Turbulent channel flow
–
DNS with conjugate heat transfer

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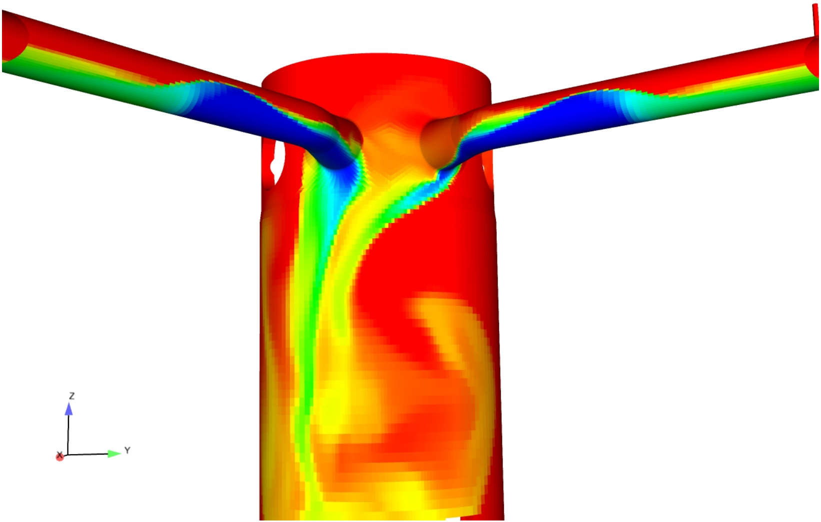
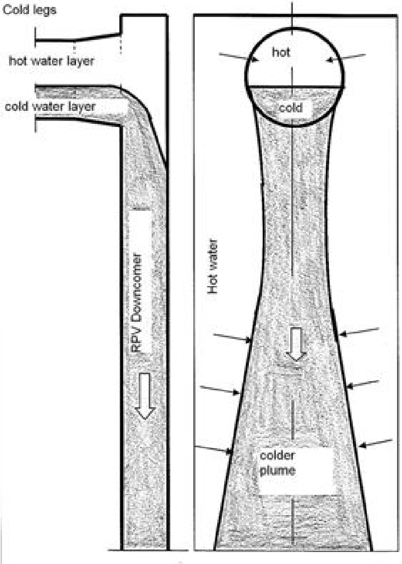
Table of contents

EDF R&D team : RPV lifespan



PTS: a scientific challenge



A multidisciplinary effort:

- Thermal analysis
- Hydraulic analysis
- Neutron field calculations
- Structural analysis

Table of contents

Wall-normal diffusive term semi-implicit

- Compact finite difference scheme

$$
\begin{aligned}
\alpha f''_{i-1} + f''_i + \alpha f''_{i+1} &= a\frac{f_{i-1} - 2f_i + f_{i+1}}{h^2} \\
&+ b\frac{f_{i-2} - 2f_i + f_{i+2}}{4h^2} \\
&+ c\frac{f_{i-3} - 2f_i + f_{i+3}}{9h^2}
\end{aligned}
$$

Wall-normal diffusive term semi-implicit

- Compact finite difference scheme
- Tri-diagonal **M** $\Rightarrow$ LU algorithm
- Septa-diagonal **B**

$$\begin{aligned}\alpha f''_{i-1} + f''_i + \alpha f''_{i+1} &= a\frac{f_{i-1} - 2f_i + f_{i+1}}{h^2} \\ &+ b\frac{f_{i-2} - 2f_i + f_{i+2}}{4h^2} \\ &+ c\frac{f_{i-3} - 2f_i + f_{i+3}}{9h^2}\end{aligned}$$

$$\mathbf{M}\partial_{yy} f = \mathbf{B} f$$

Wall-normal diffusive term semi-implicit

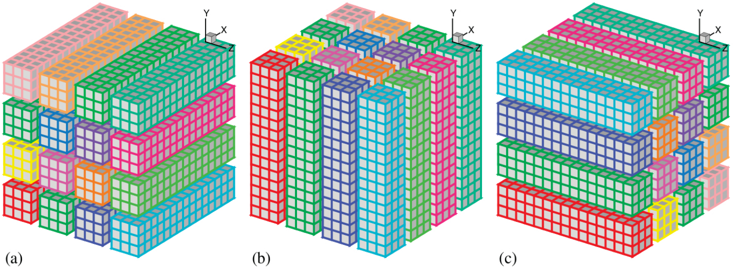
- F_i : convection + x & z diffusion

$$
\begin{aligned}
\frac{u_i^* - u_i^n}{dt} &= \frac{\partial_{yy} u_i^* + \partial_{yy} u_i^n}{2Re} + \frac{3}{2}F_i(u^n) - \frac{1}{2}F_i(u^{n-1}) - \partial_i p^n \\
\left(\frac{2Re}{dt} - \partial_{yy}\right) u_i^* &= r.h.s.(p^n, u^n, u^{n-1}) \\
\mathbf{M}\left(\frac{2Re}{dt} - \partial_{yy}\right) u_i^* &= \mathbf{M} r.h.s. \text{ and } \mathbf{M}\partial_{yy} f = \mathbf{B} f \\
\left(\mathbf{M}\frac{2Re}{dt} - \mathbf{B}\right) u_i^* &= \mathbf{M} r.h.s.
\end{aligned}
$$

- Septa-diagonal left hand side ⇒ LU algorithm

Solid thermal diffusion

- Solid on top ($y < 0$) and bottom ($y > L_y$)
- Fluid & solid : same grid in x and z
- Solid : Chebyshev grid in y



Solid thermal diffusion

Time and space discretization

- Chebyshev grid for $y \in [a, b]$ with $N + 1$ nodes :

$$y_i = \frac{a+b}{2} + \frac{b-a}{2} \cos\left(\frac{2(N-i)+1}{2N+2}\pi\right), 0 \leq i \leq N$$

