFT}}_{\alpha\beta\gamma\delta}|_{\text{amp}} = \left( i \frac{4G_F}{\sqrt{2}} V^{\text{CKM}}_{ub} V^{\text{CKM}*}_{us} \mathcal{C}_2^u \right) (\gamma_\mu P_L)_{\alpha\beta} (\gamma^\mu P_L)_{\gamma\delta} \tag{7.7}$$

Now we must compute the equivalent diagram in the UV complete theory. We state two assumptions (Buras, 2020) that we make: all external momenta are neglected and all masses from light particles are also neglected (although this last assumption will not be required in this particular example). Using the Feynman rules on Figure 7.9, along with the quark-$W$ vertex rule given in Section 5.2, we obtain the following contribution:

$$V^{\text{MSSM}}_{\alpha\delta\epsilon\tau}(p_1, p_2, p_3, p_4) = \left( \frac{ie}{\sqrt{2} s_W} V^{\text{CKM}*}_{us} \right) \left( -\frac{ie}{\sqrt{2} s_W} V^{\text{CKM}}_{ub} \right) \left( \frac{i\eta_{\mu\nu}}{(p_1 + p_3)^2 + m_W^2} \right)$$

$$
\begin{aligned}
&\times \left(\frac{-i(\gamma\cdot p_2+m_s)}{p_2^2+m_s^2}\right)_{\alpha\beta}(\gamma^\mu P_L)_{\beta\gamma}\left(\frac{-i(-\gamma\cdot p_4+m_u)}{p_4^2+m_u^2}\right)_{\gamma\delta}\\
&\times \left(\frac{-i(\gamma\cdot p_3+m_u)}{p_3^2+m_u^2}\right)_{\epsilon\rho}(\gamma^\nu P_L)_{\rho\sigma}\left(\frac{-i(-\gamma\cdot p_1+m_b)}{p_1^2+m_b^2}\right)_{\sigma\tau} \qquad (7.8)
\end{aligned}
$$

Amputating the external propagators, enforcing the assumption of vanishing external momenta and using the definition of the Fermi constant $G_F$ in (7.2), we obtain:

$$
\begin{aligned}
V^{\text{MSSM}}_{\alpha\beta\gamma\delta}|_{\text{amp}} &= \left(\frac{ie}{\sqrt{2}s_W}\right)^2 V^{\text{CKM}}_{ub}V^{\text{CKM}*}_{us}\left(\frac{i\eta_{\mu\nu}}{m_W^2}\right)(\gamma^\mu P_L)_{\alpha\beta}(\gamma^\nu P_L)_{\gamma\delta}\\
&= -i\frac{g_2^2}{2}\frac{1}{m_W^2}V^{\text{CKM}}_{ub}V^{\text{CKM}*}_{us}(\gamma^\mu P_L)_{\alpha\beta}(\gamma_\mu P_L)_{\gamma\delta}\\
&= -i\frac{4G_F}{\sqrt{2}}V^{\text{CKM}}_{ub}V^{\text{CKM}*}_{us}(\gamma_\mu P_L)_{\alpha\beta}(\gamma^\mu P_L)_{\gamma\delta} \qquad (7.9)
\end{aligned}
$$

By comparing the two expressions (7.7) and (7.9), we come to the following condition:

$$
-1 = \mathcal{C}_2^u = \mathcal{C}^{u(0)} + \mathcal{O}(\alpha_s) \qquad (7.10)
$$

By neglecting the charged Higgs contribution, justified for small $\tan\beta$ (Ewerth, 2004a), we deduce that this is the only tree level contribution (see Section 5.2) and so the up-sector Wilson coefficient of the second current-current operator is:

$$
\mathcal{C}_2^{u(0)} = -1 \qquad (7.11)
$$

This corresponds with the value found in the literature (Bobeth et al., 2000; Ewerth, 2004a) (noting the slight difference that the authors calculate $\mathcal{C}_2^{c(0)}$ and state its equality with $\mathcal{C}_2^{u(0)}$).

### 7.2.3 Example of Loop Contribution to $b \to s\ell^-\ell^+$

The second diagram that we consider for NLO matching is the "box" diagram and is presented in Figure 2. These types of diagrams contribute to $b \to s\ell^-\ell^+$ decays and at 1LPI take part in generating the $\mathcal{C}_9$ and $\mathcal{C}_{10}$ Wilson coefficients simultaneously, making it a good example to consider. As in Section 7.2.2, the change of quark flavour is due to $W$ boson interactions but these occur within the loop and so yield a more complicated matching. Observing the loop, we can see that it is composed of two bosonic and two fermionic fields. Hence, we expect the contribution from the diagram to converge in the naive order of $\sim \frac{1}{k^2}$, where $k$ is the loop momenta. Therefore, it suffices that in the following we do not prepare the integrals for dimensional regularisation.

We begin by calculating the Feynman diagram contribution from the full theory. In the following, we note that the $+i\epsilon$ terms are omitted, but that the Feynman prescription is nevertheless intended to calculate propagator integrals. This analysis is based on the box diagram analysis in (Buras, 2020), albeit for a different process and with an even

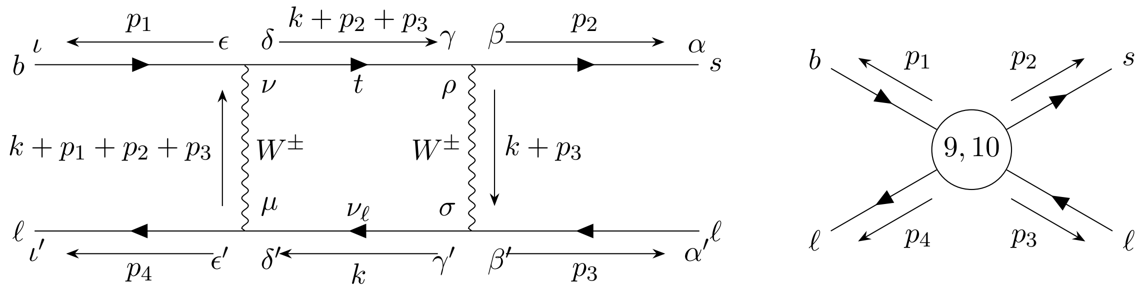



Figure 2: One-loop level box diagram contributing to the $b \to s\ell^+\ell^-$ process matching

more explicit presentation.

