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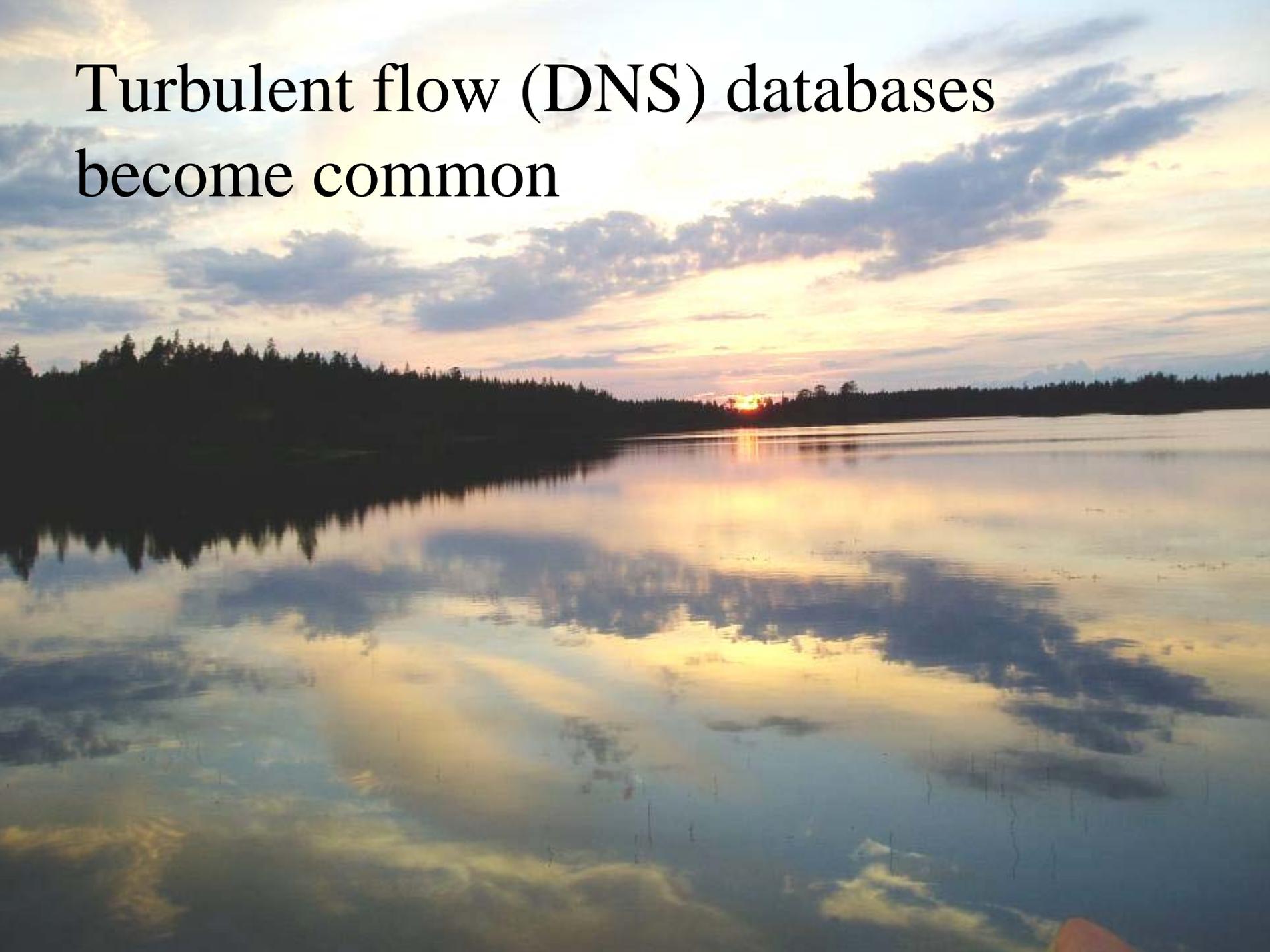
# Master Mode Set for 3D Turbulent Channel Flow

(Database of master modes is available online)

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Turbulent flow (DNS) databases  
become common



A scenic view of a river at sunset. The sky is filled with soft, pink and purple clouds, and a faint rainbow is visible in the distance. The river flows from the foreground towards the background, reflecting the colors of the sky. On the left bank, there are several kayakers with their gear, including orange and blue kayaks. The surrounding area is lush with green trees and vegetation.

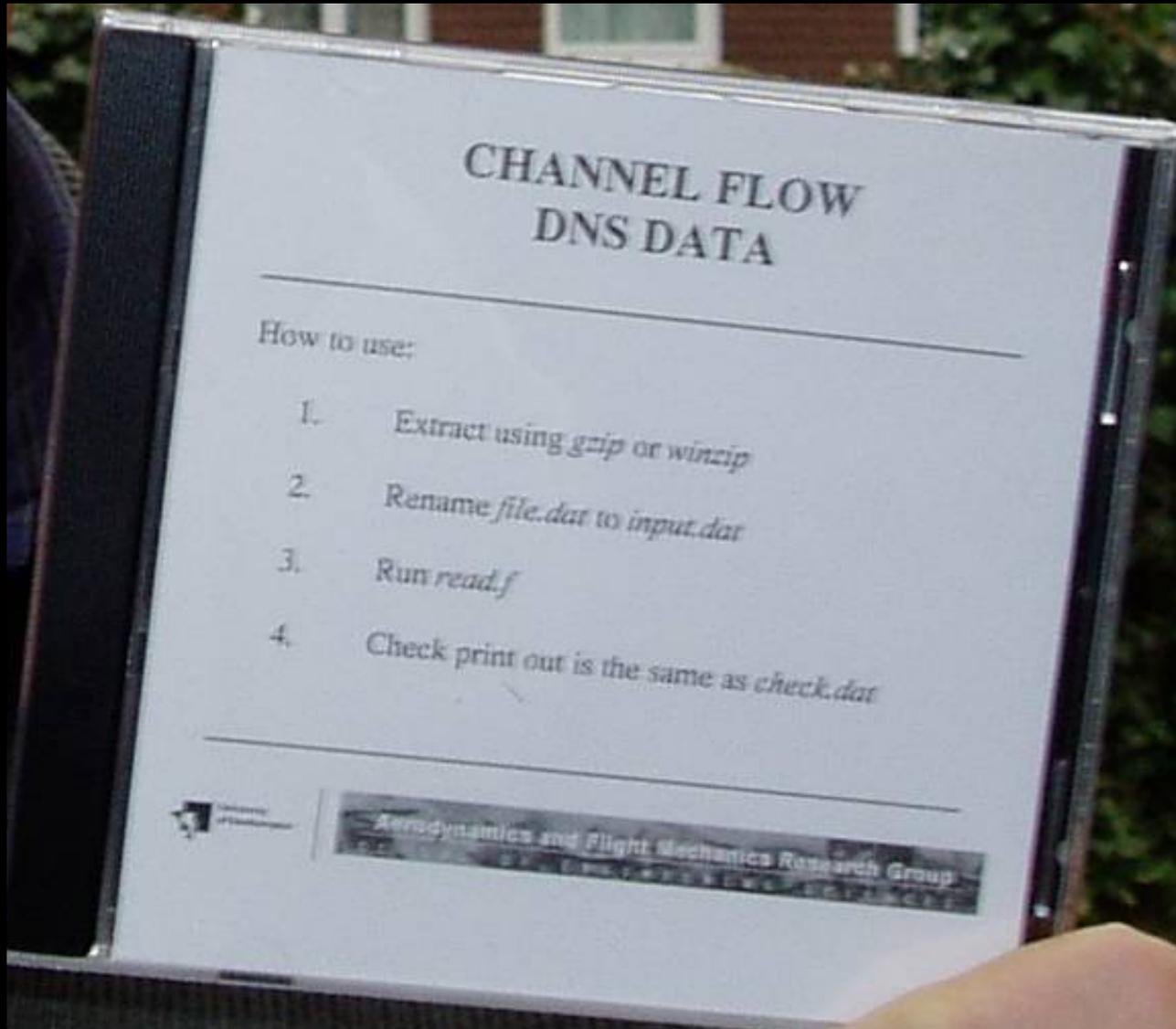
Researchers using DNS  
databases gain advantage

