Interference Channels with Correlated Receiver Side Information

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***Abstract*—The problem of joint source-channel coding in transmitting independent sources over interference channels with correlated receiver side information is studied. When each receiver has side information correlated with its own desired source, it is shown that source-channel code separation is optimal. When each receiver has side information correlated with the interfering source, sufficient conditions for reliable transmission are provided based on a joint source-channel coding scheme using the superposition encoding and partial decoding idea of Han and Kobayashi. When the receiver side information is a deterministic function of the interfering source, source-channel code separation is again shown to be optimal. As a special case, for a class of Z-interference channels, when the side information of the receiver facing interference is a deterministic function of the interfering source, necessary and sufficient conditions for reliable transmission are provided in the form of single letter expressions. As a byproduct of these joint source-channel coding results, the capacity region of a class of Z-channels with degraded message sets is also provided.**

I. Introduction

The wireless medium is shared by multiple communication systems operating simultaneously, which leads to interference among users transmitting over the same frequency band. In the simple scenario of two transmitter-receiver pairs, the interference channel [1] models two simultaneous transmissions interfering with each other. In the classical interference channel model, the sources intended for each receiver are independent of each other, and the receivers decode based only on their own received signals. In another scenario particularly relevant to sensor networks, receivers have access to their own correlated observations about the underlying source sequences as well. In principle, these correlated observations at the receivers can be exploited in the code design to improve the system performance.

A finite letter expression for the capacity region of an interference channel is unknown even when there is no side information available at the receiver terminals. We know the capacity region in the case of interference channels with statistically equivalent outputs [2]–[4], discrete additive degraded interference channels [5], a class of deterministic interference channels [6], strong interference channels [7]–[11], a class of degraded interference channels [12], and more recently for a class of Z-interference channels [13]. For general interference channels, the best known achievable rate region is the one proposed by Han and Kobayashi [9], a simplification of which is given in [14].

Shamai and Verdú consider the availability of correlated side information at the receiver in a point-to-point scenario in [15]. They show that the source-channel separation theorem applies in this setting and, moreover, that Slepian-Wolf source compression followed by optimal channel coding suffices to achieve the optimal end-to-end performance. With the availability of side information at the receiver, we can transmit the source reliably over a channel with smaller capacity than the one required when there is no receiver side information. However, it is known that the source-channel separation theorem does not generalize to multi-user channels [1], [16], and necessary and sufficient conditions for reliable transmission in the case of correlated sources and correlated receiver side information are not known in general. In [17], necessary and sufficient conditions are characterized for broadcasting a common source to multiple receivers with different correlated side information. An alternative achievability scheme for the setup of [17] is given in [18]. In [19], the results of [17] are extended to broadcast channels with degraded message sets in which the receivers have access to parts of the underlying messages. Availability of messages or message parts at the receivers of broadcast channels from the channel coding perspective is studied also in [20]–[22]. In [23], broadcasting a pair of correlated sources with correlated receiver side information is studied.

The interference channel with correlated sources is considered in [24], and a sufficient condition for reliable transmission is given. In [25], an interference channel with independent sources, in which each receiver has access to side information correlated with the interfering transmitter's source, is considered. Necessary and sufficient conditions for this setup are characterized under the strong source-channel interference conditions, which generalize the usual strong interference conditions by considering correlated side information as well. The result of [25] shows that interference cancellation is optimal even when the underlying channel interference is not

This research was supported in part by the U.S. National Science Foundation under Grants ANI-03-38807, CCF-07-28208, and CNS-06-25637, the DARPA ITMANET program under Grant 1105741-1-TFIND, and the U.S. Army Research Office under MURI award W911NF-05-1-0246.

strong, as long as the overall source-channel interference is.