The diagram in Figure 2 gives the contribution (highlighting again the implied $+i\epsilon$ terms for loop propagators):

$$\begin{aligned}
&V^{\mathrm{MSSM}}_{\alpha\iota\alpha'\iota'}(p_1,p_2,p_3,p_4)\\
&\quad=\int đ^4k\Bigg[\left(\frac{-i(\gamma\cdot p_2+m_s)}{p_2^2+m_s^2}\right)_{\alpha\beta}\left(\frac{-i(\gamma\cdot(k+p_2+p_3)+m_t)}{(k+p_2+p_3)^2+m_t^2}\right)_{\gamma\delta}\left(\frac{-i(-\gamma\cdot p_1+m_b)}{p_1^2+m_b^2}\right)_{\epsilon\iota}\\
&\qquad\times\left(\frac{-i(\gamma\cdot p_4+m_\ell)}{p_4^2+m_\ell^2}\right)_{\alpha'\beta'}\left(\frac{-i\gamma\cdot k}{k^2}\right)_{\gamma'\delta'}\left(\frac{-i(-\gamma\cdot p_3+m_\ell)}{p_3^2+m_\ell^2}\right)_{\epsilon'\iota'}\\
&\qquad\times\left(\frac{i\eta_{\mu\nu}}{(k+p_1+p_2+p_3)^2+m_W^2}\right)\left(\frac{i\eta_{\rho\sigma}}{(k+p_3)^2+m_W^2}\right)\\
&\qquad\times\left(-\frac{ie}{\sqrt{2}s_W}V^{\mathrm{CKM}}_{tb}(\gamma^\nu P_L)_{\delta\epsilon}\right)\left(\frac{ie}{\sqrt{2}s_W}V^{\mathrm{CKM}*}_{ts}(\gamma^\rho P_L)_{\beta\gamma}\right)\\
&\qquad\times\left(-\frac{ie}{\sqrt{2}s_W}V^{\mathrm{CKM}}_{tb}(\gamma^\sigma P_L)_{\delta'\epsilon'}\right)\left(\frac{ie}{\sqrt{2}s_W}V^{\mathrm{CKM}*}_{ts}(\gamma^\mu P_L)_{\beta'\gamma'}\right)\Bigg]
\end{aligned}\tag{7.12}$$

As previously discussed, we assume that the external momenta for the process can be neglected and we amputate the external propagators of the diagram to give:

$$\begin{aligned}
&V^{\mathrm{MSSM}}_{\beta\epsilon\beta'\epsilon'}|_{\mathrm{amp}}\\
&\quad=\left(\frac{ie}{\sqrt{2}s_W}\right)^4 V^{\mathrm{CKM}}_{tb}V^{\mathrm{CKM}*}_{ts}(-i)^2 i^2(\gamma^\mu P_L)_{\delta\epsilon}(\gamma_\mu P_L)_{\beta'\gamma'}(\gamma_\nu P_L)_{\beta\gamma}(\gamma_\nu P_L)_{\delta'\epsilon'}\\
&\qquad\times\int đ^4k\frac{(\gamma\cdot k+m_t)_{\gamma\delta}(\gamma\cdot k)_{\gamma'\delta'}}{(k^2+m_t^2)k^2(k^2+m_W^2)^2} &(7.13)\\
&\quad=\left(-\frac{e^2}{2s_W^2}\right)^2 V^{\mathrm{CKM}}_{tb}V^{\mathrm{CKM}*}_{ts}(-i)^2 i^2(\gamma^\mu P_L)_{\delta\epsilon}(\gamma_\mu P_L)_{\beta'\gamma'}(\gamma_\nu P_L)_{\beta\gamma}(\gamma_\nu P_L)_{\delta'\epsilon'}\\
&\qquad\times\int đ^4k\frac{((\gamma^\rho)_{\gamma\delta}k_\rho+m_t\delta_{\gamma\delta})(\gamma^\sigma)_{\gamma'\delta'}k_\sigma}{(k^2+m_t^2)k^2(k^2+m_W^2)^2} &(7.14)\\
&\quad=\frac{4G_F}{\sqrt{2}}m_W^2\left(\frac{e^2}{2s_W^2}\right)V^{\mathrm{CKM}}_{tb}V^{\mathrm{CKM}*}_{ts}(\gamma^\mu P_L)_{\delta\epsilon}(\gamma_\mu P_L)_{\beta'\gamma'}(\gamma_\nu P_L)_{\beta\gamma}(\gamma_\nu P_L)_{\delta'\epsilon'}(\gamma^\rho)_{\gamma\delta}(\gamma^\sigma)_{\gamma'\delta'}
\end{aligned}$$

$$\times \int đ^4k \frac{k_\rho k_\sigma}{(k^2+m_t^2)k^2(k^2+m_W^2)^2} \tag{7.15}$$

To proceed, we must compute the value of the integral:

$$I_{\rho\sigma} = \int đ^4k \frac{k_\rho k_\sigma}{(k^2+m_t^2)k^2(k^2+m_W^2)^2} \tag{7.16}$$

This can be done using a Passarino-Veltman type reduction argument (Passarino & Veltman, 1979): since we have neglected the external momenta $p_i$, the only tensor structure that we have available for the integral to obtain tensor indices is the Minkowski metric. Therefore we deduce that:

$$I_{\rho\sigma} = \eta_{\rho\sigma} I \tag{7.17}$$

Where $I$ is now a scalar to be calculated. By contracting both sides of (7.17), we obtain a scalar integral which can be fully calculated using Wick rotation to a Euclidean space.

$$\begin{aligned}\eta^{\rho\sigma}\eta_{\rho\sigma} I = 4I = \eta^{\rho\sigma} I_{\rho\sigma} &= \int đ^4k \frac{k^2}{(k^2+m_t^2)k^2(k^2+m_W^2)^2} \\ &= \int đ^4k \frac{1}{(k^2+m_t^2)(k^2+m_W^2)^2}\end{aligned} \tag{7.18}$$