Full DNS  
database  
requires  
too much  
storage

25  
Tb



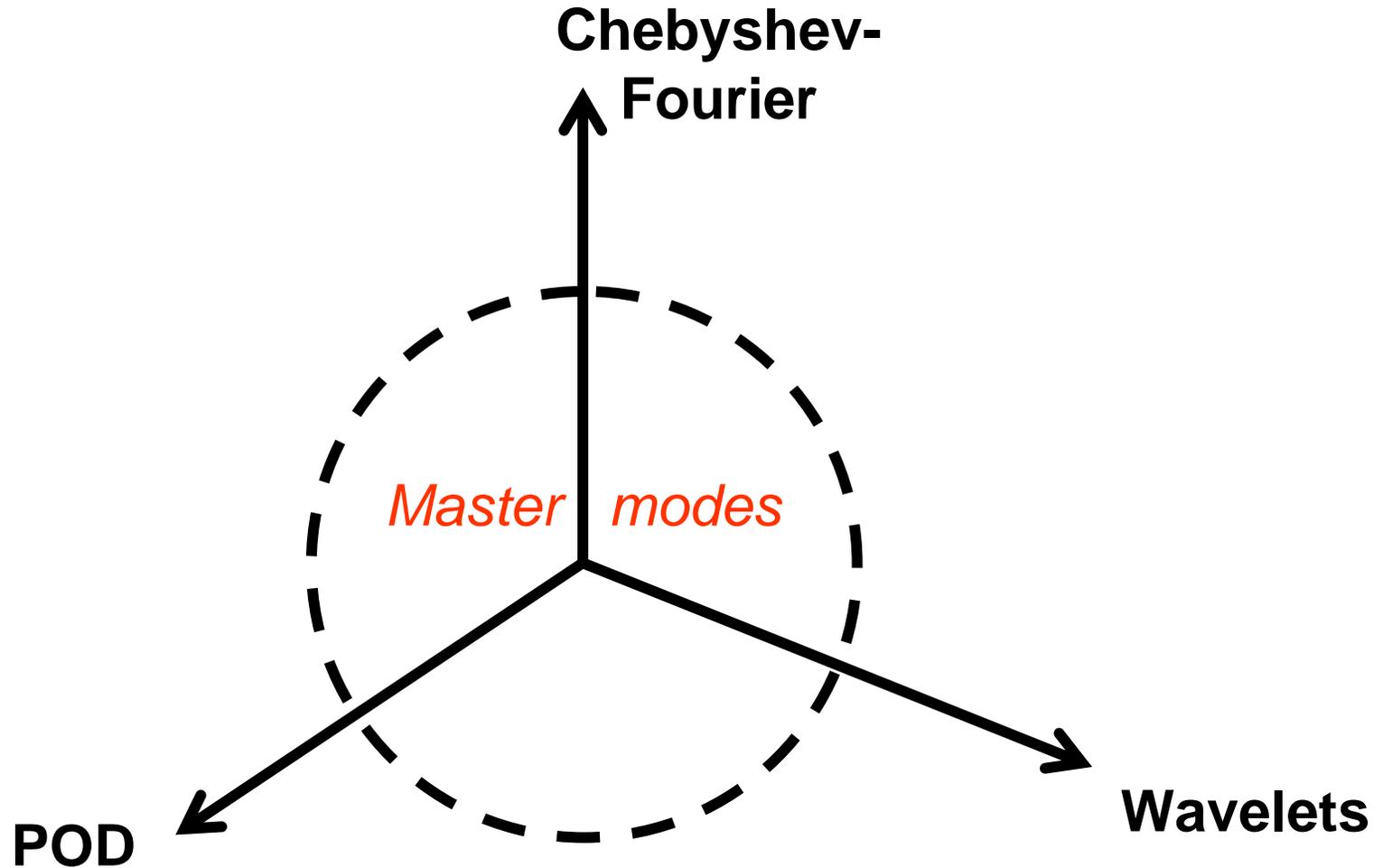
# Complete database can be small



**Storing only the master-mode set  
solves the problem**



# What master-modes are not



Master-mode set contains full information about the developed flow

$$\mathbf{u}_P = \sum_{\dot{c}=1}^{\infty} \left[ \mathbf{T}_{\dot{c}}(P) \phi_{\dot{c}}(\mathbf{x}) \mathbf{L} \right] \quad \left| \quad \begin{array}{l} \mathbf{T}_{\dot{c}}(P) = \mathbf{T}_{\dot{c}}(P) \mathbf{L} \\ 1 \leq \dot{c} \leq \infty \end{array} \right.$$

$$\mathbf{u}_T = \sum_{\dot{c}=1}^{\infty} \mathbf{T}_{\dot{c}}(P) \phi_{\dot{c}}(\mathbf{x}) \quad \left| \quad \begin{array}{l} 1 \leq \dot{c} \leq \infty \end{array} \right.$$


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$$\|\mathbf{u}_P - \mathbf{u}_T\| \rightarrow 0 \quad \text{as} \quad P \rightarrow \infty$$

**In numerical calculations master-mode sets are somewhat different**

Time marching is done by applying a recurrent formula to mode amplitudes

$$\mathbf{u}_W(\mathbf{x}) = \sum_{\zeta=1}^c \hat{\mathbf{u}}_{W\zeta} \phi_{\zeta}(\mathbf{x}) \mathbf{L} \quad \mathbf{P} = \mathbf{P}_W$$

$$\hat{\mathbf{u}}_{W+1\zeta} = [ \zeta ( \hat{\mathbf{u}}_{W\zeta 1} \mathbf{L} \hat{\mathbf{u}}_{W\zeta 2} \mathbf{L} \mathbf{S} \mathbf{S} \mathbf{L} \hat{\mathbf{u}}_{W\zeta c} )$$

# Master-mode set definition

$$\mathbf{v}_W(\mathbf{x}) = \sum_{\dot{c}=1}^c \hat{\mathbf{v}}_{W|\dot{c}} \phi_{\dot{c}}(\mathbf{x})$$

$$M = \{ \dot{c} \mid \hat{\mathbf{v}}_{W|\dot{c}} \neq \mathbf{0} \}$$

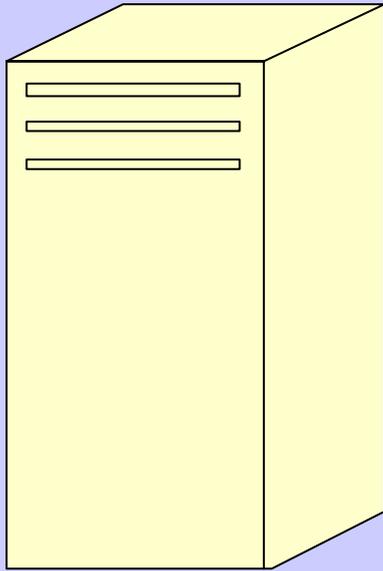
$$\hat{\mathbf{v}}_{W+1|\dot{c}} = \begin{cases} \hat{\mathbf{u}}_{W+1|\dot{c}} & \dot{c} \in M \\ [ \dot{c} (\hat{\mathbf{v}}_{W|\dot{c}} \mathbf{L} \mathbf{S} \mathbf{S} \mathbf{L} \hat{\mathbf{v}}_{W|\dot{c}}) ] & \dot{c} \notin M \end{cases}$$

$$\| \mathbf{v}_W(\mathbf{x}) - \mathbf{u}_W(\mathbf{x}) \| \rightarrow 0 \text{ as } W \rightarrow \infty \quad \forall \mathbf{v}_1(\mathbf{x})$$

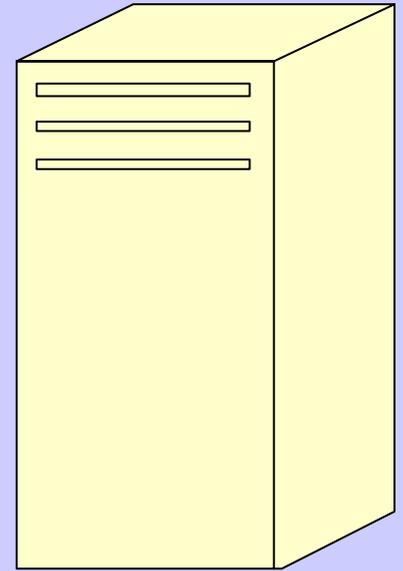
$\Leftrightarrow M$  is a master-mode set

**Master-mode set is not unique: the minimal one is of interest**

# Master code dictates master-modes to the slave code



$$\hat{\mathbf{u}}_{\text{WLe } 1} \text{LSSSL} \hat{\mathbf{u}}_{\text{WLe}} .$$