In this paper, we extend the scenario studied in [25] to more general interference channels. We first consider the case in which each receiver has side information correlated with the source sequence it wants to decode. We prove the optimality of source-channel code separation in this situation; that is, the optimal performance can be achieved by first compressing each of the sources using Slepian-Wolf coding with respect to the correlated receiver side information, and then transmitting the compressed bits over the channel using an optimal interference channel code.

Next, we consider the scenario in which each receiver has side information correlated with the interfering transmitter's source. As an example of such a model and to illustrate the benefits of side information about the interfering source, consider the extreme case in which each receiver has access to the message of the interfering transmitter. Note that this setup is equivalent to the restricted two-way channel model of Shannon, whose capacity is characterized in [1]. In this case, each receiver can excise the interference from the undesired transmitter, since its message is exactly known at the receiver. Here, we consider the more general case of arbitrary correlation between the receiver side information and the interfering source, and propose a joint source-channel coding scheme similar to that of Han and Kobayashi [9] taking the side information into account. Later, we consider the case in which the side information is a deterministic function of the interfering source, and show that source-channel code separation is again optimal. Finally, we consider a special class of interference channels called Z-interference channels, in which only one receiver faces interference. Further focusing on a special class of Z-interference channels satisfying certain conditions (which will be stated later), and the case in which the side information is a deterministic function of the interfering source, we are able to characterize necessary and sufficient conditions for reliable transmission in the form of single letter expressions. This setting also constitutes an example for which the general sufficiency conditions we provide are also necessary, proving their tightness for certain special cases.

The rest of the paper is organized as follows. In Section II we present the system model. In Section III we prove the optimality of source-channel code separation when the side information is correlated with the desired source. The case in which the side information is correlated with the interfering source is considered in Section IV. In Section IV-A, we provide sufficient conditions for reliable transmission, while in Section IV-B, we prove the optimality of source-channel code separation when the side information is a deterministic function of the interfering source. In Section IV-C we show that, for a special source and channel model, the sufficient conditions for reliable transmission proposed in Section IV-A are also necessary, and hence we give a single letter characterization of the necessary and sufficient conditions for this model. In Section V we characterize the capacity region of a class of Z-channels with degraded message sets. This is followed by conclusions in Section VI.

II. System Model

An interference channel is composed of two transmitter-receiver pairs. The underlying discrete memoryless channel is characterized by the transition probability $p(y_1, y_2|x_1, x_2)$ from finite input alphabet $\mathcal{X}_1 \times \mathcal{X}_2$ to finite output alphabet $\mathcal{Y}_1 \times \mathcal{Y}_2$. Transmitter k has access to the source sequence $\{U_{k,i}\}_{i=1}^{\infty}$, $k = 1, 2$. Consider side information sequences $\{V_{k,i}\}_{i=1}^{\infty}$, where the source and the side information sequences are independent and identically distributed (i.i.d.) and are drawn according to joint distribution $p(u_1, v_1)p(u_2, v_2)$ over a finite alphabet $\mathcal{U}_1 \times \mathcal{V}_1 \times \mathcal{U}_2 \times \mathcal{V}_2$; that is, the two source-side information pairs are independent of each other.

For $k = 1, 2$, Transmitter k observes U_k^n and wishes to transmit it noiselessly to Receiver k over n uses of the channel[1]. The encoding function at Transmitter k is

$$f_k^n : \mathcal{U}_k^n \to \mathcal{X}_k^n.$$

We assume that the side information $V_{\pi(k)}^n$ is available at receiver k, where $\pi(\cdot)$ is a permutation of $\{1, 2\}$. Depending on the scenario, we will specify whether the side information is correlated with the desired source or with the interfering source.

The decoding function at receiver k reconstructs its estimate $\hat{U}_k$ from its channel output and side information vector using the decoding function

$$g_k^n : \mathcal{Y}_k^n \times \mathcal{V}_{\pi(k)}^n \to \mathcal{U}_k^n.$$

The probability of error for this system is defined as

$$P_e^n = \Pr\{(U_1^n, U_2^n) \neq (\hat{U}_1^n, \hat{U}_2^n)\},$$

Definition 1: We say that a source pair (U_1, U_2) can be reliably transmitted over a given interference channel if there exist a sequence of encoders and decoders $(f_1^n, f_2^n, g_1^n, g_2^n)$ such that $P_e^n \to 0$ as $n \to \infty$.