- Semi-implicit in y, explicit in x and z :

$$\begin{aligned} \rho C_p \partial_t T_s &= \kappa \nabla_y^2 T_s + \kappa \nabla_{xz}^2 T_s \\ \frac{T_s^{n+1} - T_s^n}{dt} &= \frac{\kappa}{\rho c_p} \left(\gamma \nabla_y^2 T_s^{n+1} + (1-\gamma) \nabla_y^2 T_s^n\right) \\ &+ \frac{\kappa}{\rho c_p} \left(\frac{3}{2} \nabla_{xz}^2 T_s^n - \frac{1}{2} \nabla_{xz}^2 T_s^{n-1}\right) \end{aligned}$$

Solid thermal diffusion

- Lagrange interpolation on Chebyshev nodes :

$$
\begin{aligned}
l_i(y) &= \prod_{j=0, j\neq i}^{k} \frac{y - y_j}{y_i - y_j} \\
T_s(y) &= \sum_{i=0}^{k} T_s(y_i) l_i(y)
\end{aligned}
$$

- Diffusive term in $y \Rightarrow 2^{nd}$ derivative

$$
\begin{aligned}
\partial_{yy} T_s(y_j) &= \sum_{i=0}^{k} T_s(y_i) l_i''(y_j) \\
\partial_{yy} T_s &= \mathbf{M} T_s
\end{aligned}
$$

Solid thermal diffusion

- Non-compact finite difference schemes for x and z diffusion
- 6^{th} order, avoid global MPI-communication bottleneck

$$\frac{T_s^{n+1} - T_s^n}{dt} = \frac{\kappa}{\rho c_p}\left(\gamma \nabla_y^2 T_s^{n+1} + (1-\gamma)\nabla_y^2 T_s^n\right) + \frac{\kappa}{\rho c_p}\left(\frac{3}{2}\nabla_{xz}^2 T_s^n - \frac{1}{2}\nabla_{xz}^2 T_s^{n-1}\right)$$

$$\left(\frac{\rho C_p}{\kappa dt} - \gamma \mathbf{M}\right) T_s^{n+1} = r.h.s.(T_s^n, T_s^{n-1})$$

- Full $(N+1) \times (N+1)$ left hand side $\Rightarrow$ LAPACK

Conjugate heat transfer

- Fluid & Solid domain (G ratio of thermal diffusivity)

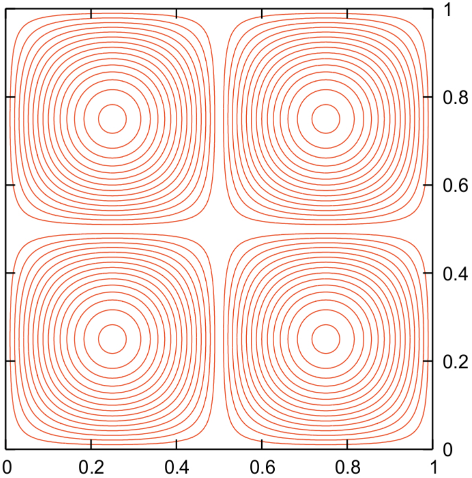
$$\begin{aligned} \partial_t T &= -\partial_i (T u_i) + \frac{1}{RePr} \nabla^2 T \text{ in } \Omega \\ \partial_t T_s &= \frac{1}{GRePr} \nabla^2 T_s \text{ in } \Omega_s \end{aligned}$$

- Fluid-solid interface (α ratio of thermal conductivity)

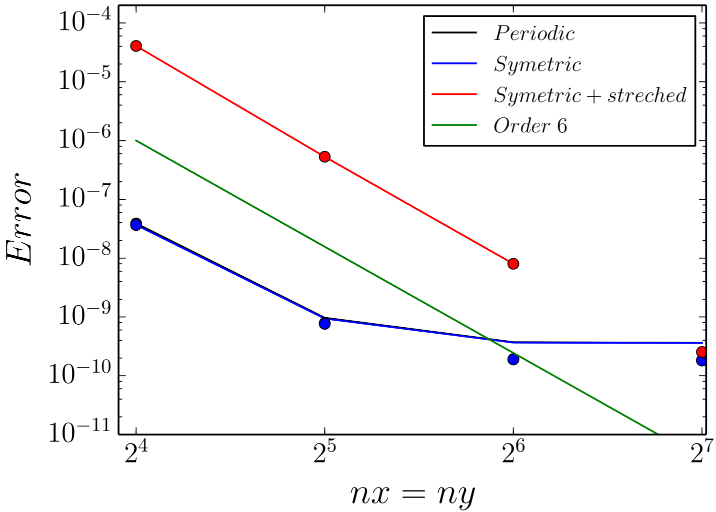
$$\begin{aligned} T &= T_s \text{ in } \partial\Omega \cap \partial\Omega_s \\ \alpha \partial_n T &= \partial_n T_s \text{ in } \partial\Omega \cap \partial\Omega_s \end{aligned}$$

- First T^{n+1}, Dirichlet : $T^{n+1} = \frac{T^n + T_s^n}{2}$ in $\partial\Omega \cap \partial\Omega_s$
- Then T_s^{n+1}, Neumann : $\partial_n T_s^{n+1} = \alpha \partial_n T^{n+1}$ in $\partial\Omega \cap \partial\Omega_s$