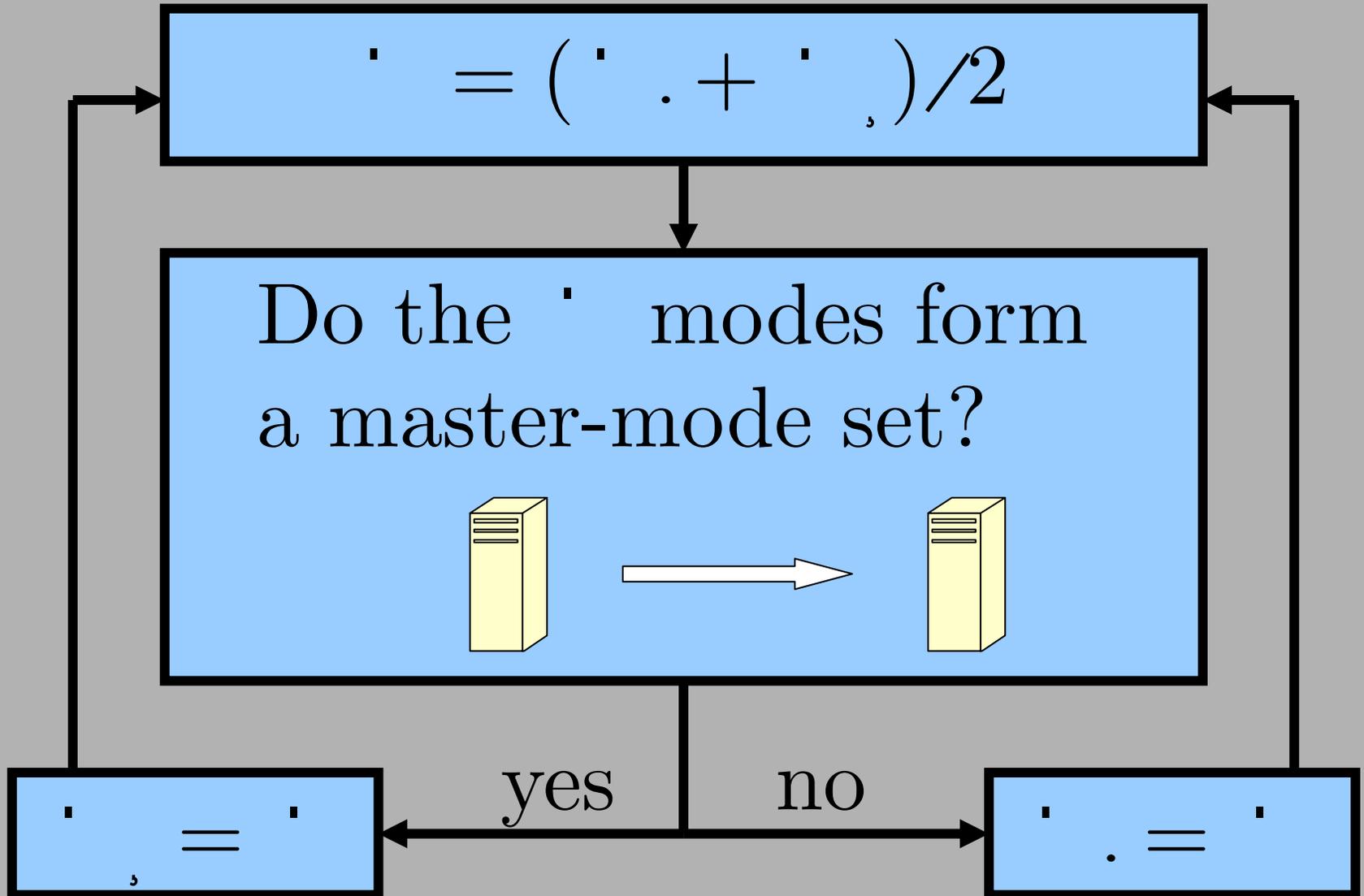


Master code

Slave code

$$\| \mathbf{u}_{\text{master}} - \mathbf{u}_{\text{slave}} \| \xrightarrow{?} 0$$

# Dichotomy is faster than trial-and-error



**Modes need to be ordered**

Olson and Titi (2003) ordered modes by wavenumbers

$$\mathbf{u} = \sum_{\hat{L}_- \hat{L}_\parallel} \hat{\mathbf{u}}(\hat{L}_- \hat{L}_\parallel) \#^W(\hat{L}_-^2 + \hat{L}_\parallel^2)$$

$$\hat{L}_-^2 + \hat{L}_\parallel^2 \text{ ( )}$$

Ordering by mean amplitude is similar to ordering by energy

$$\mathbf{u} = \sum_{\hat{L}_-, \hat{L}_+, \hat{L}_f} \hat{\mathbf{u}}_{\hat{L}_-, \hat{L}_+, \hat{L}_f} (\mathbf{P}) \#^{W(\hat{L}_-, \hat{L}_+, \hat{L}_f)} \dots_{\hat{L}_f} (\mathbf{f})$$

$$\langle \hat{c}_{\hat{L}_-, \hat{L}_+, \hat{L}_f}^2 \rangle > )$$

Can enstrophy-based ordering be better?

$$\boldsymbol{\omega} = \nabla \times \mathbf{u}$$

$$\boldsymbol{\omega} = \sum_{\substack{\hat{L}_- \hat{L}_\parallel \hat{L}_f}} \hat{\omega}_{\hat{L}_- \hat{L}_\parallel \hat{L}_f} (\mathbf{P}) \#^{W(\hat{L}_- + \hat{L}_\parallel)} \dots_{\hat{L}_f} (\mathbf{f})$$

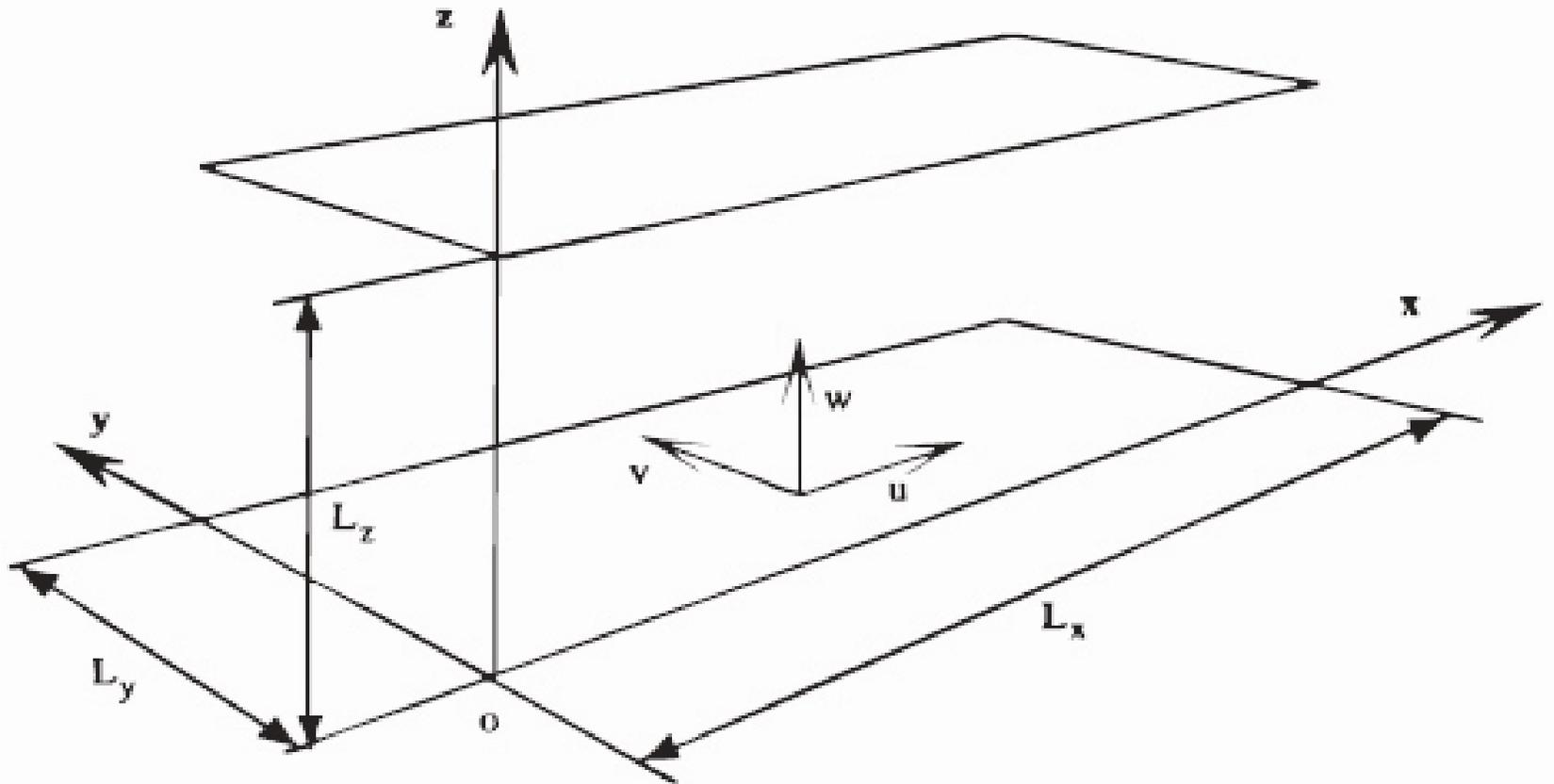
$$\langle \hat{\omega}_{\hat{L}_- \hat{L}_\parallel \hat{L}_f}^2 \rangle$$

# Ordering by mean amplitude is best

$$\# \#_K = 180L u_{\perp} \times u_{\parallel} \times u_{\text{f}} = 4 \times 3 \times 2$$

Method	
Wavenumber	$K > 5000$
Energy-based	$2800 < K < 3500$
Enstrophy-based	$K \sim 4800$