In the following sections, we consider two cases in particular. In the first case, each receiver has side information correlated with its desired source, i.e., $\pi(k) = k$, $k = 1, 2$. In the second case, each receiver has side information correlated with the interfering source, i.e., $\pi(1) = 2$ and $\pi(2) = 1$. In both cases, we want to exploit the availability of correlated side information at the receivers. In the first case, each transmitter needs to transmit less information to its intended receiver due to the availability of correlated side information. In the latter case, the side information is used to mitigate the effects of interference.

For notational convenience, we drop the subscripts on probability distributions unless the arguments of the distributions are not lower case versions of the corresponding random variables.

[1]Here we use the notation $U_k^n = (U_{k,1}, \ldots, U_{k,n})$, and similar notation for other length-n sequences.

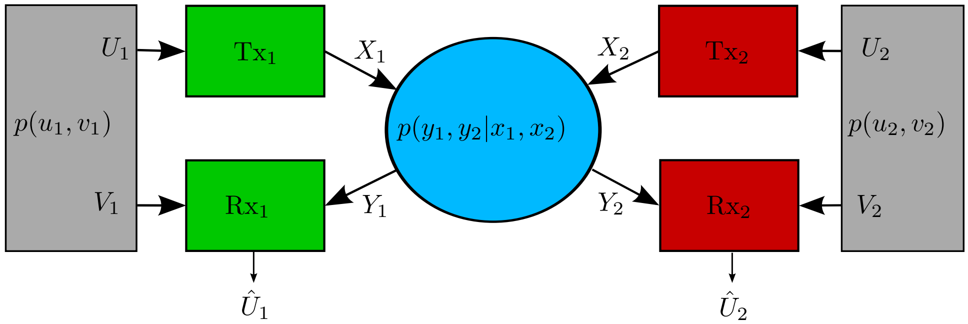

Fig. 1. Interference channel model in which the receivers have access to side information correlated with the source they want to receive.

III. Side Information Correlated with the Desired Source

In this section, we consider an interference channel in which each receiver has side information correlated with the source it wants to decode, i.e., receiver k has access to side information V_k (see Fig. 1). For this special case, we prove that the source-channel separation theorem applies; that is, it is optimal for the transmitters first to apply Slepian-Wolf source coding to compress their sources conditioned on the side information at the corresponding receiver, and then to transmit the compressed bits over the channel using an optimal interference channel code. Note that, in the general case, we do not have a single-letter characterization of the capacity region of the interference channel, yet we can still prove the optimality of source-channel code separation. In the proof, we use the n-letter expression for the capacity region, which was also used in [26] to prove the optimality of source-channel code separation for a multiple access channel with receiver side information and feedback. The main result of this section is the following theorem.

Theorem 1: Sources U_1 and U_2 can be transmitted reliably to their respective receivers over the discrete memoryless interference channel $p(y_1, y_2|x_1, x_2)$ with side information V_k at receiver k, $k = 1, 2$, if

$$(H(U_1|V_1), H(U_2|V_2)) \in int(\mathcal{C}) \tag{1}$$

where $int(\cdot)$ denotes the *interior*, and $\mathcal{C}$ denotes the capacity region of the underlying interference channel.

Conversely, if $(H(U_1|V_1), H(U_2|V_2)) \notin \mathcal{C}$, then sources U_1 and U_2 cannot be transmitted reliably.

Proof: Due to space limitations, the proof is not included here and can be found in [27]. ■

IV. Side Information Correlated with the Interfering Source

In this section we consider the case in which Receiver 1 has access to V_2 while Receiver 2 has access to V_1, i.e., each receiver has side information about the interfering transmitter's source (see Fig. 2). We investigate how the side information about the interference helps in decoding the desired information.