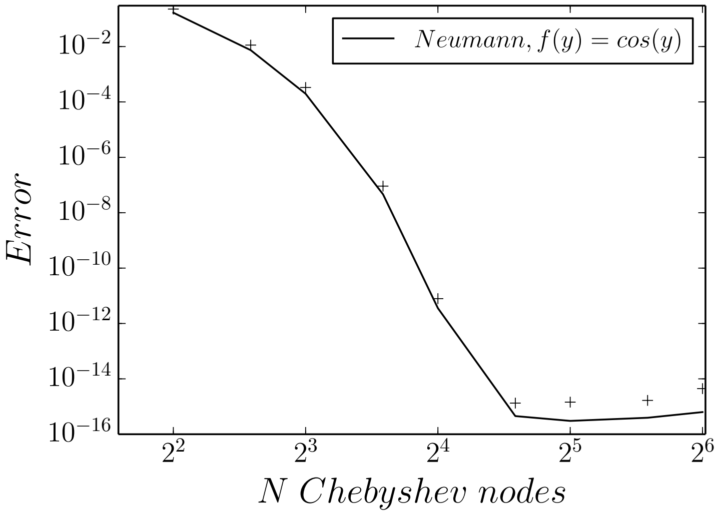
Analytical benchmark : Taylor-Green vortex



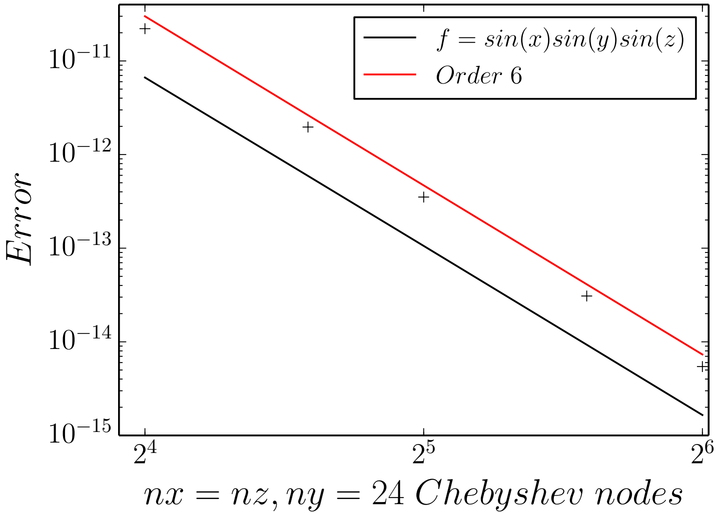
Analytical benchmark : Taylor-Green vortex



Analytical benchmark : solid thermal diffusion



Analytical benchmark : solid thermal diffusion

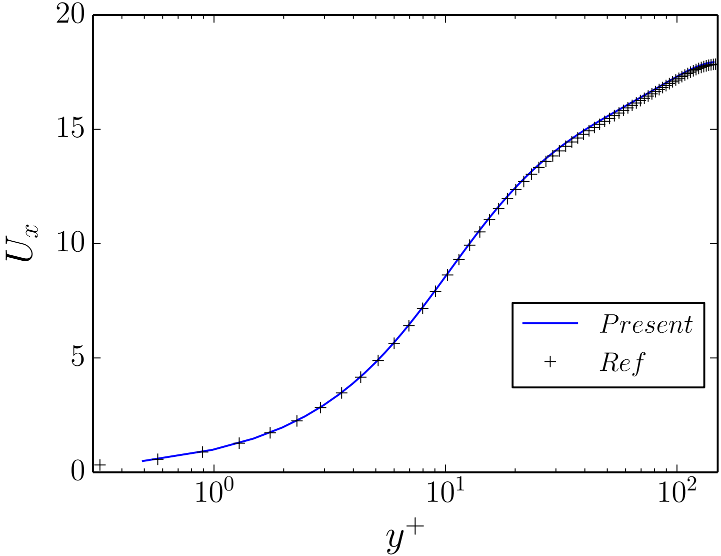


Academic benchmark : channel flow, $Re_\tau = 150$

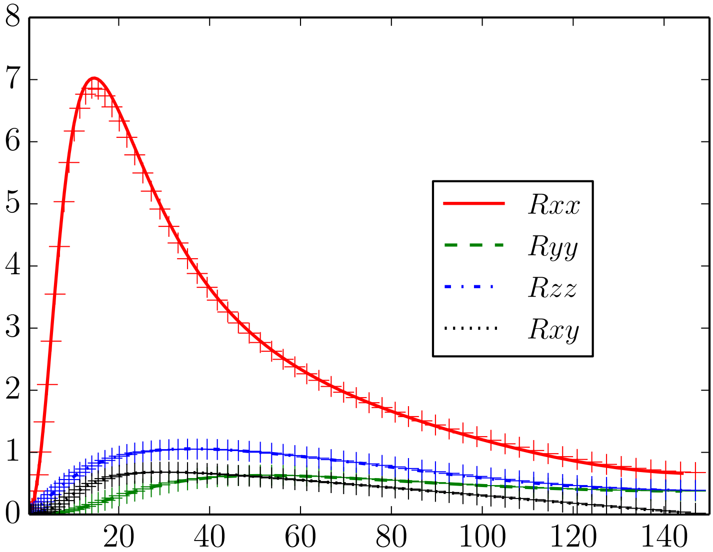
Numerical setup, Pr=0.71

	Present	Kasagi et al. (1991)
Domain	$[12.8; 2; 4.26]$	$[5\pi; 2; 2\pi]$
Grid	$[256; 193; 256]$	$[128; 97; 128]$
Re_τ	148.8	150
dy^+ $[min, max]$	$[0.49; 4.8]$	$[0.08; 4.9]$
$[dx^+, dz^+]$	$[7.4; 2.5]$	$[18.4; 7.36]$
dt^+ $(\frac{\nu}{u_\tau^2})$	2.10^{-4}	?
Final time	160	2100