3D channel flow has a master-mode set



# Our code works

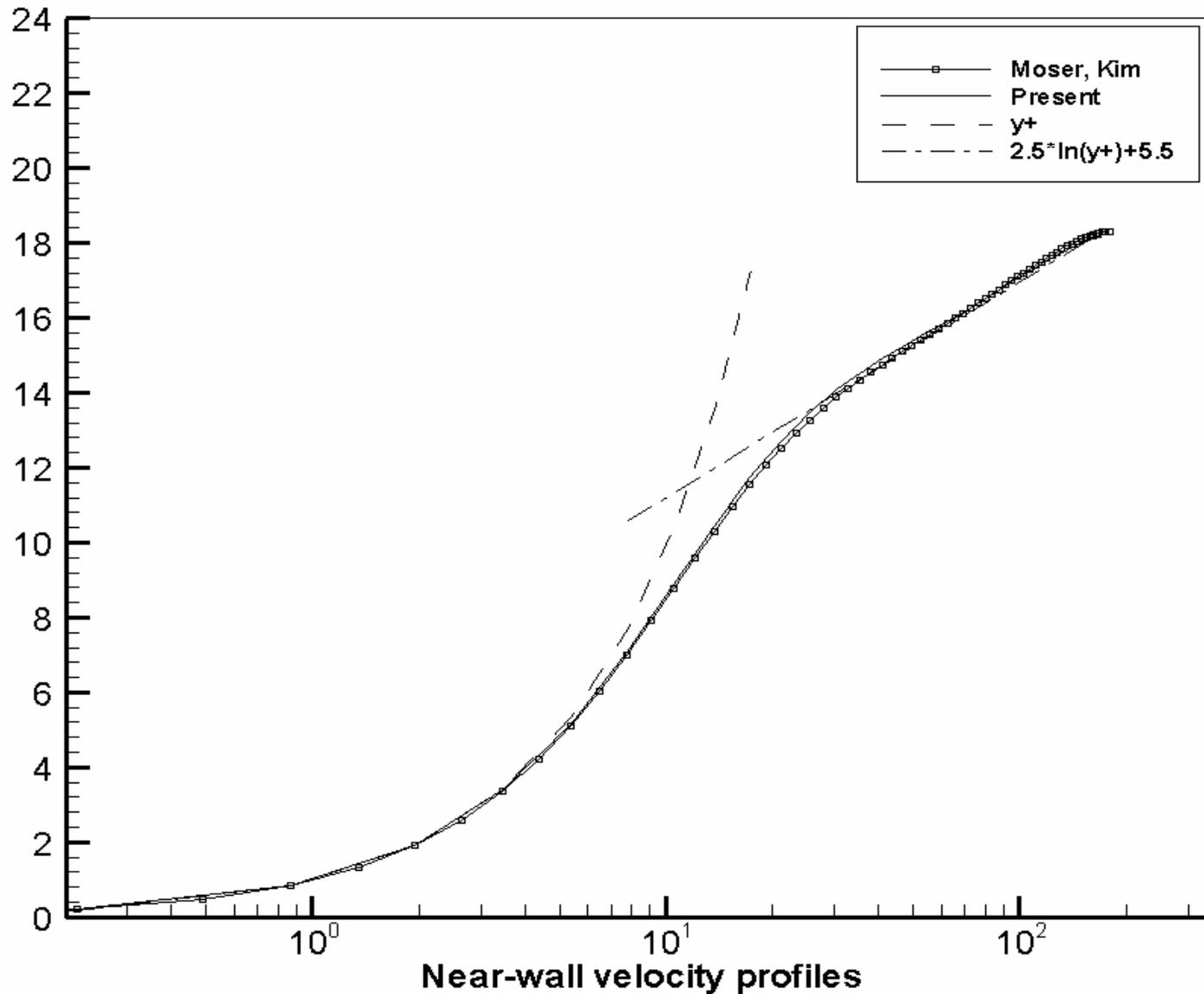
- Sandham and Howard pseudo-spectral code is reliable
- We test codes by adding a body force
- Standard comparisons were also made

We test codes by adding a body force

$$\frac{H\dot{\zeta}}{HP} - B\Delta\dot{\zeta} + (\dot{\zeta} \cdot \nabla\dot{\zeta}) + \nabla^{\sim} = w$$

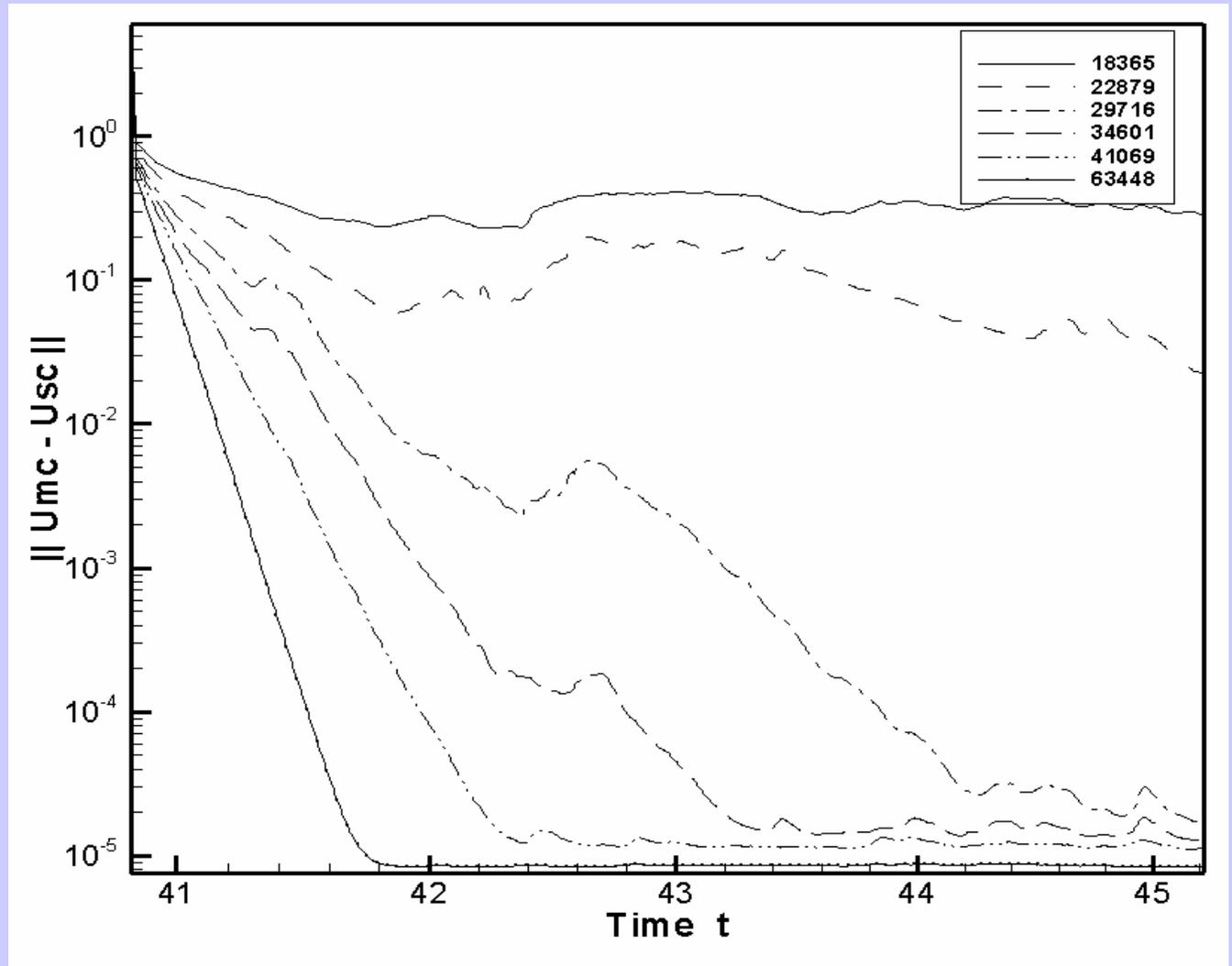
$$\nabla \cdot \dot{\zeta} = 0 \quad \underline{\quad} \in \Omega$$