A. Sufficient Conditions for Reliable Transmission

We first provide sufficient conditions for reliable transmission of the sources. In the spirit of the Han-Kobayashi scheme for the classical interference channel, we propose a joint source-channel coding scheme that requires the receivers to decode part of the interference with the help of their side information. In the Han-Kobayashi scheme, each transmitter splits its message into two pieces to allow the non-intended receiver to decode part of the interference. In our scheme, each transmitter enables a quantized version of its source to be decoded by both receivers, where the unintended receiver uses its correlated side information as well as the channel output to decode the interference corresponding to this quantized part. Sufficient conditions for reliable transmission in this setup are given in the following theorem.

Theorem 2: Sources U_1 and U_2 can be transmitted reliably over the interference channel $p(y_1, y_2|x_1, x_2)$ with side information V_1 at Receiver 2 and V_2 at Receiver 1 if there exist random variables W_1 and W_2 such that

$$\begin{aligned}
H(U_1) &< I(X_1; V_2, Y_1|W_2, Q),\\
H(U_2) &< I(X_2; V_1, Y_2|W_1, Q),\\
H(U_1) &< I(W_2, X_1; V_2, Y_1|Q) - I(U_2; W_2|Q),\\
H(U_2) &< I(W_1, X_2; V_1, Y_2|Q) - I(U_1; W_1|Q),\\
H(U_1) + H(U_2) &< I(X_1; V_2, Y_1|W_1, W_2, Q)\\
&\quad + I(W_1, X_2; V_1, Y_2|Q),\\
H(U_1) + H(U_2) &< I(X_2; V_1, Y_2|W_1, W_2, Q)\\
&\quad + I(W_2, X_1; V_2, Y_1|Q),\\
H(U_1) + H(U_2) &< I(W_1, X_2; V_1, Y_2|W_2, Q)\\
&\quad + I(W_2, X_1; V_2, Y_1|W_1, Q),\\
H(U_1) + H(U_2) &< I(W_2, X_1; V_2, Y_1|Q)\\
&\quad + I(W_1, X_2; V_1, Y_2|W_2, Q) - I(U_1; W_1|Q),\\
H(U_1) + H(U_2) &< I(W_1, X_2; V_1, Y_2|Q)\\
&\quad + I(W_2, X_1; V_2, Y_1|W_1, Q) - I(U_2; W_2|Q),\\
2H(U_1) + H(U_2) &< I(W_2, X_1; V_2, Y_1|Q)\\
&\quad + I(X_1; V_2, Y_1|W_1, W_2, Q) + I(W_1, X_2; V_1, Y_2|W_2, Q),\\
H(U_1) + 2H(U_2) &< I(W_1, X_2; V_1, Y_2|Q)\\
&\quad + I(X_2; V_1, Y_2|W_1, W_2, Q) + I(W_2, X_1; V_2, Y_1|W_1, Q),
\end{aligned}$$

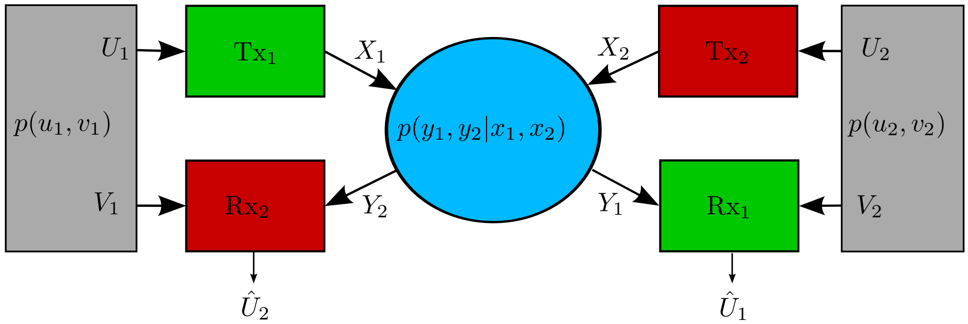

Fig. 2. Interference channel model in which the receivers have access to side information correlated with the source of the interfering transmitter.