Firstly we note that, due to the contraction, the factor of $k^2$ producing an IR divergence in the denominator of the integral has disappeared, making this integral fully convergent upon Wick rotation. Secondly, this particular integral presents some complexity but is a well-known and common integral in the literature: it is part of the Inami-Lim family of weak-process integrals and has both been calculated explicitly in (Inami & Lim, 1981) and ('t Hooft & Veltman, 1979). Performing the contraction and using the result found in the literature:

$$\implies \frac{\eta_{\rho\sigma}}{4} \int đ^4k \frac{1}{(k^2+m_t^2)(k^2+m_W^2)^2} = -\frac{i\eta_{\rho\sigma}}{64\pi^2 m_W^2}\left(4B_0\left(\frac{m_t^2}{m_W^2}\right)+1\right) \tag{7.19}$$

Where $B_0(x)$ is part of the Inami-Lim functions and is defined explicitly as:

$$B_0(x) = \frac{1}{4(1-x)} + \frac{x}{4(1-x)^2}\log x \tag{7.20}$$

Now, we can return to computation of the full contribution of equation (7.15). Using the properties of the Gamma matrices, we can rearrange to the following contribution for the amputated diagram:

$$\begin{aligned}&V^{\text{MSSM}}{}_{\alpha\beta\alpha'\beta'}|_{\text{amp}} \\ &\quad = -i\frac{4G_F}{\sqrt{2}} m_W^2 \left(\frac{e^2}{2s_W^2}\right) V_{tb}^{\text{CKM}} V_{ts}^{\text{CKM}*} \frac{\eta_{\rho\sigma}}{64\pi^2 m_W^2} \\ &\quad\quad \times \left(4B_0\left(\frac{m_t^2}{m_W^2}\right)+1\right)(\gamma^\nu\gamma^\rho\gamma^\mu P_L)_{\alpha\beta}(\gamma_\mu\gamma^\sigma\gamma_\nu P_L)_{\alpha'\beta'}\end{aligned}$$

$$= - i\frac{4G_F}{\sqrt{2}}V_{tb}^{\rm CKM}V_{ts}^{\rm CKM*}\frac{e^2}{s_W^2}\frac{1}{16\pi^2}\frac{1}{4}\left(4B_0\left(\frac{m_t^2}{m_W^2}\right)+1\right)(\gamma_\mu P_L)_{\alpha\beta}(\gamma^\mu(\mathbb{1}+i\gamma_5))_{\alpha'\beta'} \tag{7.21}$$

This is the final result that will be used for LEFT matching. Now considering the diagrams in 2 with the effective vertices $\mathcal{O}_9$ and $\mathcal{O}_{10}$, we obtain the contribution:

$$\begin{aligned}
&V_{\alpha\delta\alpha'\delta}^{\rm LEFT}(p_1,p_2,p_3,p_4)\\
&\quad=\left(i\frac{4G_F}{\sqrt{2}}V_{tb}^{\rm CKM}V_{ts}^{\rm CKM*}\mathcal{C}_9^t\right)\frac{e^2}{g_3^2}\left(\frac{-i(\gamma\cdot p_2+m_s)}{p_2^2+m_s^2}\right)_{\alpha\beta}(\gamma_\mu P_L)_{\beta\gamma}\left(\frac{-i(-\gamma\cdot p_1+m_b)}{p_1^2+m_b^2}\right)_{\gamma\delta}\\
&\quad\times\left(\frac{-i(-\gamma\cdot p_3+m_\ell)}{p_3^2+m_\ell^2}\right)_{\alpha'\beta'}(\gamma^\mu)_{\beta'\gamma'}\left(\frac{-i(\gamma\cdot p_4+m_\ell)}{p_4^2+m_\ell^2}\right)_{\gamma'\delta'}\\
&\quad+\left(i\frac{4G_F}{\sqrt{2}}V_{tb}^{\rm CKM}V_{ts}^{\rm CKM*}\mathcal{C}_{10}^t\right)\frac{e^2}{g_3^2}\left(\frac{-i(\gamma\cdot p_2+m_s)}{p_2^2+m_s^2}\right)_{\alpha\beta}(\gamma_\mu P_L)_{\beta\gamma}\left(\frac{-i(-\gamma\cdot p_1+m_b)}{p_1^2+m_b^2}\right)_{\gamma\delta}\\
&\quad\times\left(\frac{-i(-\gamma\cdot p_3+m_\ell)}{p_3^2+m_\ell^2}\right)_{\alpha'\beta'}(\gamma^\mu(-i\gamma_5))_{\beta'\gamma'}\left(\frac{-i(\gamma\cdot p_4+m_\ell)}{p_4^2+m_\ell^2}\right)_{\gamma'\delta'}
\end{aligned} \tag{7.22}$$

We can proceed to amputate the external propagators to obtain:

$$\begin{aligned}
V_{\alpha\beta\alpha'\beta'}^{\rm LEFT}|_{\rm amp}&=\left(i\frac{4G_F}{\sqrt{2}}V_{tb}^{\rm CKM}V_{ts}^{\rm CKM*}\mathcal{C}_9^t\right)\frac{e^2}{g_3^2}(\gamma_\mu P_L)_{\alpha\beta}(\gamma^\mu)_{\alpha'\beta'}\\
&+\left(i\frac{4G_F}{\sqrt{2}}V_{tb}^{\rm CKM}V_{ts}^{\rm CKM*}\mathcal{C}_{10}^t\right)\frac{e^2}{g_3^2}(\gamma_\mu P_L)_{\alpha\beta}(\gamma^\mu(-i\gamma_5))_{\alpha'\beta'}
\end{aligned} \tag{7.23}$$

In the above, the separation with the $\gamma_5$ comes from expansion of the left-handed projection operator. Now, we can finally compare the full theory and LEFT expressions to calculate the contribution of the $W$ boson box diagram to the Wilson coefficients at NLO:

$$\frac{e^2}{g_3^2}\mathcal{C}_9^t=\frac{e^2}{g_3^2}\mathcal{C}_9^{t(0)}+\frac{e^2}{16\pi^2}\mathcal{C}_9^{t(1)}+\cdots=-\frac{e^2}{16\pi^2}\frac{1}{s_W^2}\frac{1}{4}\left(4B_0\left(\frac{m_t^2}{m_W^2}\right)+1\right) \tag{7.24}$$

$$\frac{e^2}{g_3^2}\mathcal{C}_{10}^t=\frac{e^2}{g_3^2}\mathcal{C}_{10}^{t(0)}+\frac{e^2}{16\pi^2}\mathcal{C}_{10}^{t(1)}+\cdots=\frac{e^2}{16\pi^2}\frac{1}{s_W^2}\frac{1}{4}\left(4B_0\left(\frac{m_t^2}{m_W^2}\right)+1\right) \tag{7.25}$$