# Standard comparisons were also made



# Slave solution converges to the master solution

$\ddagger \#_K = 360$



$6 \times 3 \times 2$  box

$\ddagger \sim 30000$

Master-mode set size is close to the attractor dimension

$$\} \%_{PP}, \sim 10^{-6} u_{-} u_{\parallel} Re^{9/4}$$

$$\approx 20000$$

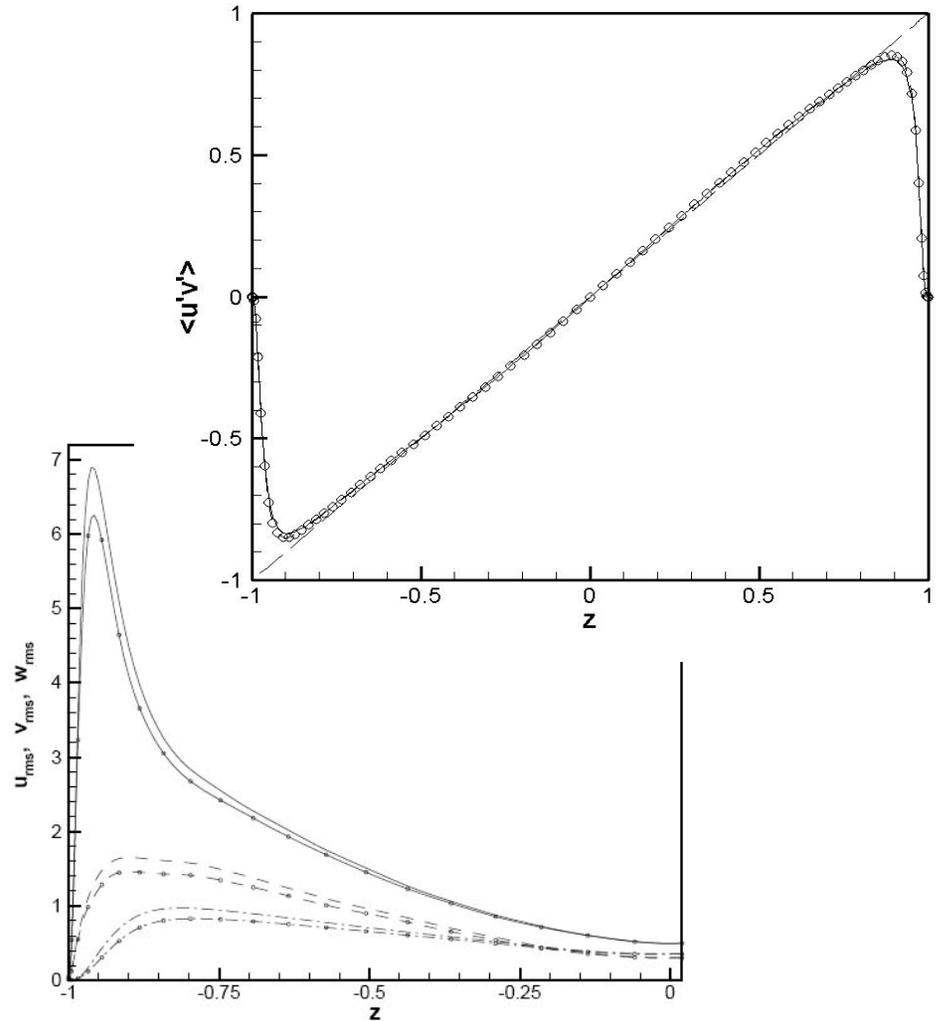
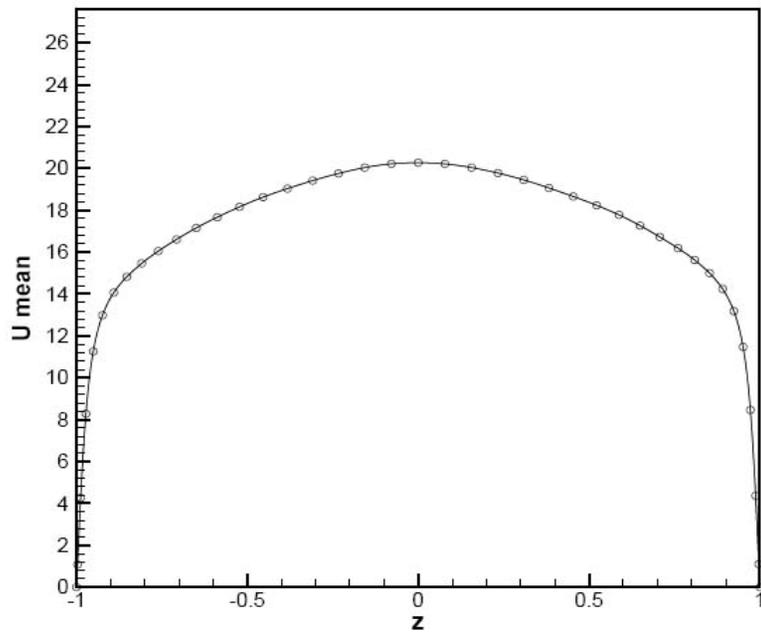
Re based on mean velocity and channel width

# Master-mode-based database is 100 times smaller

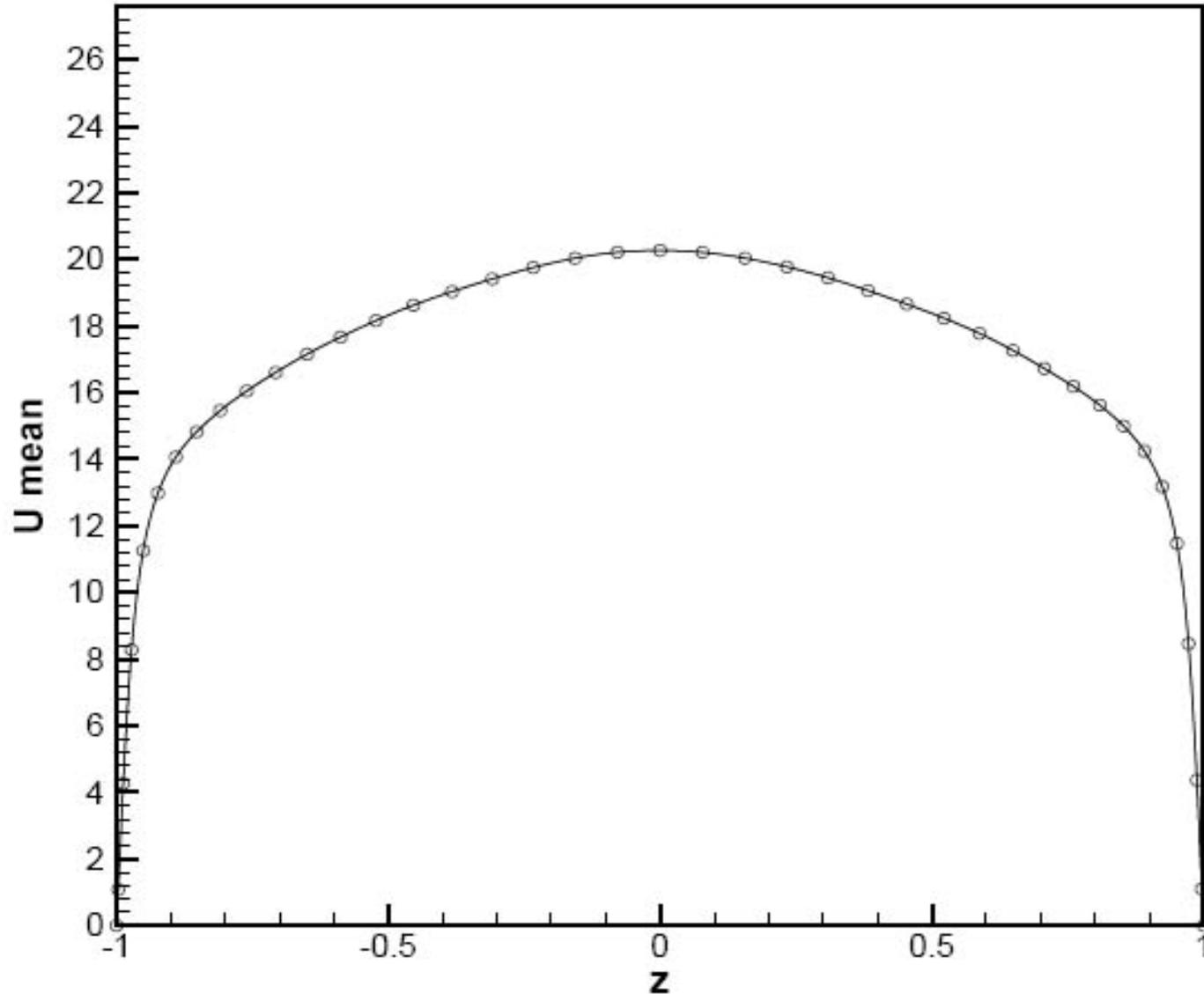
- We had 2621440 modes in total
- Master-mode set size is less than 30000
- ~1%!

**Master-modes alone provide a decent approximation**

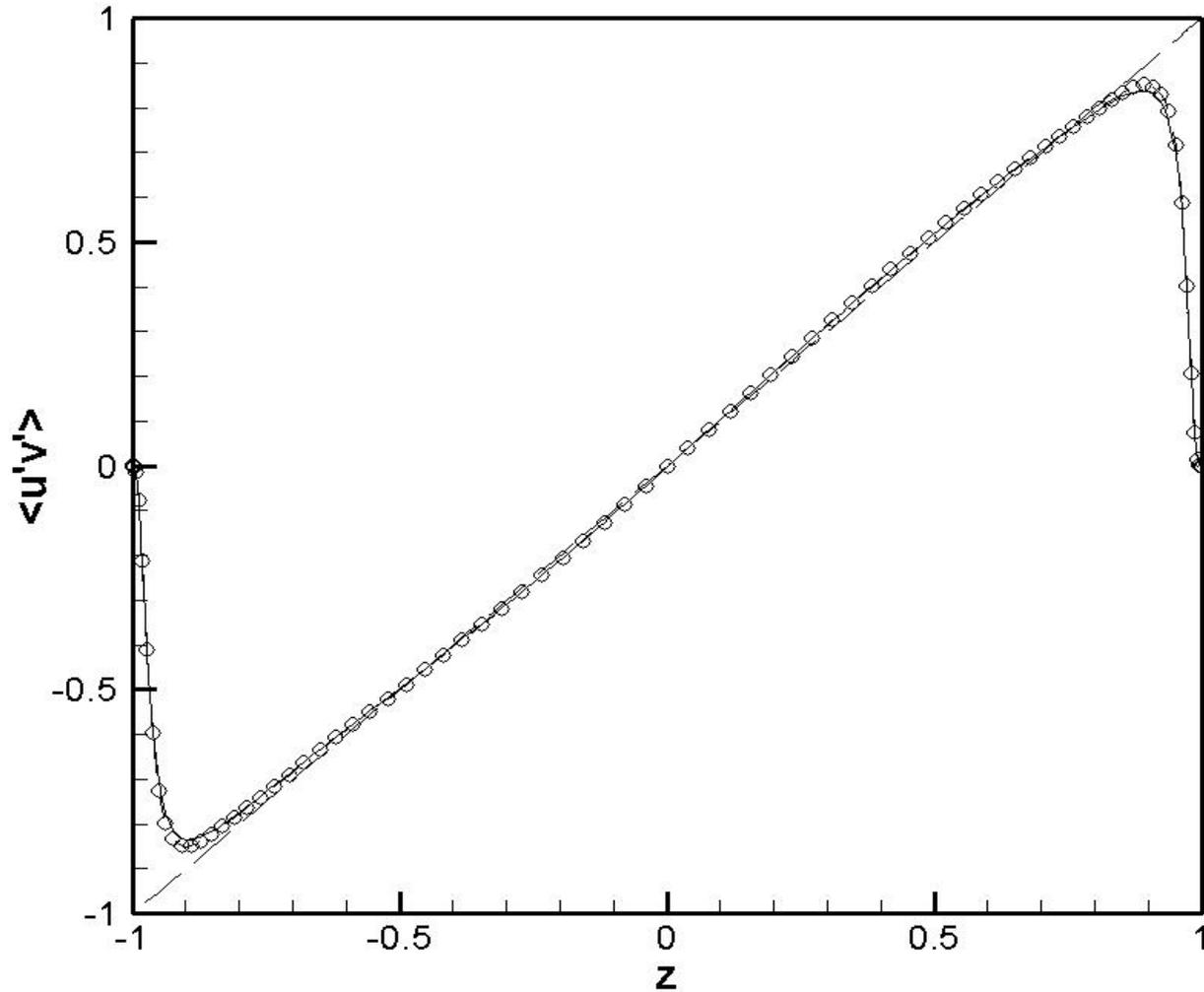
# Many mean quantities are within a few per cent