for some $p(q)$, $p(w_1,x_1|u_1,q)$, and $p(w_2,x_2|u_2,q)$, where the entropies and mutual information terms are evaluated using the joint distribution

$$p(q,u_1,v_1,u_2,v_2,w_1,w_2,x_1,x_2,y_1,y_2)=p(q)p(u_1,v_1)\cdot p(u_2,v_2)p(w_1,x_1|u_1,q)p(w_2,x_2|u_2,q)p(y_1,y_2|x_1,x_2). \quad (2)$$

Proof: The encoding/decoding scheme is described in the Appendix and the complete proof can be found in [27]. ■

We remark here that the achievability scheme in the proof of Theorem 2 uses joint source-channel coding and hence, similarly to [16] and [28], the expressions involve joint distribution of the source and channel variables, which potentially increases the achievable rate region by enlarging the set of possible joint distributions. Below in Corollary 1, we provide a sufficient condition for reliable transmission based on separate source and channel codes in the spirit of "operational separation" as in [17], [25], which can be obtained as a special case of Theorem 2. Note that operational separation is different from the classical ("informational") separation, in which each source is first assigned to an index and then these indices are transmitted using an optimal channel code for the underlying channel. Operational separation corresponds to separation of the source and the channel variables as in Corollary 1 without using the optimal source or the channel codes (see [25] for further details and examples).

Corollary 1: Sources U_1 and U_2 can be transmitted reliably over the interference channel $p(y_1,y_2|x_1,x_2)$ with side information V_1 at Receiver 2 and V_2 at Receiver 1 if there exist random variables $\overline{W}_1,\widetilde{W}_1$ and $\overline{W}_2,\widetilde{W}_2$ such that inequalities at the bottom of the page are satisfied for some $p(q)$, $p(\overline{W}_1|u_1,q)$, $p(\overline{W}_2|u_2,q)$, $p(\widetilde{W}_1,x_1|q)$ and $p(\widetilde{W}_2,x_2|q)$, where the entropies and mutual information terms are evaluated using joint distribution

$$p(q,u_1,v_1,u_2,v_2,\overline{w}_1,\overline{w}_2,\widetilde{w}_1,\widetilde{w}_2,x_1,x_2,y_1,y_2)=p(q)\cdot p(u_1,v_1)p(\overline{w}_1|u_1,q)p(u_2,v_2)p(\overline{w}_2|u_2,q)p(\widetilde{w}_1,x_1|q)\cdot p(\widetilde{w}_2,x_2|q)p(y_1,y_2|x_1,x_2). \quad (3)$$

Proof: Corollary 1 follows directly from Theorem 2 by letting $W_k=(\overline{W}_k,\widetilde{W}_k)$ and fixing the distributions as $p(w_k,x_k|u_k,q)=p(\overline{w}_k|u_k,q)p(\widetilde{w}_k,x_k|q)$, for $k=1,2$. ■

The sufficient conditions in Corollary 1 are looser than those in Theorem 2. However, it is not clear whether they are strictly looser.

Remark 1: In the special case of no receiver side information, i.e., $V_1=V_2=\emptyset$, by fixing $\overline{W}_1=\overline{W}_2=\emptyset$, and defining