We note that the equality between left and right sides is not a strict equality, in the sense that these are only partial contributions to the $\mathcal{C}_{9,10}$ coefficients coming from the specific diagram. A further simplification can be made. Due to the Glashow-Iliopoulos-Maiani (GIM) mechanism (Glashow et al., 1970), any constant term in these contributions will disappear by the unitarity of the CKM matrix used in the definition of the effective Hamiltonian (7.1).[2] Using this, we can finally conclude with a one loop expression for the partial contribution to the Wilson coefficients:

$$\mathcal{C}_9^{t(1)}=-\frac{1}{s_W^2}B_0\left(\frac{m_t^2}{m_W^2}\right) \tag{7.26}$$

[2]The author would like to thank Prof. Andrzej J. Buras (of the Technical University of Munich) for his correspondence and help pertaining to such results.

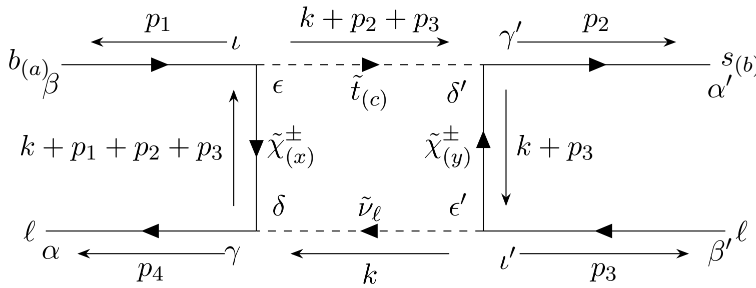



Figure 3: Chargino box diagram depicting an MSSM contribution to $b \to s\ell^+\ell^-$

$$\mathcal{C}_9^{t(1)} = \frac{1}{s_W^2} B_0 \left( \frac{m_t^2}{m_W^2} \right) \tag{7.27}$$

This result is in fact in agreement with the literature (Bobeth et al., 2000).

### 7.2.4 An MSSM Contribution to $b \to s\ell^-\ell^+$

Now that the matching of a box diagram has been presented, this method can be applied in a fundamentally identical manner to computing box diagrams composed of the MSSM fields. As depicted in Diagram 2, it is in fact the sum of these 1LPI diagrams that determines the full expressions for the Wilson coefficients.

The chosen diagram is chargino-squark-sneutrino diagram taking part in the $b \to s\ell^-\ell^+$ subprocess, as seen in Diagram 3. While evaluating the full contribution is of too great a complexity (from the integrals, superpartner mass-diagonalisation matrices and decoupled gluinos assumption (Bobeth et al., 2005)), we derive the expression for the Feynman diagram and state the resulting Wilson coefficient contribution from the literature. An observation to be made is that, as the previous $W$ boson box, the naive degree of divergence of the integral is $\sim \frac{1}{k^2}$, coming from the virtual two fermion and two scalar lines, hence we expect the integral to converge when Wick rotated to Euclidean space and not require renormalisation. Using the Feynman rules, the MSSM vertices in Section 5 and the explicit contributions in (Rosiek, 1990)[3]:

$$\begin{aligned}
&V_{\alpha\beta\alpha'\beta' ab}(p_1, p_2, p_3, p_4) \\
&\quad = \int \text{đ}^4 k \Bigg[ \left( \frac{-i(\gamma \cdot p_4 + m_\ell)}{p_4^2 + m_\ell^2} \right)_{\alpha\gamma} \left( \frac{-i(-\gamma \cdot (k + p_1 + p_2 + p_3) + m_{\tilde{\chi}_x^\pm})}{(k + p_1 + p_2 + p_3)^2 + m_{\tilde{\chi}_x^\pm}^2} \right)_{\delta\epsilon} \left( \frac{-i(-\gamma \cdot p_1 + m_b)}{p_1^2 + m_b^2} \right)_{\iota\beta} \\
&\qquad \times \left( \frac{-i(\gamma \cdot p_2 + m_s)}{p_2^2 + m_s^2} \right)_{\alpha'\gamma'} \left( \frac{-i(-\gamma \cdot (k + p_3) + m_{\tilde{\chi}_x^\pm})}{(k + p_3)^2 + m_{\tilde{\chi}_x^\pm}^2} \right)_{\delta'\epsilon'} \left( \frac{-i(-\gamma \cdot p_3 + m_\ell)}{p_3^2 + m_\ell^2} \right)_{\iota'\beta'} \\
&\qquad \times \left( \frac{-i}{(k + p_2 + p_3)^2 + m_{\tilde{t}}^2} \right) \left( \frac{-i}{k^2 + m_{\tilde{\nu}_\ell}^2} \right)
\end{aligned}$$

[3]The $Z^X$ and $Y^X_{(Y)}$ matrices in this diagram are part of the notation in (Rosiek, 1990) which we will not reintroduce to avoid unnecessary complexity.

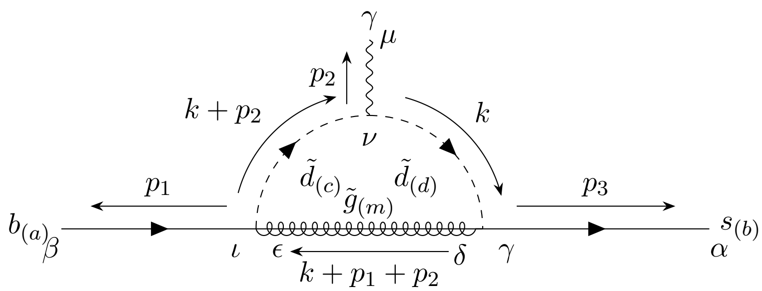



Figure 4: Gluino penguin diagram depicting an MSSM contribution to $b \to s\gamma$