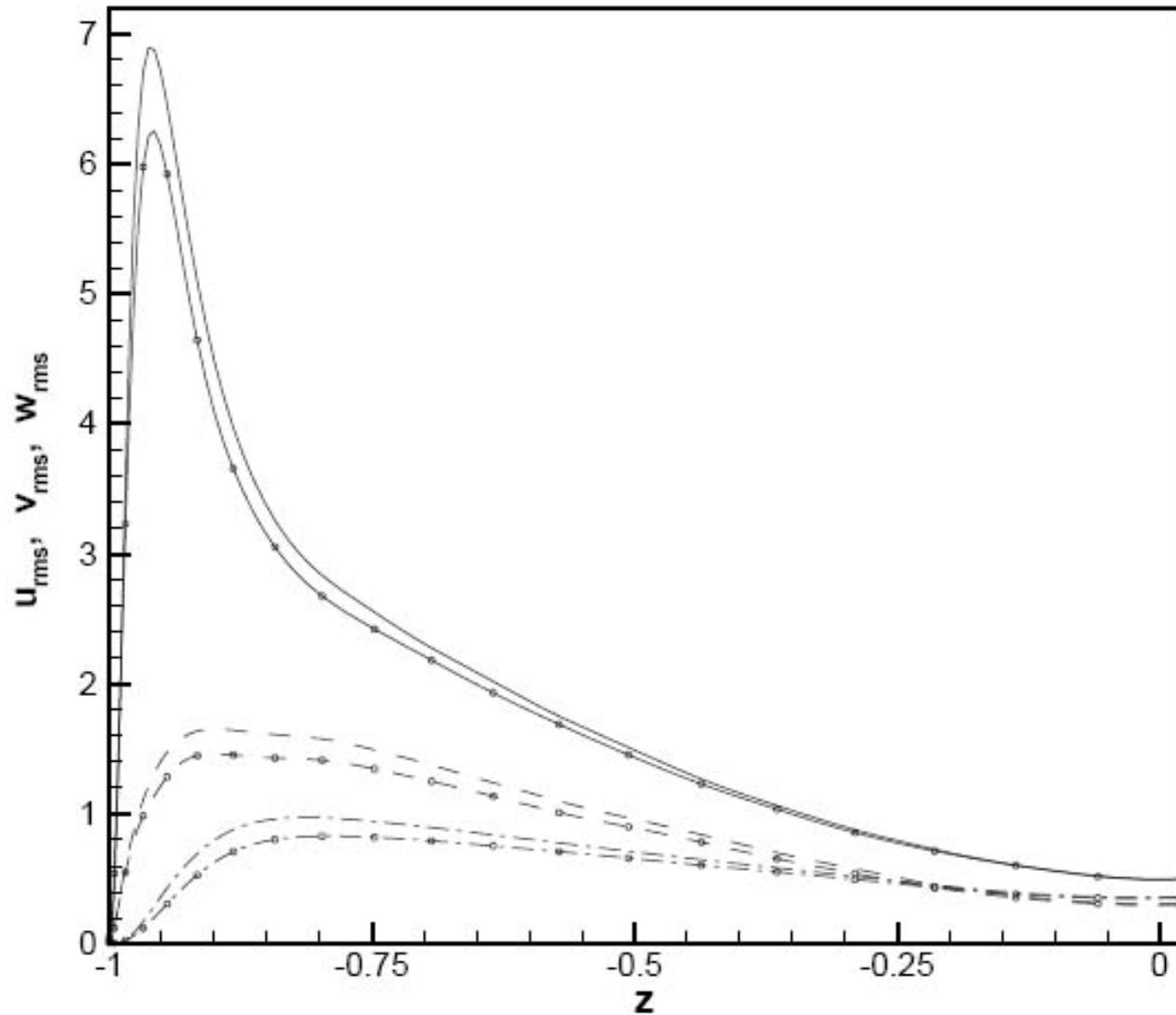
# Mean velocity is reproduced closely



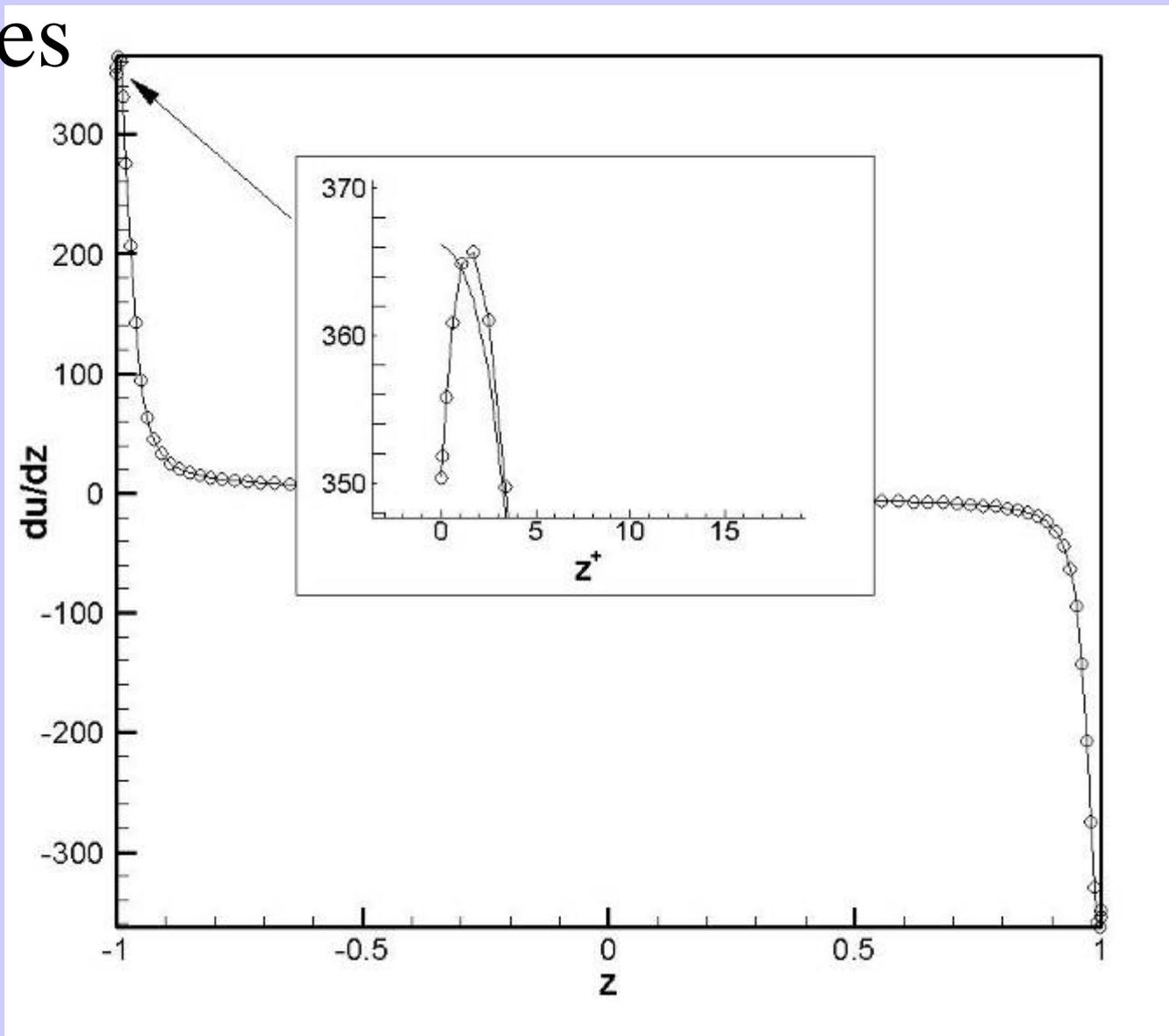
Shear stress is reproduced closely, too



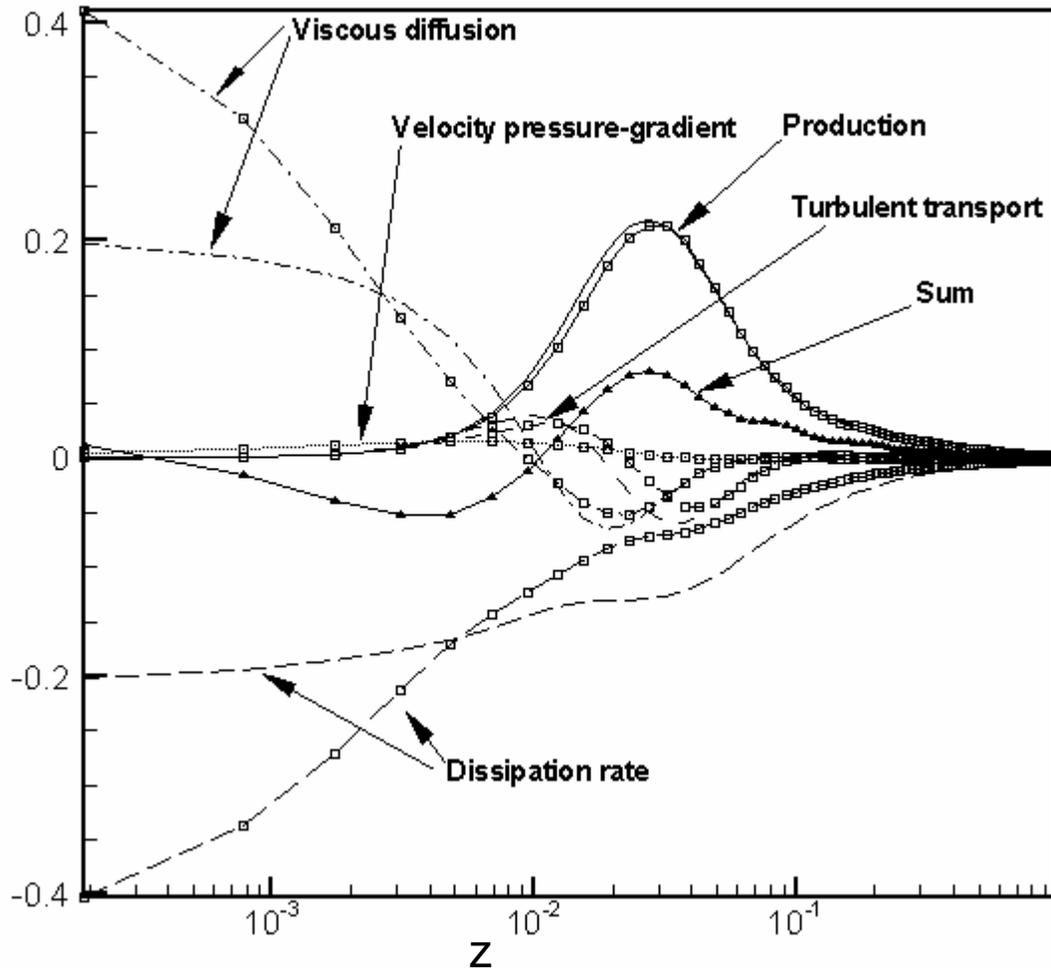
Fluctuation intensity is reasonably good



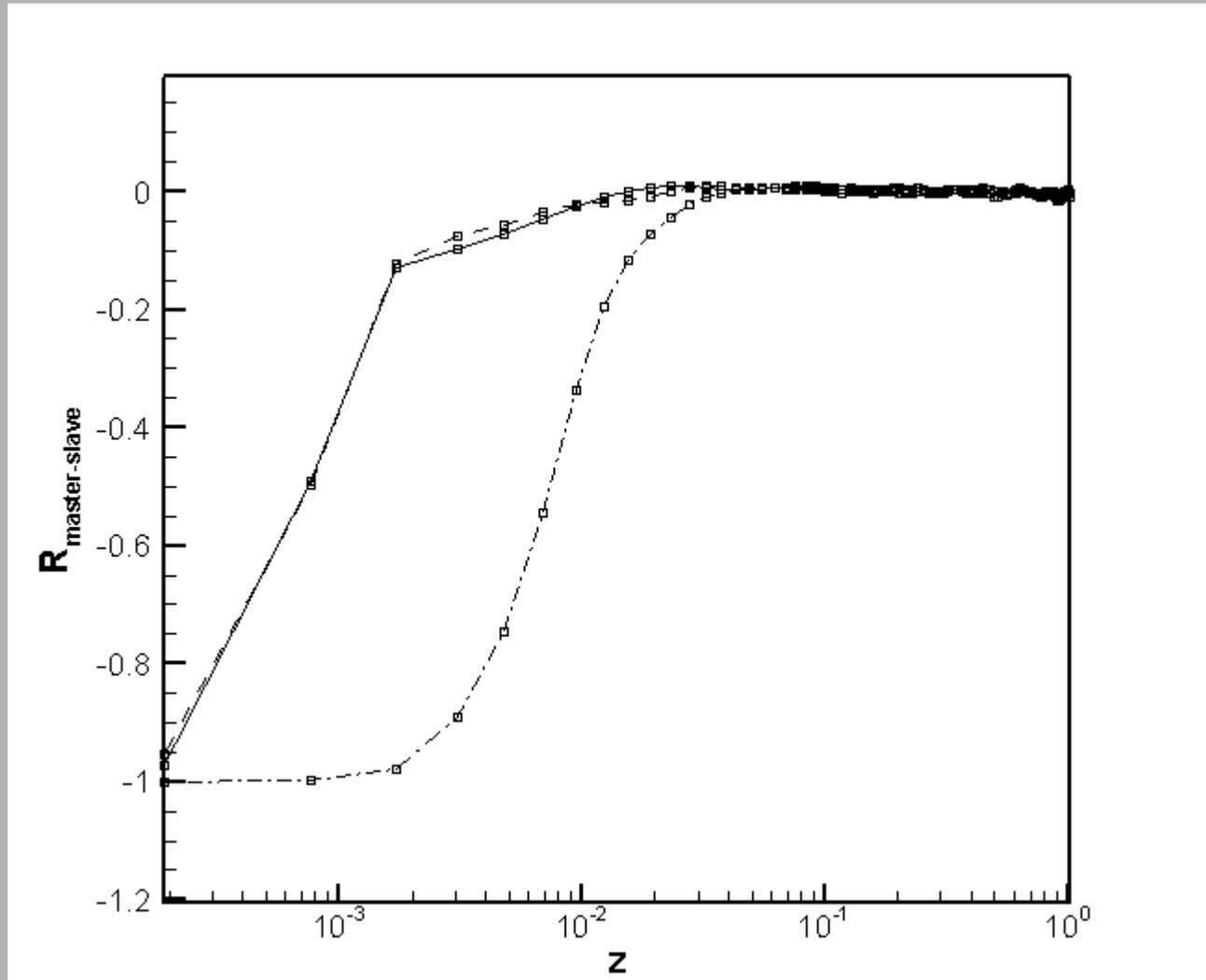