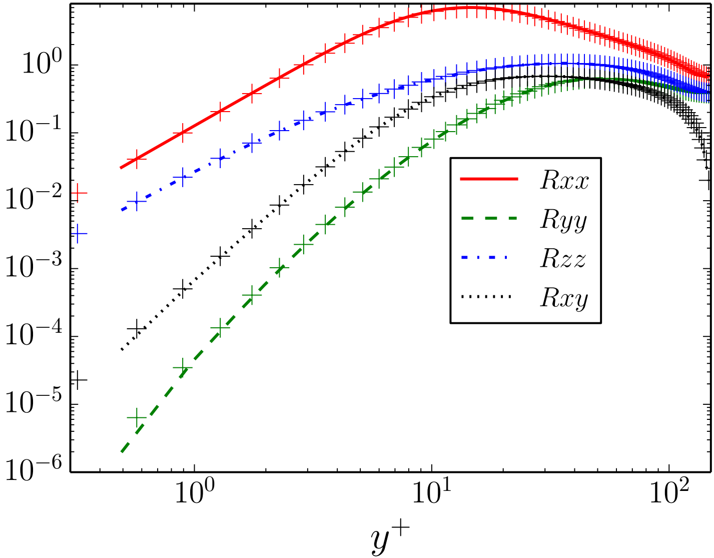
Academic benchmark : channel flow, $Re_\tau = 150$



Academic benchmark : channel flow, $Re_\tau = 150$



Academic benchmark : channel flow, $Re_\tau = 150$



Kasagi et al. (1991), Budget R_{xx}

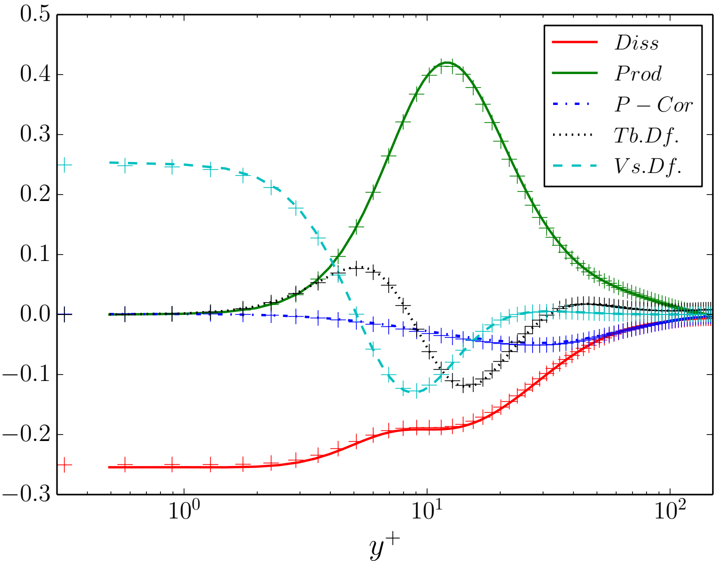
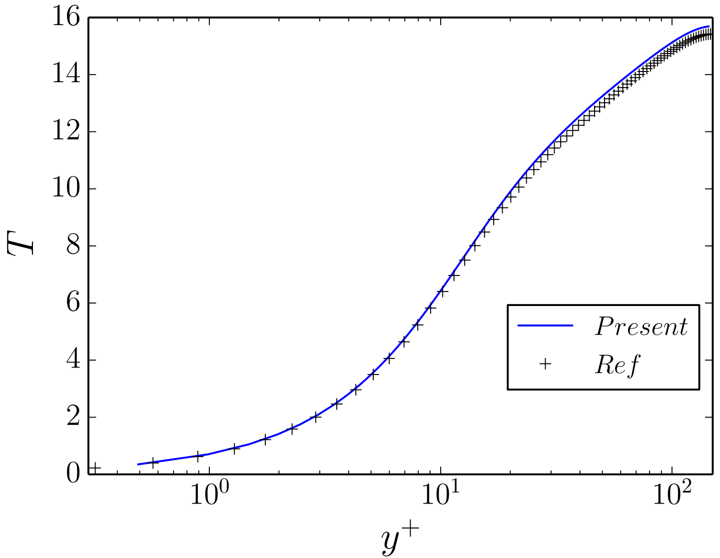
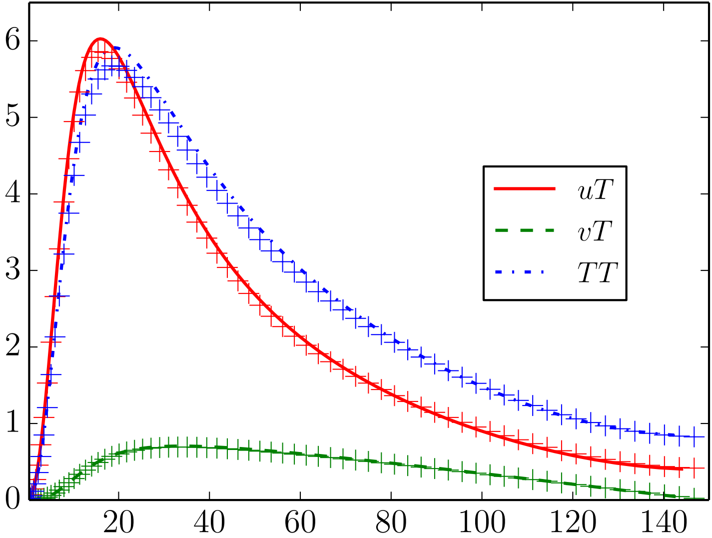