$$\begin{aligned}
H(U_1) &< I(X_1;Y_1|\widetilde{W}_2,Q),\\
H(U_1)+I(\overline{W}_2;U_2|V_2,Q) &< I(X_1,\widetilde{W}_2;Y_1|Q),\\
H(U_2) &< I(X_2;Y_2|\widetilde{W}_1,Q),\\
H(U_2)+I(\overline{W}_1;U_1|V_1,Q) &< I(X_2,\widetilde{W}_1;Y_2|Q),\\
H(U_1)+H(U_2)-I(\overline{W}_1;V_1|Q) &< I(X_1;Y_1|\widetilde{W}_1,\widetilde{W}_2,Q)+I(\widetilde{W}_1,X_2;Y_2|Q),\\
H(U_1)+H(U_2)-I(\overline{W}_2;V_2|Q) &< I(X_2;Y_2|\widetilde{W}_1,\widetilde{W}_2,Q)+I(\widetilde{W}_2,X_1;Y_1|Q),\\
H(U_1)+H(U_2)-I(\overline{W}_1;V_1|Q)-I(\overline{W}_2;V_2|Q) &< I(\widetilde{W}_1,X_2;Y_2|\widetilde{W}_2,Q)+I(\widetilde{W}_2,X_1;Y_1|\widetilde{W}_1,Q),\\
H(U_1)+H(U_2)+I(\overline{W}_1;U_1|V_1,Q)-I(\overline{W}_2;V_2|Q) &< I(\widetilde{W}_2,X_1;Y_1|Q)+I(\widetilde{W}_1,X_2;Y_2|\widetilde{W}_2,Q),\\
H(U_1)+H(U_2)+I(\overline{W}_2;U_2|V_2,Q)-I(\overline{W}_1;V_1|Q) &< I(\widetilde{W}_1,X_2;Y_2|Q)+I(\widetilde{W}_2,X_1;Y_1|\widetilde{W}_1,Q),\\
2H(U_1)+H(U_2)-I(\overline{W}_1;V_1|Q)-I(\overline{W}_2;V_2|Q) &< I(\widetilde{W}_2,X_1;Y_1|Q)+I(X_1;Y_1|\widetilde{W}_1,\widetilde{W}_2,Q)+I(\widetilde{W}_1,X_2;Y_2|\widetilde{W}_2,Q)\\
H(U_1)+2H(U_2)-I(\overline{W}_1;V_1|Q)-I(\overline{W}_2;V_2|Q) &< I(\widetilde{W}_1,X_2;Y_2|Q)+I(X_2;Y_2|\widetilde{W}_2,\widetilde{W}_1,Q)+I(\widetilde{W}_2,X_1;Y_1|\widetilde{W}_1,Q),
\end{aligned}$$

$R_1 = H(U_1)$ and $R_2 = H(U_2)$, the sufficiency conditions in Corollary 1 boils down to the Han-Kobayashi rate region in the form expressed in [14, Theorem 2].

We do not know whether the sufficient conditions for reliable transmission provided in Theorem 2 are too strong, leading to pessimistic results in general. However, in Section IV-C, we show that for some special cases, the sufficient conditions obtained through separate source and channel coding in Corollary 1 are also necessary, which shows that at least for certain special cases, Theorem 2 is tight.

B. Deterministic Side Information

In this subsection, we focus on the special case in which the side information sequences V_1 and V_2 are deterministic functions of the sources U_1 and U_2, respectively, i.e.,

$$V_{k,i} = h_k(U_{k,i}), \qquad k = 1,2, \quad i = 1,2,\cdots \tag{4}$$

for some deterministic functions h_1 and h_2, or equivalently we have $H(V_k|U_k) = 0$ for $k = 1,2$.

The main result of this subsection is that when the side information is a deterministic function of the interfering source, the source-channel separation theorem applies; that is, it is optimal to first perform source coding and encode V_k^n into message W_{ks}, and the remaining part of U_k^n, denoted by $U_k^n|V_k^n$, into message W_{kp}, $k = 1,2$, and then to transmit these messages optimally over the underlying interference channel $p(y_1, y_2|x_1, x_2)$ with side information W_{1s} at Receiver 2, and side information W_{2s} at Receiver 1.