Near the wall there is a problem with derivatives



# Near the wall turbulence energy balance is not reproduced



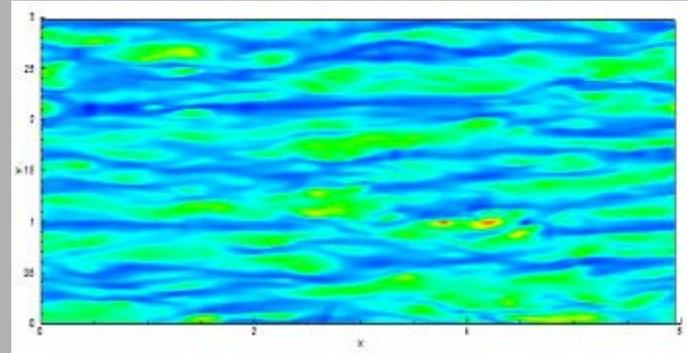
Near the wall master-slave correlation is high



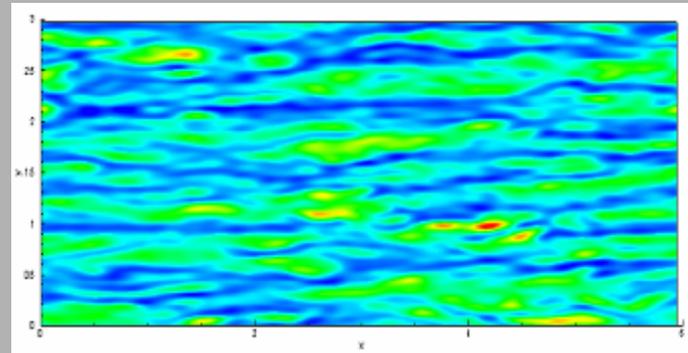
Streaks and vortices largely remain in  
master part