Table of contents

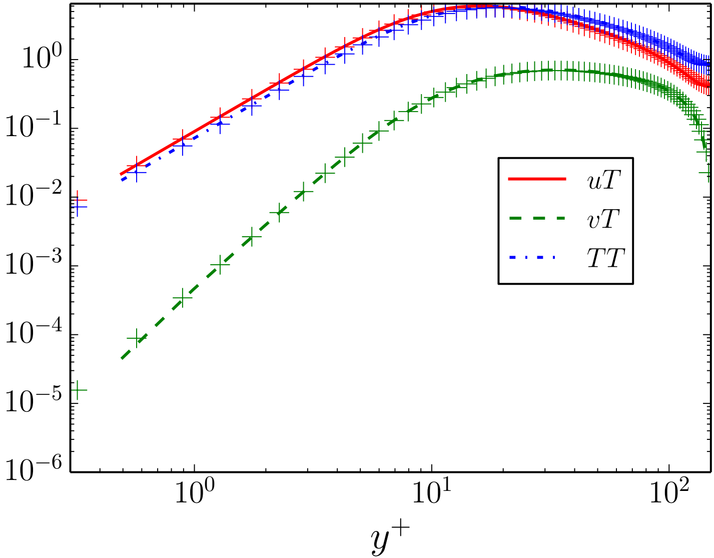
Channel flow, $Re_\tau = 150$, Kasagi et al. (1991)



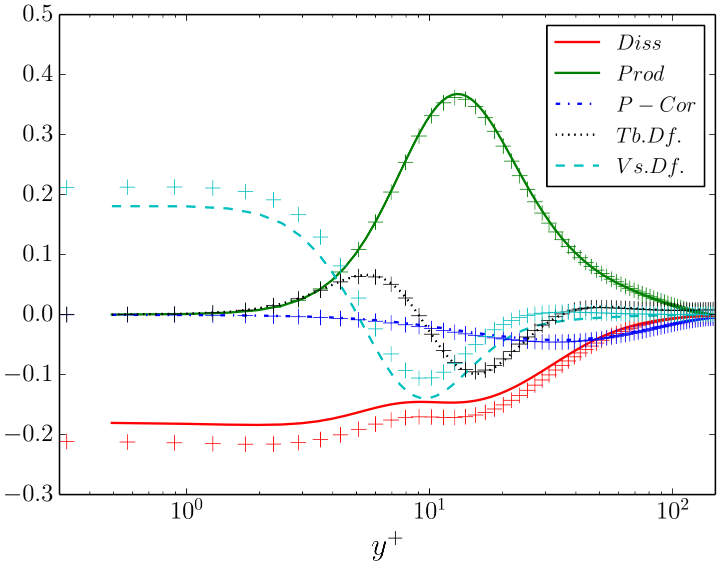
Channel flow, $Re_\tau = 150$, Kasagi et al. (1991)



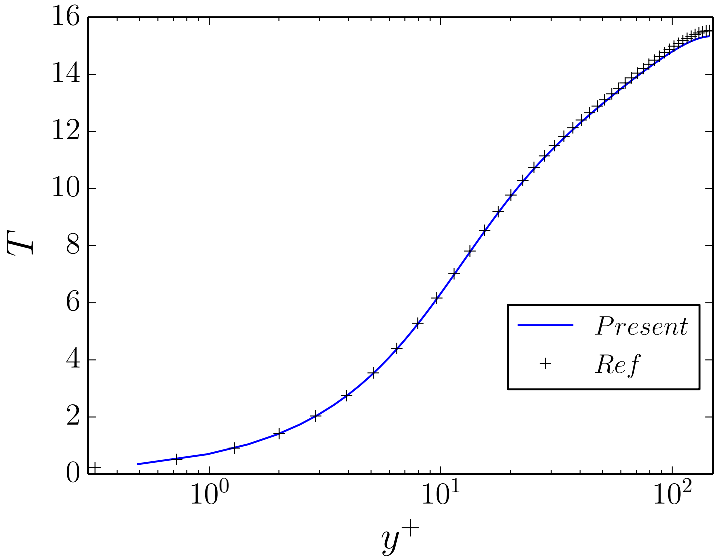
Channel flow, $Re_\tau = 150$, Kasagi et al. (1991)