$$\times \left(-i\left(\frac{e}{s_W}Z^{+}_{1y}P_L + Y^{\ell}_{(\ell)}Z^{-*}_{2y}P_R\right)_{\epsilon'\iota'} Z^{\tilde{\nu}*}_{\ell\nu}\right)\left(-i\left(\frac{e}{s_W}Z^{+}_{1x}P_L + Y^{\ell}_{(\ell)}Z^{-*}_{2x}P_R\right)_{\gamma\delta} Z^{\tilde{\nu}*}_{\ell\nu}\right)^{*}$$
$$\times \left(i\left(\left(-\frac{e}{s_W}Z^{\tilde{u}*}_{X_L\tilde{t}}Z^{+}_{1x} + Y^{X}_{(u)}Z^{\tilde{u}*}_{X_R\tilde{t}}Z^{+*}_{2x}\right)(P_L)_{\epsilon\iota} - Y^{b}_{(d)}Z^{\tilde{u}*}_{X_L\tilde{t}}Z^{-*}_{2x}(P_R)_{\epsilon\iota}\right)V^{\text{CKM}}_{Xb}\right)$$
$$\times \left(i\left(\left(-\frac{e}{s_W}Z^{\tilde{u}*}_{X_L\tilde{t}}Z^{+}_{1y} + Y^{X}_{(u)}Z^{\tilde{u}*}_{X_R\tilde{t}}Z^{+*}_{2y}\right)(P_L)_{\gamma'\delta'} - Y^{s}_{(d)}Z^{\tilde{u}*}_{X_L\tilde{t}}Z^{-*}_{2y}(P_R)_{\gamma'\delta'}\right)V^{\text{CKM}}_{Xs}\right)^{*}\Bigg] \tag{7.28}$$

By evaluating the expression and equating the contribution to the previously-derived LEFT diagram expression (7.23) using the $\mathcal{O}_{9,10}$ operators, the Wilson coefficient contributions found in the literature (Bobeth et al., 2005), with the functions defined within it, are:

$$\delta\mathcal{C}^{t(1)}_{9} = -\frac{1}{s_W^2}[B^{\ell\bar{\ell}}_{9}]^{(0)}_{\tilde{X}} \tag{7.29}$$

$$\delta\mathcal{C}^{t(1)}_{10} = \frac{1}{s_W^2}[B^{\ell\bar{\ell}}_{10}]^{(0)}_{\tilde{X}} \tag{7.30}$$

The $\delta\mathcal{C}$ notation has been used this time to stress that these are only a parts of the full Wilson coefficients. This procedure can be repeated for all Feynman box diagrams of the MSSM theory to produce the full NLO contribution to the $\mathcal{C}^{q(0)}_{9,10}$ and $\mathcal{C}^{q(1)}_{9,10}$.

### 7.2.5 A Further MSSM Contributions in $b \to s\gamma$

To conclude with the discussion on matching, we very briefly present photon "penguin" diagrams. These constitute one-loop flavour changing interactions that emit a photon from the internal virtual loop and can be regarded as the elementary beauty-to-strange flavour changing neutral current. These contribute to $b \to s\ell^-\ell^+$ decays indirectly, producing a dilepton (one lepton and one anti-lepton) from the outgoing photon line. Diagram 5 shows how the sum of possible photon penguin diagrams in the MSSM contribute to the $\mathcal{C}_{2,7}$ Wilson coefficients of the $\mathcal{O}_{2,7}$ operators in (7.3b) and (7.3g).

An exemplary photon penguin MSSM diagram is the virtual squark-gluino mediated

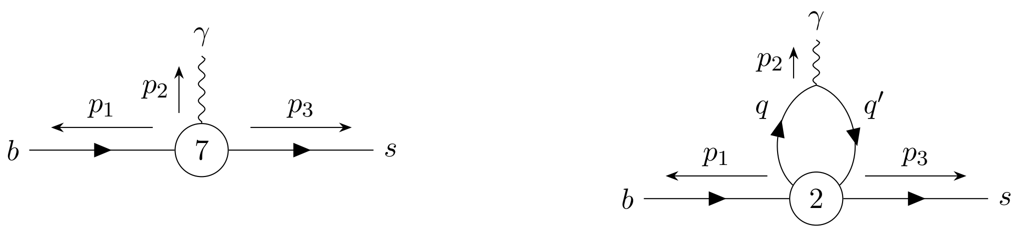



Figure 5: Effective $\mathcal{O}_2$ and $\mathcal{O}_7$ operators that participate in the photon penguin matching

diagram, in 4. As in Section 7.2.4, we only limit to giving the Feynman diagram contribution using the Feynman rules and stating that the LEFT matching can be done in a very similar fashion to that in (7.23) with the $\mathcal{O}_{2,7}$ vertices. This specific diagram has been chosen for three reasons. The first is the peculiarity of including a massless particle on one of the external lines. This breaks the possibility of neglecting external momenta, and hence matching has to be done keeping both momenta and external masses (Buras, 2020). The second is the inclusion of a top squark, which in many analyses can be taken to be light compared to the other squark flavours (Lunghi et al., 2000). The third is the inclusion of the gluino line, which on the contrary can be taken to be one of the heaviest particles in the decoupled gluino approximation (Ewerth, 2004a).

From the Feynman diagram 4, the derived expression is:

$$
\begin{aligned}
&V_{\alpha\beta\mu ab}(p_1, p_2, p_3, p_4) \\
&\quad = \int d\!\!{}^-{}^4 k \Bigg[ \left( \frac{-i(\gamma \cdot p_3 + m_s)}{p_3^2 + m_s^2} \right)_{\alpha\gamma} \left( \frac{-i(-\gamma \cdot (k + p_1 + p_2) + m_{\tilde{g}})}{(k + p_1 + p_2)^2 + m_{\tilde{g}}^2} \right)_{\delta\epsilon} \left( \frac{-i(-\gamma \cdot p_1 + m_b)}{p_1^2 + m_b^2} \right)_{\iota\beta} \\
&\qquad \times \left( \frac{i\eta_{\mu\nu}}{p_2^2} \right) \left( \frac{-i}{(k + p_2)^2 + m_{\tilde{t}}^2} \right) \left( \frac{-i}{k^2 + m_{\tilde{t}}^2} \right) \\
&\qquad \times \left( \frac{1}{3} ie \right) (2k + p_2)^\nu \\
&\qquad \times \left( ig_3 \sqrt{2} Y_{ca}^m \left( -Z_{d_L \tilde{d}}^{\tilde{d}} P_L + Z_{d_R \tilde{d}}^{\tilde{d}} P_R \right)_{\epsilon\iota} \right) \left( ig_3 \sqrt{2} Y_{db}^m \left( -Z_{s_L \tilde{d}}^{\tilde{d}} P_L + Z_{s_R \tilde{d}}^{\tilde{d}} P_R \right)_{\gamma\delta} \right)^* \Bigg]
\end{aligned}
\tag{7.31}
$$

There is one final observation we can make regards the $\mathcal{O}_2$ operator as in Diagram 5. This is part of a divergent loop, and so plays a part in the complication of renormalising Wilson coefficients discussed in the literature.