# Streaks are in the master-mode set

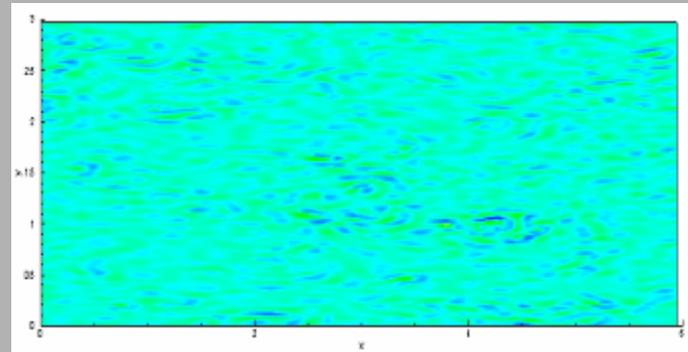
Full



Master

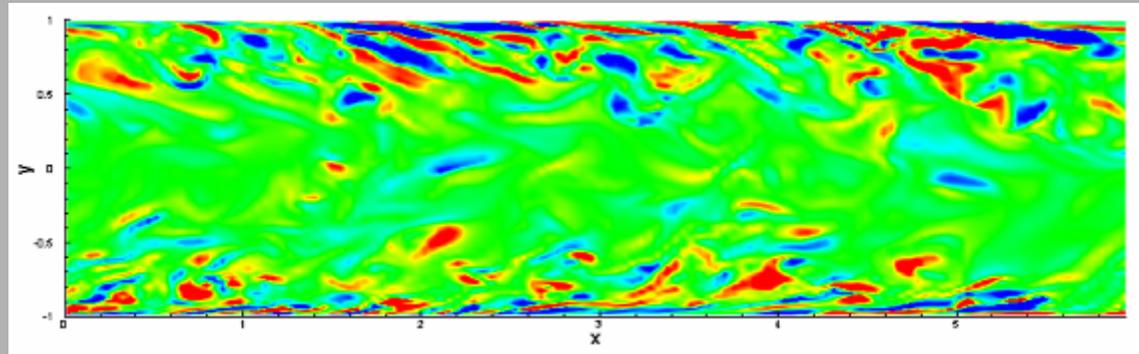


Slave

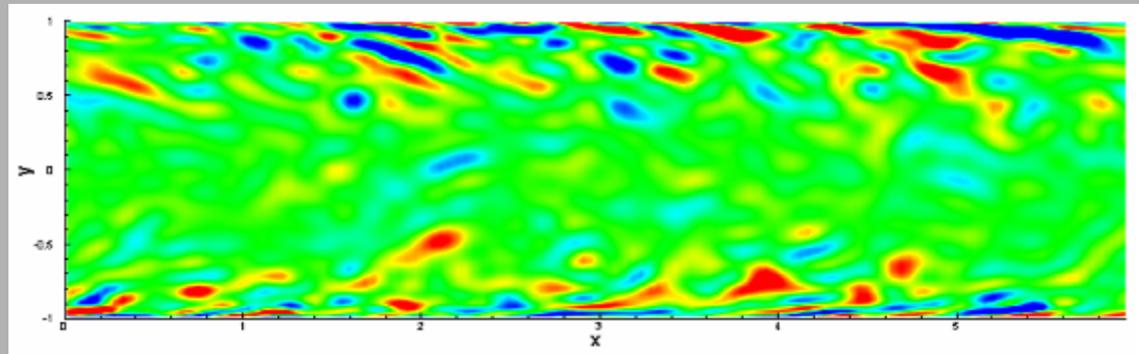


# Vortices are mostly in master-mode set

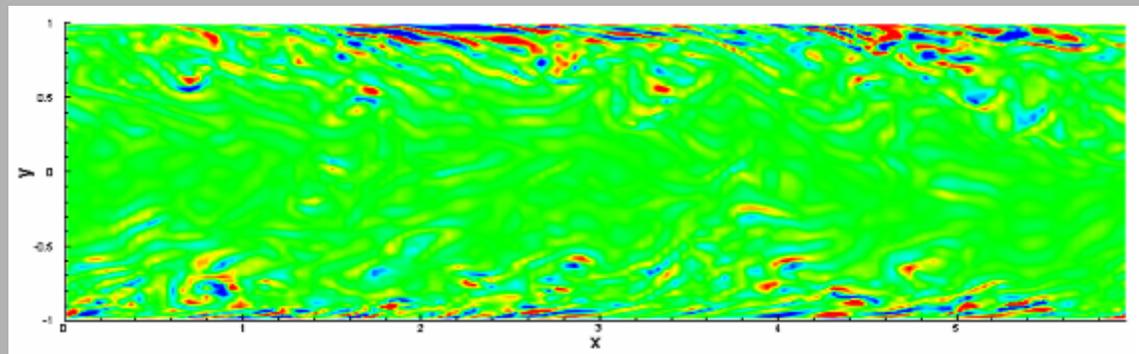
Full



Master



Slave



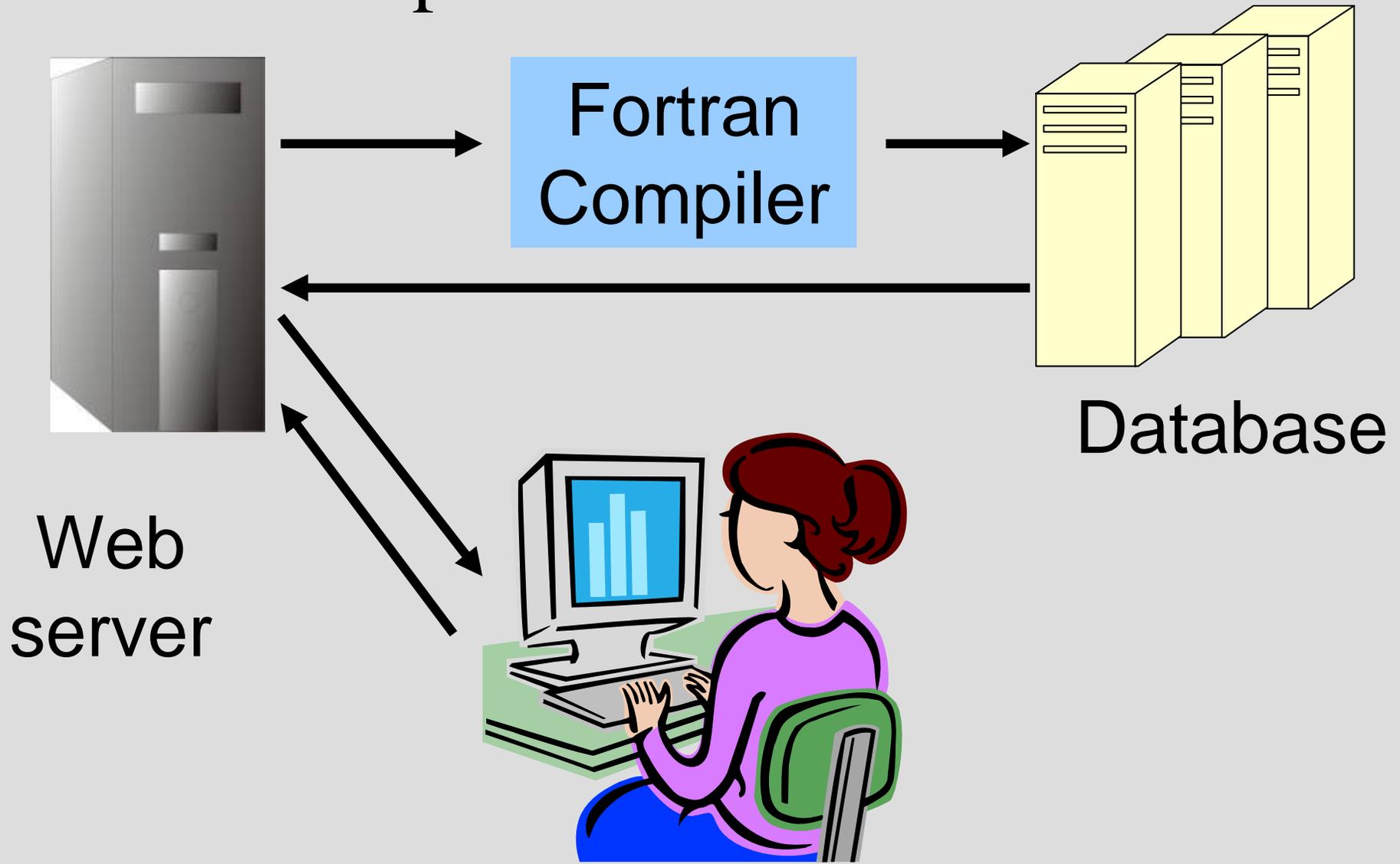
Master-mode database is best suited  
for catching rare events

Can you have access to the database?  
Right now, today?

One could use a full database but they are too large!



# Storing only the master-mode set solves the problem



# Master-mode database is available online

The screenshot shows a Microsoft Internet Explorer browser window with the following details:

- Title Bar:** DNS - Microsoft Internet Explorer
- Menu Bar:** File, Edit, View, Favorites, Tools, Help
- Address Bar:** http://www.dnsdata.afm.ses.soton.ac.uk/
- Search Bar:** Google G
- Navigation Bar:** Home, Login, Cases, Applications, Web server, Help, Contact us
- Page Content:**
  - Left Sidebar:** Aerodynamics and Flight Mechanics Research Group, University of Southampton logo, and a list of links: Home, Login, Profile, Cases, Applications, Web server, Help, Forum, Contact us, Logout.
  - Main Content:**

## DNS Database of Turbulent Flows

Welcome to the DNS database of turbulent flows at the Aerodynamics and Flight Mechanics (AFM) Research Group, School of Engineering Sciences, University of Southampton. Data on this server are provided free for academic usage by different contributors. Users are advised to check related paper for simulation details and numerical methods, and should cited listed references to give credit to the contributors.

Apart from ascii files for turbulence statistics, binary files for full three-dimensional flowfields and other data have also been archived. The web-based service built on this server allows users manipulate those files with own code. Currently only FOTRAN90 code (.f90) can be uploaded and run.
- Status Bar:** http://www.dnsdata.afm.ses.soton.ac.uk/index.php

Join us online in using master-modes

**Welcome**

to the

**University of Southampton**  
**Highfield Campus**

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