First, we define the capacity region of the interference channel with message side information at the receivers (see Fig. 3). In this communication scenario, Transmitter k has two messages W_{ks} and W_{kp}, of rates R_{ks} and R_{kp} respectively, to transmit with negligible probability of error to Receiver k, $k = 1,2$, while Receiver 2 has access to W_{1s}, and Receiver 1 has access to W_{2s}. All messages are independent. A $\left(2^{nR_{1s}}, 2^{nR_{1p}}, 2^{nR_{2s}}, 2^{nR_{2p}}, n\right)$ code for this channel consists of two encoding functions, $f_1^n : \{1,2,\cdots,2^{nR_{1s}}\} \times \{1,2,\cdots,2^{nR_{1p}}\} \rightarrow \mathcal{X}_1^n$ and $f_2^n : \{1,2,\cdots,2^{nR_{2s}}\} \times \{1,2,\cdots,2^{nR_{2p}}\} \rightarrow \mathcal{X}_2^n$, and two decoding functions $g_1^n : \mathcal{Y}_1^n \times \{1,2,\cdots,2^{nR_{2s}}\} \rightarrow \{1,2,\cdots,2^{nR_{1s}}\} \times \{1,2,\cdots,2^{nR_{1p}}\}$ and $g_2^n : \mathcal{Y}_2^n \times \{1,2,\cdots,2^{nR_{1s}}\} \rightarrow \{1,2,\cdots,2^{nR_{2s}}\} \times \{1,2,\cdots,2^{nR_{2p}}\}$.

The average probability of error for the $\left(2^{nR_{1s}}, 2^{nR_{1p}}, 2^{nR_{2s}}, 2^{nR_{2p}}, n\right)$ code is defined as

$$\begin{aligned} P_e^n = &\frac{1}{2^{n(R_{1s}+R_{1p}+R_{2s}+R_{2p})}} \sum_{w_{1s}=1}^{2^{nR_{1s}}} \sum_{w_{1p}=1}^{2^{nR_{1p}}} \sum_{w_{2s}=1}^{2^{nR_{2s}}} \sum_{w_{2p}=1}^{2^{nR_{2p}}} \\ &\Pr\{g_1^n(Y_1^n, w_{2s}) \neq (w_{1s}, w_{1p}) \text{ or} \\ &g_2^n(Y_2^n, w_{1s}) \neq (w_{2s}, w_{2p}) | (w_{1s}, w_{1p}, w_{2s}, w_{2p}) \text{ is sent}\}. \end{aligned}$$

Definition 2: A rate quadruplet $(R_{1s}, R_{1p}, R_{2s}, R_{2p})$ is said to be achievable if there exists a sequence of $\left(2^{nR_{1s}}, 2^{nR_{1p}}, 2^{nR_{2s}}, 2^{nR_{2p}}, n\right)$ codes for which $P_e^n \rightarrow 0$ as $n \rightarrow \infty$. The capacity region is defined as the closure of the set of achievable rate quadruplets $(R_{1s}, R_{1p}, R_{2s}, R_{2p})$, and is denoted by $\mathcal{C}_I$.

In order to show the optimality of source-channel code separation, similarly to Theorem 1, we will use the n-letter characterization of $\mathcal{C}_I$ provided in the next lemma. Define $\mathcal{G}^n$ as

$$\begin{aligned} \mathcal{G}^n = \Bigg\{ &(R_{1s}, R_{1p}, R_{2s}, R_{2p}) : R_{1p} \leq \frac{1}{n} I(X_1^n; Y_1^n | S_{1s}^n, S_{2s}^n), \\ &R_{1s} + R_{1p} \leq \frac{1}{n} I(X_1^n; Y_1^n | S_{2s}^n), R_{2p} \leq \frac{1}{n} I(X_2^n; Y_2^n | S_{1s}^n, S_{2s}^n), \\ &R_{2s} + R_{2p} \leq \frac{1}{n} I(X_2^n; Y_2^n | S_{1s}^n), \\ &\text{for any } p^n(s_{1s}^n) p^n(s_{2s}^n) p^n(x_1^n | s_{1s}^n) p^n(x_2^n | s_{2s}^n) \Bigg\} \end{aligned} \tag{5}$$

Lemma 1: The capacity region of the interference channel with message side information W_{1s} at Receiver 2, and message side information W_{2s} at Receiver 1 is

$$\mathcal{C}_I = \lim_{n \rightarrow \infty} \quad \mathcal{G}^n \tag{6}$$

where the limit of the region is as defined in [1, Theorem 5].