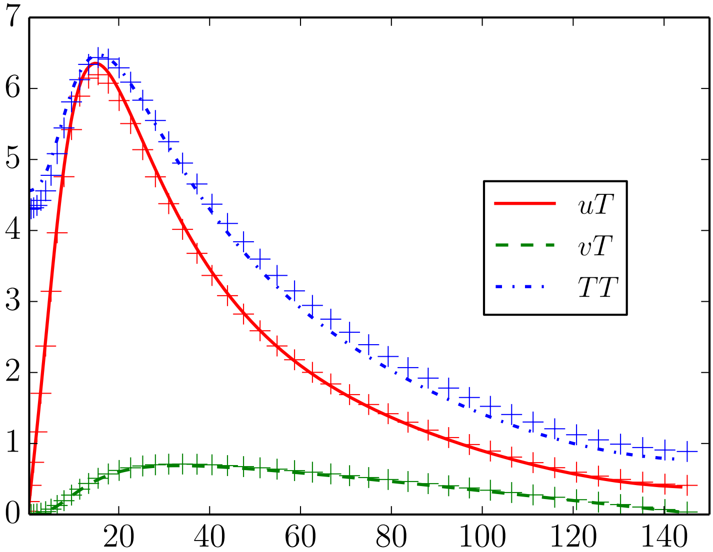
Kasagi et al. (1991), Budget *ut*



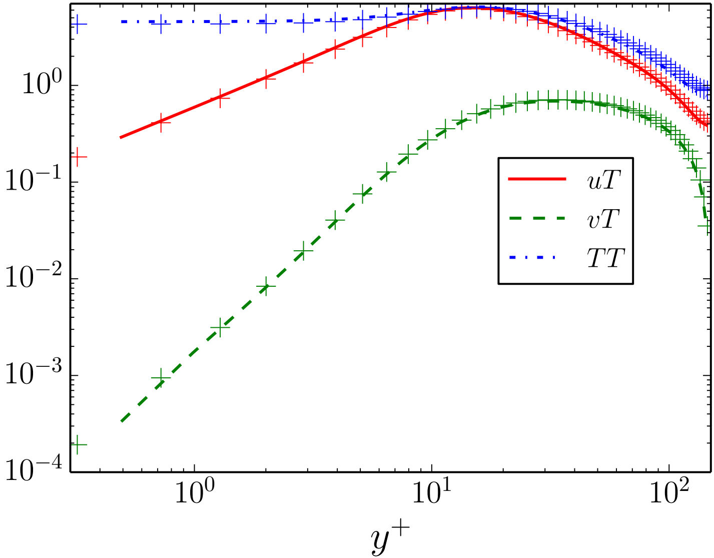
Channel flow, $Re_\tau = 150$, Tiselj et al. (2001)



Channel flow, $Re_\tau = 150$, Tiselj et al. (2001)



Channel flow, $Re_\tau = 150$, Tiselj et al. (2001)



Setup

- Solid domain (128 Chebyshev nodes)

$$-\delta < y_{solid} < 0 < \quad y_{fluid} \quad < 2\delta < y_{solid} < 3\delta$$

- Outer solid boundary condition : heat flux imposed
- Fluid & Solid diffusivity ($G = 1$)

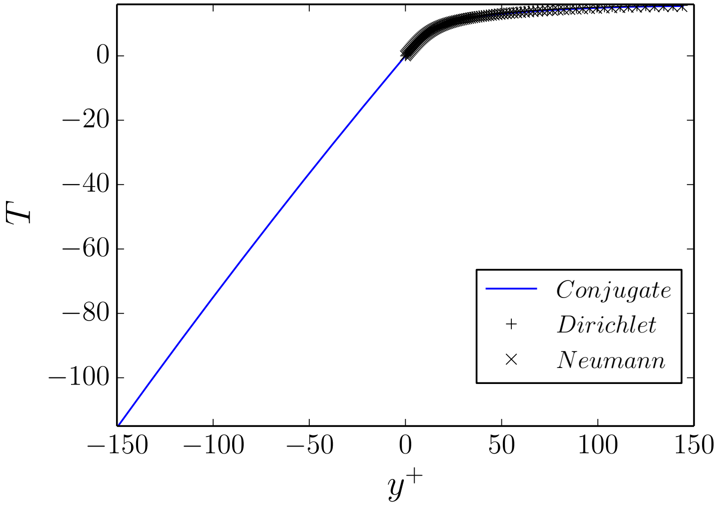
$$\partial_t T_s \quad = \quad \frac{1}{GRePr} \nabla^2 T_s \text{ in } \Omega_s$$

- Fluid & Solid conductivity ($\alpha = 1$)

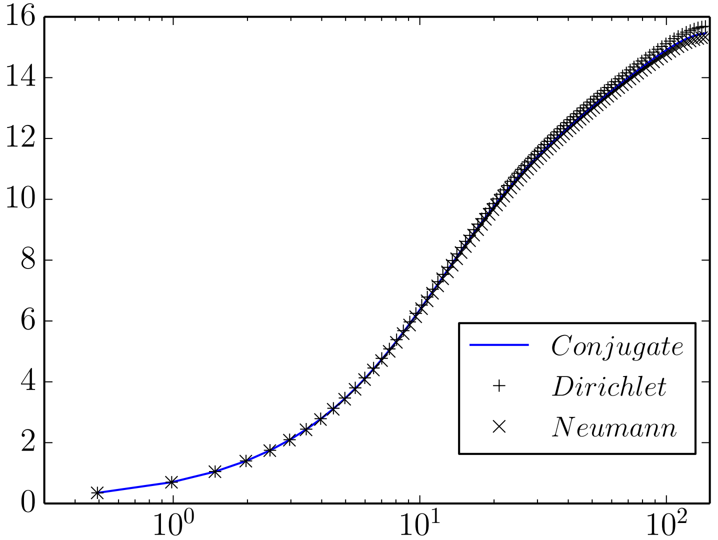
$$\alpha \partial_n T \quad = \quad \partial_n T_s \text{ in } \partial\Omega \cap \partial\Omega_s$$

- 3-way comparison : conjugate, iso-flux, iso-thermal

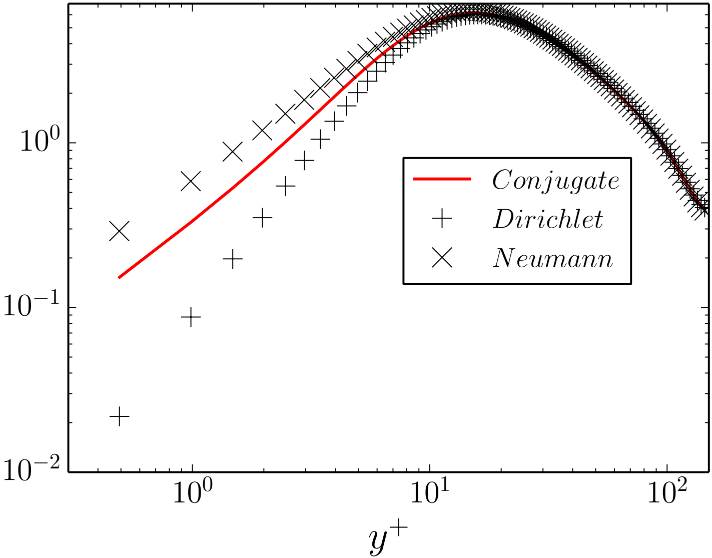
$< T >$


0
−20
−40
−60
−80
−100
T
−150
−100
−50
0
50
100
150
y+
Conjugate
Dirichlet
Neumann

