Mathematics Aptitude Test (MAT)

Sample Paper 2

[30 minutes]

There are **30 questions** in the MAT.
The solutions for each question is provided at the end of the question paper.

The questions are based on the standard A-level Maths syllabus. They are designed to test problem-solving abilities.

Instructions

1. You are not expected to answer all questions, unanswered questions are not penalised. Each incorrect answer deducts 25% of the marks available for this question.

2. The test consists of 30 questions — 20 are worth one mark, 10 are worth two marks. You will be able to see how many marks each question is worth during the test.

3. All questions are multiple choice. Only one is correct.

4. You are allowed to use plain A4 papers for working out your answers.

5. This is a closed-book examination, any access to resources and calculators are not allowed.

1. Solve the following for x : [1 mark]

$$2e^{2x} + 8e^{x} = 10,\ x \in \mathbb{R}$$

   A. $\ln(1)$ and $\ln(-5)$
   B. 1
   C. 0
   D. 1 and $\ln(-5)$

2. Simplify the following : [2 marks]

$$\log_a(a^5) = \log_a(e^{\ln(2)-\ln(a)}) - 2\log_a(4a)$$

   A. $5 = \log_a(\frac{2}{a} - 16a^2)$
   B. $5 = 3\log_a(2a)$
   C. $5 = -\log_a(2a^3)$
   D. $5 = -3\log_a(2a)$

3. Simplify the following: [1 mark]

$$\sqrt{\frac{2\ln(2) + \ln(8)}{\ln(e) - \ln(1)}}$$

   A. $\sqrt{5}\ln(2)$
   B. $\sqrt{\ln(32)}$
   C. $\frac{5}{2}\ln(2)$
   D. $\ln(16)$

4. Given that 6^{5x-3} can be written as 216^a, what is a in terms of x? [1 mark]
   A. $15x - 3$
   B. $\frac{5x-3}{3}$
   C. $5x - \frac{8}{3}$
   D. $\sqrt[3]{5x-3}$

5. Let [1 mark]

$$f(x) = x\sqrt{x\sqrt{x\sqrt{x\sqrt{x \ldots \infty}}}}$$

   What is $f(3)$?
   A. 1
   B. 27
   C. $3\sqrt{3}$
   D. 9

6. Solve for x: [1 mark]

$$\ln\left(\frac{3}{5}\right) = \log_{10}(x)$$

   A. $(\frac{3}{5})^{\frac{1}{\log_{10}(e)}}$
   B. $\frac{\log_{10}(\frac{3}{5})}{\log_{10}(e)}$
   C. $(\frac{5}{3})^{\frac{1}{\log_{10}(e)}}$
   D. None of the above

7. Using a suitable substitution, evaluate the following: [1 mark]

$$\int_0^{\frac{\pi}{2}} \cos(x)\sin(x)\sqrt{1+\sin(x)}\,dx$$

   A. $\frac{4(\sqrt{2}-1)}{15}$
   B. $\frac{2\sqrt{2}\pi^5}{5} - \frac{\sqrt{2}\pi^3}{3}$
   C. $\sqrt{2}\pi^3(\frac{2\pi^2}{5} + \frac{1}{3})$
   D. $\frac{4\sqrt{2}}{15} + \frac{4}{15}$

8. What is the gradient of the curve defined by [1 mark]

$$y = (x^2 + 7x + 2)^7$$

   at $x = 1$?
   A. 6.3×10^5
   B. 63
   C. 6.3×10^7
   D. 63×10^{-7}

9. What is the following expression equivalent to? [1 mark]

$$I = e^x \ln x - \int \frac{e^x}{x}\,dx$$

   A. $I = \int x^{-1}e^x\,dx$
   B. $I = \int e^x(\ln x)^2\,dx$
   C. $I = \int e^x \ln x\,dx$
   D. None of the above

10. The volume of a curve rotated around the x-axis can be calculated using the formula: [2 marks]

$$\pi \int_{x_1}^{x_2} y^2 \, dx$$

The following curve has been rotated around the x-axis. What is the volume between the x values corresponding to $y = 3$ and $y = 5$, where $y = 2\sin(x)+3$ and $0 \leq x < \pi$?

A. $\frac{11}{2}\pi^2 + 24\pi$

B. $\frac{\pi}{2}(24 + 11\pi)$

C. $\frac{11}{2}\pi + 12\pi$

D. $\pi(24 + 11\pi)$

11. The Taylor series of a real-valued function $f(x)$ that is infinitely differentiable at a real number a is the power series $\sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$, where $n!$ denotes the factorial of n and $f^{(n)}(a)$ denotes the n^{th} derivative of f evaluated at point a. [1 mark]

Approximate the function

$$\ln(x) \text{ centered at } a = 1$$

using three terms in the Taylor expansion (ignore x^4 and higher order terms).