# 8 Implications, Discussion and Conclusion

In this report, we have begun with the description of supersymmetry as a justified and important hypothetical symmetry of nature and have examined one of its possible realisations in the theory of the SM, through the Minimally Supersymmetric Standard Model. We have discussed its less-trivial components, such as R-parity symmetry or the emergence of a scalar potential, and developed the technology required to be able to perform calculations on it, by exploiting the superspace and superfield formalism that arises naturally from the Super-Poincaré symmetry.

In order to understand the MSSM's effect and phenomenological interplay in the context of beauty decays, processes that are of fundamental importance for the contemporary probing of new physics, we have developed the framework of effective field theory and explained how the SM can be seen as the remnant, renormalisable part of a SMEFT incorporating BSM effects. By discussing renormalisation group evolution, we have presented the LEFT as the relevant EFT for processes occurring under the EWSB scale. Within the LEFT, we have demonstrated how higher-scale physics, such as that arising from virtual loop MSSM interactions, can be matched to lower-energy processes through the calculation of example Wilson coefficients at LO and NLO. In summary, we have re-traced the developed of a robust and phenomenologically relevant framework in which the MSSM can be both challenged by indirect experimental observations and can benefit from a constraining of its unobserved parameters.

To conclude this report, we would like to include a brief discussion on two cases in which LEFT analysis of MSSM effects has been successfully applied in the literature[4], to provide a starting point for potential future work and avenues for exploration.

The first application of the framework developed in this report is the use of Wilson coefficients to constrain squark flavour-violating parameters, as presented in (Behring et al., 2012). As has been seen, virtual squark loops participate to flavour changing neutral currents in the MSSM. Taking this one step further, by assuming that the squark mass matrices that give rise to the mixing have small off-diagonal entries, we can more directly tie Wilson coefficients to single parameters appearing in this non-diagonal structure. In order to do this, we use the mass insertion approximation (MIA) (Hall et al., 1986; Gabbiani & Masiero, 1989) for which the up squark matrix, for example, takes the following form (as seen in the publication):

$$\mathcal{M}^2_{\tilde{u}} = \begin{pmatrix} (\mathcal{M}^2_{\tilde{u}})_{LL} & (\mathcal{M}^2_{\tilde{u}})_{LR} \\ (\mathcal{M}^2_{\tilde{u}})^\dagger_{LR} & (\mathcal{M}^2_{\tilde{u}})_{RR} \end{pmatrix}, \quad (\mathcal{M}^2_{\tilde{u}})_{LR} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & (\Delta^u_{23})_{LR} \\ 0 & 0 & (\Delta^u_{33})_{LR} \end{pmatrix} \tag{8.1}$$

[4]The author thanks Prof. Gudrun Hiller (of the Technical University of Dortmund) for the helpful insight and elucidation relating to the cases discussed in this section.

We also define the following MIA parameter:

$$(\delta_{ij}^{X})_{PQ} = \frac{(\Delta_{ij}^{X})_{PQ}}{\sqrt{(m^2_{\tilde{X}i})_{PP}(m^2_{\tilde{X}j})_{QQ}}} \tag{8.2}$$

In this approximation, we can employ a Taylor expansion and the GIM mechanism to extract relevant MIA parameters from Wilson coefficient contributions. For example, the chargino loop contribution (which exchanges virtual squarks) to $\mathcal{C}_7$ in the light stop approximation can be approximated to:

$$\mathcal{C}_7^{\tilde{\chi}^{\pm}} = \frac{V_{cs}^{\text{CKM}*}}{V_{ts}^{\text{CKM}*}} \frac{\lambda_t}{g_2} \frac{m_W^2}{m_{\tilde{q}}^2} (\delta_{23}^{u})_{LR} F \tag{8.3}$$

Where $\lambda_t$ and $F$ are defined in the appendix of (Behring et al., 2012). The publication argues, for example, that the parameter $(\delta_{23}^{d})_{LR}$ can in this way be constrained from $\mathcal{C}_7$ observed in $\bar{B} \to X_s\gamma$ decays, or that $(\delta_{23}^{u})_{LR}$ can be constrained mostly by measuring $\mathcal{C}_9$ and $\mathcal{C}_{10}$ coefficients. This has phenomenological implications on the branching fraction of the $\bar{B}_s \to \mu^+\mu^-$ process, which can be pursued experimentally.

A second interesting example, pertaining less to constraining MSSM parameters and more on a higher-level view of the theory, can be found in discussions on lepton flavour universality. We have seen that current hints of anomalies in the $R_K$ and $R_{K^*}$ branching fraction ratios have suggested a violation of LFU, and effort has been directed in suggesting new BSM physics responsible for this. In this landscape, leptoquark models (containing particles with both lepton and baryon numbers) have been some of the most promising (Aaij et al., 2022), but a potential connection can also be made to extensions of the MSSM. In fact, as discussed in (Hiller & Schmaltz, 2014), it is possible for scalar leptoquark effects to be equivalently given by superpartners of left-handed quark doublets interacting in such a way to generate R-parity violating terms (as in Section 4.2.3). An EFT analysis of this hypothetical scenario shows that this would induce effects in the $\mathcal{C}'_9$ and $\mathcal{C}'_{10}$ Wilson coefficients for the flipped chirality operators[5]:

$$\mathcal{O}'_9 = (\bar{s}\gamma_\mu P_R b)(\bar{\ell}\gamma^\mu \ell), \quad \mathcal{O}'_{10} = (\bar{s}\gamma_\mu P_R b)(\bar{\ell}\gamma^\mu \gamma_5 \ell) \tag{8.4}$$

While this is disfavoured in the way that this effect affects the $R_K, R_{K^*}$ ratios, it can be used to constrain R-parity violating couplings while allowing experimentally interesting discussions on extending the MSSM into R-parity violating models. One might speculate that combining this with future Hyper-Kamiokande measurements on proton decay, for example, might allow us to understand the real possibility for extensions to the MSSM arising from R-parity violation.