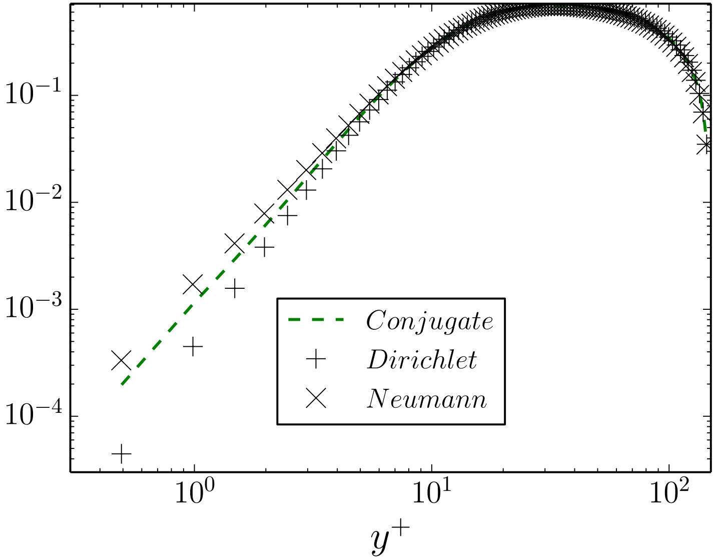
$< T >$



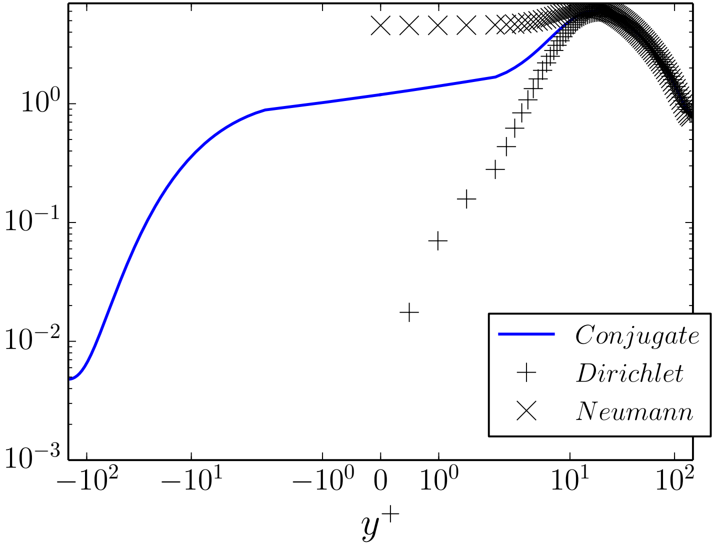
$< ut >$



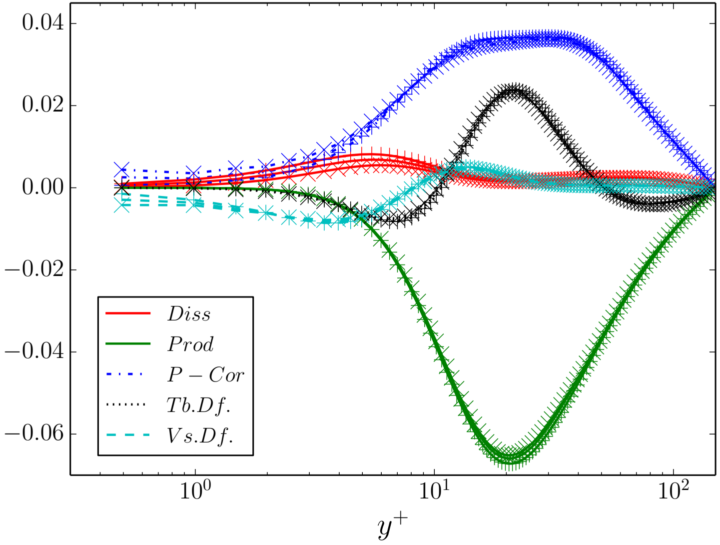
$< vt >$



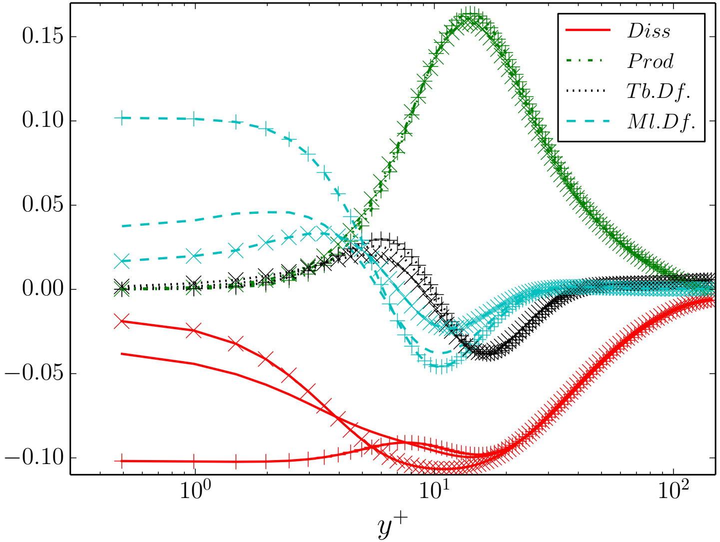
< tt >


10^0
10^{-1}
10^{-2}
10^{-3}
-10^2
-10^1
-10^0
0
10^0
10^1
10^2
y^+
Conjugate
Dirichlet
Neumann