A. $(x-1) + \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3}$

B. $(x-1) - \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3}$

C. $x + \frac{x^2}{2} + \frac{x^3}{3}$

D. $x - \frac{x^2}{2} + \frac{x^3}{3}$

12. Which of the following statements are true? [1 mark]

1. $\cos(-\theta) = -\cos(\theta)$
2. $\tan(180° - \theta) = -\tan(\theta)$
3. $-\tan(\theta) = \frac{\sin(180° - \theta)}{\cos(-\theta)}$

A. none are true

B. 1 and 3 only

C. 2 only

D. 2 and 3 only

13. What is the coefficient of the x^2 term in the expansion of the following? [1 mark]

$$\frac{x^2 - 1}{\sqrt{x+1}}$$

Hint:

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + ...$$

where n is rational and $|x| < 1$.

A. $\frac{3}{8}$

B. $\frac{5}{8}$

C. $-\frac{3}{8}$

D. $-\frac{5}{8}$

14. Which range of values satisfy the following inequalities? [1 mark]

$$2x^2 > 3x + 2$$

$$x^2 < 4$$

A. $-2 < x < 2$

B. $-\frac{1}{2} < 2$

C. $-2 < x < -\frac{1}{2}$

D. $x < 2$

15. A curve is defined by the parametric equations: [2 marks]

$$x = a(t^3 + 1) \qquad y = 9t^2 - 2a \qquad t < 0$$

Given that $a = 3$, where does the curve intercept the x-axis?

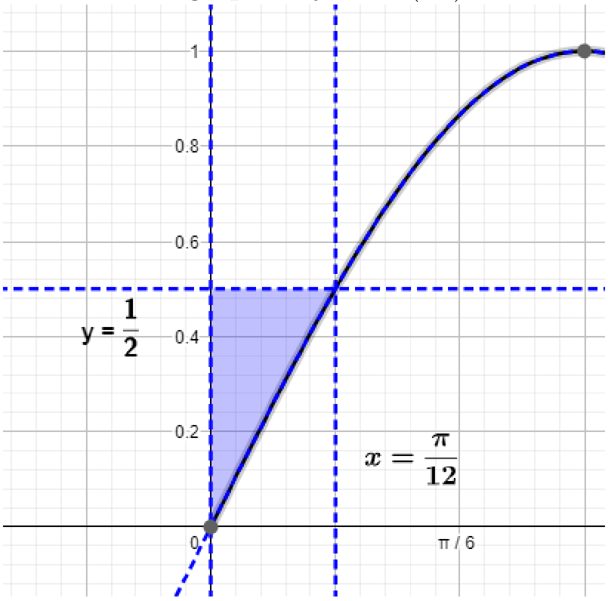
A. $\frac{9-2\sqrt{6}}{3}$

B. 5

C. $1 - \sqrt{6}$

D. 3

16. Consider the graph of $y = \sin(2x)$ and determine the shaded area: [1 mark]



A. $\frac{\pi - 6\sqrt{3} + 12}{24}$
B. $\frac{\pi + 6\sqrt{3} - 12}{24}$
C. $\frac{\pi - 6\sqrt{3} - 12}{24}$
D. None of the above

17. Simplify the following : [1 mark]

$$\frac{\sin(\theta)}{\sin(4\theta)} = -0.5$$

A. $1 = 2\cos(\theta) - 4\cos^3(\theta)$
B. $1 = 2\cos(\theta) + 4\cos^3(\theta)$
C. $1 = -2\cos(\theta) - 4\cos^3(\theta)$
D. $1 = -2\cos(\theta) + 4\cos^3(\theta)$

18. Consider the graph of $y = e^x$ and determine which of the following has the greatest value? [2 marks]

A. $\sum_{n=1}^{100} e^n$
B. $\sum_{n=1}^{100} e^{n-1}$
C. $\int_0^{100} e^x \, dx$
D. $\int_0^{100} 2^x \, dx$

19. Functions f and g are defined as: [1 mark]

$$f(x) = x^2 + 2x - 4, x \in \mathbb{R}$$

$$g(x) = 3x - 4, x \in \mathbb{R}$$

What is the range of $f(g(x))$?

A. $f(g(x)) \geq 1$

B. $f(g(x)) \geq -5$

C. $f(g(x)) \geq -1$

D. $f(g(x)) \geq -19$

20. Evaluate the following limit where $|\pi y| < e$: [2 marks]

$$\lim_{n\to\infty} \frac{x(1-y^n)}{1-y}$$

A. 0

B. x

C. $\frac{x}{1-y}$

D. Undefined

21. Given that the sum of the first 5 terms of an arithmetic series is 30 and that the common difference of the series is 3. What is the 5^{th} term of the series? [1 mark]

A. 18

B. 9

C. 12

D. 15

22. Evaluate the following limit and $f(1)$ where [2 marks]

$$f(x) = \lim_{a\to 0} \frac{e^{(x+a)^2} - e^{x^2}}{a}$$

A. $2e$

B. e

C. e^2

D. None of the above

23. Which of the following is equivalent to: [2 marks]

$$\sum_{r=1}^{n}(3r^3 - 4r^2 + 1)$$

A. $n(n+1)\left[\frac{3n}{4}(n+1) - \frac{2}{3}(2n+1) + \frac{1}{(n+1)}\right]$

B. $n\left[\frac{3n}{4}(n+1)^2 - \frac{2}{3}(n+1)(n+2) + 1\right]$

C. $n(n+1)\left[\frac{n}{2}(n+1) - (n+2) + \frac{1}{(n+1)}\right]$

D. $n\left[\frac{n}{2}(n+1)^2 - (n+1)(2n+1) + 1\right]$

24. Derive the iterative formula to solve the following equation using the Newton-Raphson Method: [2 marks]

$$e^x = x$$

if the Newton-Raphson iterative formula for computing solutions to equations of the form $f(x) = 0$ is given by