These two cases represent only a fraction of the efforts in understanding supersymmetric extensions of the SM, but provide important examples of theory and observation working

[5]These are introduced in a more conventional notation than that adopted in the rest of this report.

together to advance our knowledge of the fundamental interactions. One can hope that, through theoretical advancements and valuable phenomenological analyses like these, a more complete picture of our theories of reality will become more and more clear in time.

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# A Derivation of MSSM Higgs Vevs

The vevs of the two Higgs doublets in the MSSM are obtained by minimising the Higgs potential, given by the self-interactions of the two Higgs doublets. The terms that expand to give the Higgs potential are the $F$ terms in the $\mu$ self-interaction, the $D$ terms from the Higgs kinetic term and the soft-breaking terms in $\mathcal{L}_{\text{soft}}$. Collecting these contributions as given in (Kuroda, 1999):

$$\begin{aligned}
-V_{H_1,H_2} =& -\mu(H_1^\dagger H_1 + H_2^\dagger H_2) \\
& - \frac{1}{8}(g_1^2+g_2^2)(H_1^\dagger H_1 - H_2^\dagger H_2)^2 - \frac{g_2^2}{2}|H_1^\dagger H_2|^2 \\
& - m^2_{\tilde{H}_1} H_1^\dagger H_1 - m^2_{\tilde{H}_2} H_2^\dagger H_2 + (b(H_1 \cdot H_2) + \text{h.c.}) \qquad \text{(A.1)} \\
=& -\Big[(m^2_{\tilde{H}_1}+\mu^2)H_1^\dagger H_1 + (m^2_{\tilde{H}_2}+\mu^2)H_2^\dagger H_2 + (b(H_1\cdot H_2)+\text{h.c.}) \\
& + \frac{1}{8}(g_1^2+g_2^2)(H_1^\dagger H_1 - H_2^\dagger H_2)^2 + \frac{g_2^2}{2}|H_1^\dagger H_2|^2\Big] \qquad \text{(A.2)}
\end{aligned}$$

To minimise, we proceed with complex differentiation (which should be intended by components):

$$\begin{aligned}
\frac{\partial V_{H_1,H_2}}{\partial H_1^\dagger} &= 0 \\
&= (m^2_{\tilde{H}_1}+\mu^2)H_1 + b\epsilon H_2^* + \frac{1}{4}(g_1^2+g_2^2)(H_1^\dagger H_1 - H_2^\dagger H_2)H_1 + g_2^2(H_2^\dagger H_1)H_2 \qquad \text{(A.3)} \\
\frac{\partial V_{H_1,H_2}}{\partial H_2^\dagger} &= 0 \\
&= (m^2_{\tilde{H}_2})H_2 - b\epsilon H_1^* - \frac{1}{4}(g_1^2+g_2^2)(H_1^\dagger H_1 - H_2^\dagger H_2)H_2 + g_2^2(H_1^\dagger H_2)H_1 \qquad \text{(A.4)}
\end{aligned}$$

By picking $H_1$ and $H_2$ to be explicitly orthogonal, the final $(H_2^\dagger H_1)H_2$ and $(H_1^\dagger H_2)H_1$ mixed terms disappear and each condition fixes the single free entry in each doublet as a constant. We can pick those constants to simply be $\frac{1}{\sqrt{2}}v_1$ and $\frac{1}{\sqrt{2}}v_2$ such that the vevs are given by:

$$H_1 = \frac{1}{\sqrt{2}}\begin{pmatrix} v_1 \\ 0 \end{pmatrix} \qquad H_2 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_2 \end{pmatrix} \qquad \text{(A.5)}$$

# B Example Derivation of MSSM Mass Matrices

All of the mass matrices have been re-derived from the relevant terms coming from the fully expanded MSSM Lagrangian. To show how they are derived, we consider two different examples: that of the chargino mass matrix and that of the up-type squark matrix. All MSSM Lagrangian terms used in this section are found in (Kuroda, 1999) or are extensions of the terms found therein when adding quark and lepton generations (which the original publication does not consider).

## B.1 Charginos

The chargino mass matrix calculation is useful because it combines mass contributions coming from the Higgsino fields, gauginos (both in the flavour and mass bases) and Higgsino-gaugino interactions. The relevant terms that generate chargino mass terms are the Higgs $\mu$ term, the soft-breaking term and the Kähler potential Higgs kinetic terms (for both $\tilde{H}_1$ and $\tilde{H}_2$). The relevant parts are:

$$\mathcal{L}_{H_1 \cdot H_2} = -\mu\left[\tilde{H}_1^- \tilde{H}_2^+ + \text{h.c.} + \dots\right] + \dots \tag{B.1}$$

$$\mathcal{L}_{\text{soft}} = -\frac{1}{2}[m_{\tilde{G}} \tilde{G}^{\vartheta} \tilde{G}^{\vartheta} + m_{\tilde{W}} \tilde{W}^i \tilde{W}^i + m_{\tilde{B}} \tilde{B}\tilde{B} + \text{h.c.}] + \dots \tag{B.2}$$

$$\mathcal{L}_{H_1, H_2 \text{ kin}} = -\sqrt{2} m_W \left[\cos\beta \tilde{H}_1^- \tilde{W}^+ + \sin\beta \tilde{H}_2^+ \tilde{W}^- + \text{h.c.}\right] + \dots \tag{B.3}$$

First, we transform the electroweak gauginos using $\tilde{W}^{\pm} = \frac{1}{\sqrt{2}}(\tilde{W}^1 \mp i\tilde{W}^2)$ (and that the two are complex conjugates of each other), so that we can re-express the contraction in the soft breaking term as:

$$\tilde{W}^i \tilde{W}^i = \tilde{W}^1 \tilde{W}^1 + \tilde{W}^2 \tilde{W}^2 + \tilde{W}^3 \tilde{W}^3 = \tilde{W}^3 \tilde{W}^3 + 2\tilde{W}^+ \tilde{W}^- \tag{B.4}$$