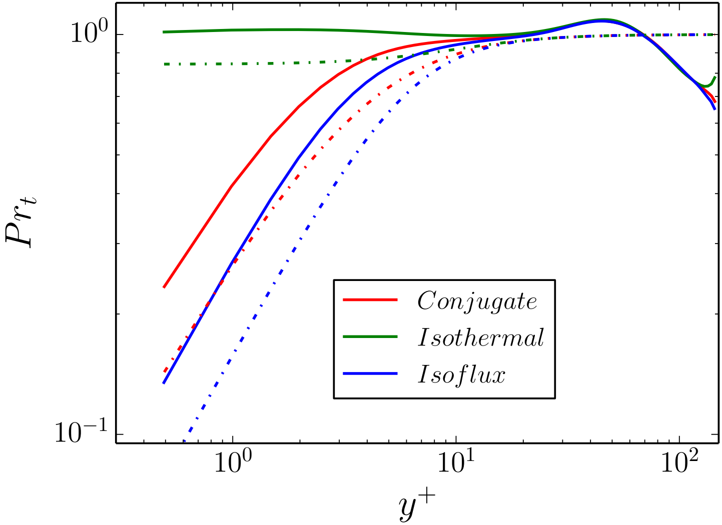
Budget $< vt >$



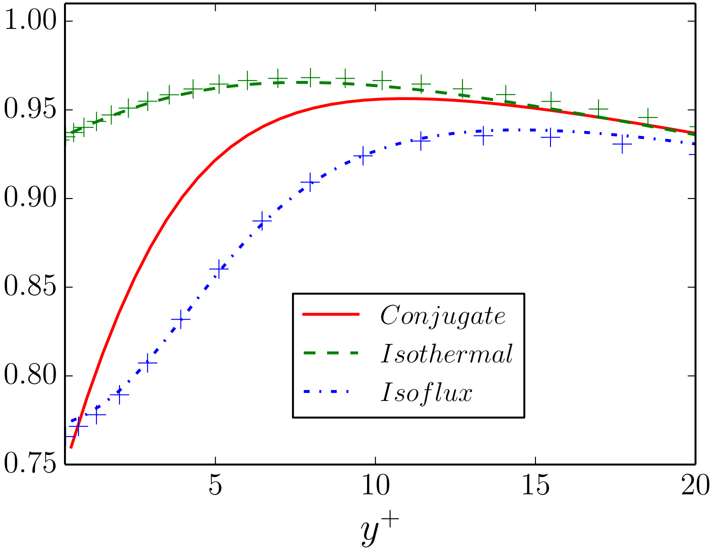
Budget $< tt >$



Turbulent Prandtl number



Correlation uT



Correlation vT

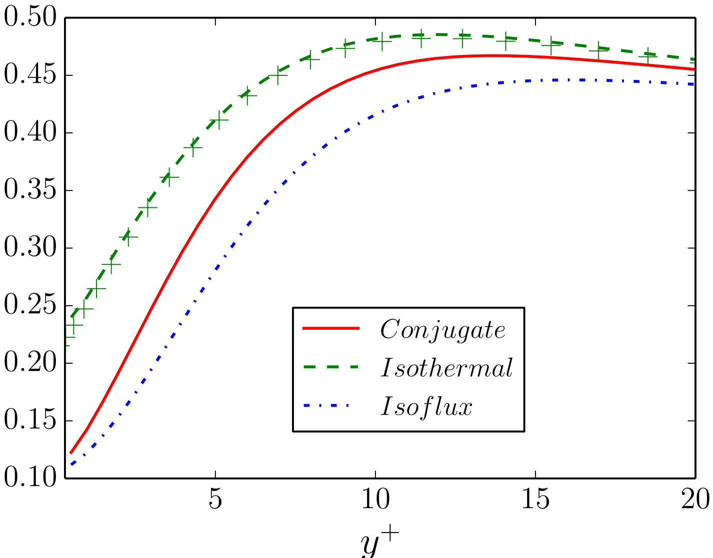
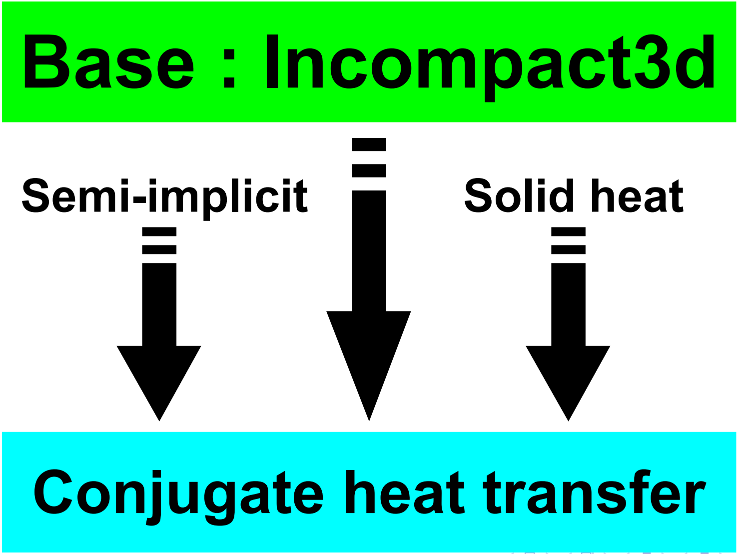


Table of contents

Developments



Explore new configurations

Present

- Imposed temperature *(Kasagi, 1991)*
- Imposed heat flux *(Tiselj, 2001)*
- Conjugate heat transfer *(new)*

Ongoing

- Frequency analysis
- Pulsating channel
- Buoyancy

Thank you for your attention.

Remarks, questions and suggestions are welcome.