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

A. $x_{n+1} = x_{n-1} - \frac{e^{x_n} - x_n}{e^{x_n} - 1}$

B. $x_{n+1} = x_n + \frac{x_n - e^{x_n}}{e^{x_n} - 1}$

C. $x_{n+1} = x_n - \frac{e^{x_n} - 1}{e^{x_n} - x_n}$

D. None of the above

25. Find all solutions to the following equation in the interval $0^\circ \leq \theta \leq 180^\circ$. [1 mark]

$$2\cos^2(2\theta) - \cos(2\theta) - 1 = 0$$

A. 0°, 120°

B. 0 rads, $\frac{\pi}{3}$ rads, $\frac{2\pi}{3}$ rads

C. 0°, 120°, 180°

D. 0 rads, $\frac{\pi}{3}$ rads, $\frac{2\pi}{3}$ rads, π rads

26. A curve is defined by the following parametric equations: [2 marks]

$$x = \ln(t^2 + 2t + 1), \qquad y = \frac{1}{t+4}, \qquad t > \sqrt{5}$$

What is the Cartesian equation of the curve?

A. $y = \frac{1}{e^{2x}+3}$

B. $y = \frac{1}{e^{0.5x}+3}$

C. $y = \frac{1}{e^{\sqrt{x}}+3}$

D. $y = \frac{1}{e^{x^2}+3}$

27. $\lfloor x \rfloor$ represents the floor function: this rounds the input down to the nearest integer, e.g. $\lfloor 0.9 \rfloor = 0$. Solve for n: [2 marks]

$$\lfloor \sqrt{8} - \sqrt{12} + \sqrt{3} - \sqrt{2} + 1 \rfloor = n$$

A. $n = 1$

B. $n = 2$

C. $n = 3$

D. None of the above

28. Function f is defined as: [1 mark]

$$f : x \mapsto \mid x \mid^3 - 3x^2 - 10x + 24$$

How many roots $x \in \mathbb{R}$ does the function have?

A. 0

B. 1

C. 2

D. 3

29. Differentiate the following expression with respect to x: [1 mark]

$$\cos\left(\ln(x+1)\right) + e^{x^5}$$

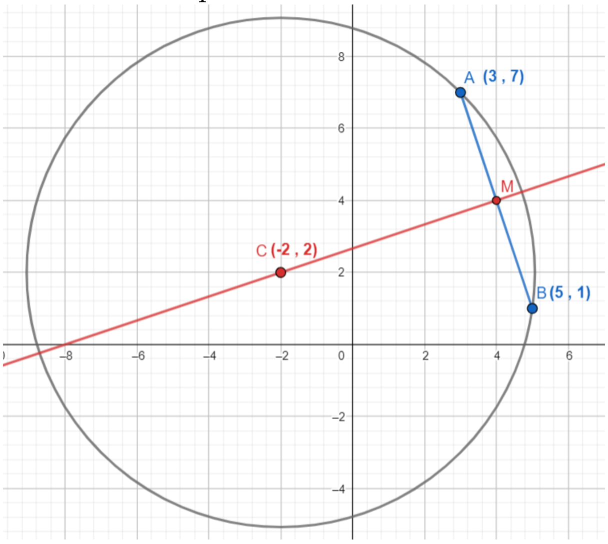
A. $5x^4e^{x^5} - \ln(x+1)\sin\left(\ln(x+1)\right)$

B. $5x^4e^{x^5} - \frac{\cos(\ln(x+1))}{x+1}$

C. $5x^4e^{x^5} - \frac{\ln(x+1)}{x+1}\sin\left(\ln(x+1)\right)$

D. $5x^4e^{x^5} - \frac{\sin(\ln(x+1))}{x+1}$

30. Point M is the midpoint of the line segment AB. [1 mark]
Line l passes through points C and M, where C is the center of the circle.
What is the equation of the circle?



A. $(x+2)^2 + (y+14)^2 = 466$

B. $(x-2)^2 + (y-14)^2 = 50$

C. $(x-2)^2 + (y+2)^2 = 82$

D. $(x+2)^2 + (y-2)^2 = 50$

Question	Points	Answer
1	1	C
2	2	D
3	1	B
4	1	B
5	1	D
6	1	A
7	1	D
8	1	C
9	1	C
10	2	B
11	1	B
12	1	C
13	1	B
14	1	C
15	2	A
16	1	B
17	1	A
18	2	A
19	1	B
20	2	C
21	1	C
22	2	A
23	2	A
24	2	B
25	1	D
26	2	B
27	2	D
28	1	C
29	1	D
30	1	D

End of MAT