Now we collect the terms for each pair of fields, keeping in mind that there is also a hermitian conjugation part:

$$\begin{aligned} \tilde{W}^+, \tilde{W}^- &: -m_{\tilde{W}} \\ \tilde{W}^+, \tilde{H}_1^- &: -\sqrt{2} m_W \cos\beta \\ \tilde{H}_2^+, \tilde{W}^- &: -\sqrt{2} m_W \sin\beta \\ \tilde{H}_1^-, \tilde{H}_2^+ &: -\mu \end{aligned} \tag{B.5}$$

In matrix form (and adding the hermitian conjugation contribution), we retrieve the result in Section 4.3 from:

$$-\begin{pmatrix} \tilde{W}^+ & \tilde{H}_2^+ \end{pmatrix} \begin{pmatrix} m_{\tilde{W}} & \sqrt{2} m_W \cos\beta \\ \sqrt{2} m_W \sin\beta & \mu \end{pmatrix} \begin{pmatrix} \tilde{W}^- \\ \tilde{H}_1^- \end{pmatrix} + \text{h.c.} \tag{B.6}$$

## B.2 Up-Squarks

The derivation for up-type squarks, especially introducing multiple generations, is more complicated. The first mass contribution we consider is that from the soft-breaking term (both the additional explicit mass terms and the trilinear mixing angle terms):

$$\mathcal{L}_{\text{soft}} = \dots - [M_{\tilde{q}}^2]_{fg} \tilde{q}_L^{\dagger f} \tilde{q}_L^g - [M_{\tilde{u}}^2]_{fg} \tilde{u}_R^{c*f} \tilde{u}_R^g + [A_{\tilde{u}}]_{fg} (\tilde{H}_2 \cdot \tilde{q}_L^f) \tilde{u}_R^{cg} + \dots \tag{B.7}$$

In the above, summation is assumed. Rotating into super-CKM basis, the direct mass terms yield diagonal (with respect to "left-handedness" and "right-handedness" of squarks) matrix contributions of $(M_{\tilde{u}}^2)_{LL}$ for left squarks and $(M_{\tilde{u}}^2)_{RR}$ for right squarks (the matrices are in flavour space). To find the contribution from the mixing angle, we now use the vev for the second Higgs doublet $H_2$ and the result:

$$(\tilde{H}_2 \cdot \tilde{q}_L^f) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & v_2 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} \tilde{u}_L^f \\ \tilde{d}_L^f \end{pmatrix} = -\frac{v_2}{\sqrt{2}} \tilde{u}_L^f \tag{B.8}$$

In super-CKM basis, combining with $\tilde{u}_R^{cg}$ this gives a L-R mixing (off-diagonal in the mass matrix) contribution proportional to $(M_{\tilde{u}}^2)_{LR}$.

To work out all remaining contributions we must turn to the scalar potential generated by integrating out $D$ and $F$ fields, as in (4.22). Expanding fully, the part of the scalar potential contributing to up squark masses involves the terms:

$$W_{\text{MSSM}} = \cdots - [y_u]_{fg} (\tilde{H}_2 \cdot \tilde{q}_L^f) \tilde{u}_R^{cg} + \cdots = \cdots - [y_u]_{fg} \frac{-v_2}{\sqrt{2}} \tilde{u}_L^f \tilde{u}_R^{cg} + \ldots \tag{B.9}$$

Following the prescription to calculate the terms in the scalar potential, we proceed with differentiating:

$$\frac{\partial W_{\text{MSSM}}}{\partial \tilde{u}_L^f} = -[y_u]_{fg} \frac{-v_2}{\sqrt{2}} \tilde{u}_R^{cg} \tag{B.10}$$

$$\frac{\partial W_{\text{MSSM}}}{\partial \tilde{u}_R^{cg}} = -[y_u]_{fg} \frac{-v_2}{\sqrt{2}} \tilde{u}_L^f \tag{B.11}$$

Squaring the contributions as in $|F|^2$ and rotating into super-CKM basis, we obtain left-/right diagonal contributions of exactly the $M_u^2$ minimal mass matrix squared. Repeating the procedure by differentiating with the Higgs terms (of which only $H_1^0$ and $H_2^0$ give nonzero contributions), we obtain the following contribution:

$$|F|^2 \propto \cdots - \mu \cot \beta [y_u]_{fg} \tilde{u}_L^f \tilde{u}_R^{cg} + \text{h.c.} + \ldots \tag{B.12}$$

Rotating into super-CKM basis, the Yukawa coupling matrix becomes the $M_u$ matrix. We now move to using the $D$ term contributions to produce more mass terms. We start with left handed up squarks:

$$|D|^2 = -\frac{1}{3} \frac{g_1^2}{4} \tilde{u}_L^{f*} \tilde{u}_L^f v^2 \cos 2\beta \tag{B.13}$$

$$|D^i|^2 = \frac{g_2^2}{4} \tilde{u}_L^{f*} \tilde{u}_L^f v^2 \cos 2\beta \tag{B.14}$$

Combined, these give a contribution in flavour space proportional to the diagonal matrix:

$$\propto \frac{1}{6} (4 m_W^2 - m_Z^2) \cos 2\beta \mathbb{1} \tag{B.15}$$

Repeating the calculation for the ($\text{SU(2)}_\text{L}$ neutral) right up squarks:

$$|D|^2 = -\frac{g_1^2}{4} (-\frac{2}{3}) v^2 \cos 2\beta = \frac{2}{3} m_Z^2 s_W^2 \cos 2\beta \tilde{u}_R^{f*} \tilde{u}_R^f \tag{B.16}$$

Finally, collecting all of the derived terms into a single matrix combining flavour space and left-right handedness, we obtain the matrix present in Section 4.3.2:

$$\mathcal{M}_{\tilde{u}}^2 = \begin{pmatrix} (M_{\tilde{u}}^2)_{LL} + M_u^2 + \frac{1}{6}(4m_Z^2 - m_Z^2)\cos 2\beta \mathbb{1} & (M_{\tilde{u}}^2)_{LR} - M_u \mu \cot\beta \\ (M_{\tilde{u}}^2)_{LR}^\dagger - M_u \mu^* \cot\beta & (M_{\tilde{u}}^2)_{RR} + M_u^2 + \frac{2}{3}m_Z^2 s_W^2 \cos 2\beta \mathbb{1} \end{pmatrix} \tag{B.17}$$