Proof: A proof of Lemma 1 can be found in [27]. ■

Now that we have the n-letter characterization of the capacity region of interference channels with message side information at the receivers, we are ready to show that the source-channel separation theorem holds when the receivers' side information sequences are deterministic functions of the interfering sources.

Theorem 3: Sources U_1 and U_2 can be transmitted reliably to their respective receivers over the discrete memoryless interference channel $p(y_1, y_2|x_1, x_2)$ with side information $V_1 = h_1(U_1)$ at Receiver 2, and side information $V_2 = h_2(U_2)$ at Receiver 1, if

$$(H(V_1), H(U_1|V_1), H(V_2), H(U_2|V_2)) \in int(\mathcal{C}_I), \tag{7}$$

where $\mathcal{C}_I$ denotes the capacity region of the interference channel with message side information at receivers.

Conversely, if $(H(V_1), H(U_1|V_1), H(V_2), H(U_2|V_2)) \notin \mathcal{C}_I$, then sources U_1 and U_2 cannot be transmitted reliably.

Proof: A proof of Theorem 3 can be found in [27]. ■

The benefits of considering the side information samples as deterministic functions of the source samples are two-fold. Firstly, the transmitters also know the side information and they can use this knowledge to minimize the amount of interference they cause. Due to this fact, we are able to achieve any point in the capacity region of the interference channel with message side information. Secondly, encoding the information of V_k, $k = 1,2$ into the codebook at Transmitter k not only helps reduce the interference at the other receiver, but also does not place any extra burden on Receiver k to decode V_k, as V_k is a deterministic function of U_k. This fact enables the converse proof of the source-channel separation theorem.

C. Necessary and Sufficient Conditions for Reliable Transmission for a Special Case

In Section IV-B, we have shown that source-channel separation is optimal when the side information is a deterministic

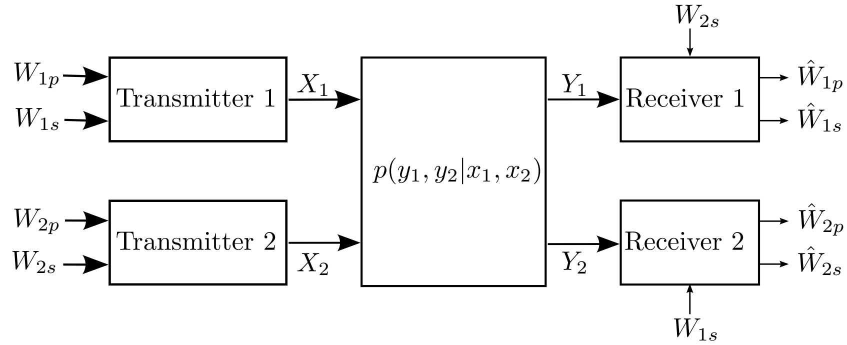

Fig. 3. Interference channel with message side information at the receivers.

function of the interfering source. Thus, for these cases, if the single-letter characterization of the capacity region of the corresponding interference channel with message side information, i.e., $\mathcal{C}_I$, is known, we would have necessary and sufficient conditions for reliable transmission in a single-letter form. However, a single-letter characterization of $\mathcal{C}_I$ is not known in general as it is a generalization of the capacity region of the classical interference channel.

In this subsection, we consider the class of interference channels studied in [13]. We show that the Han-Kobayashi scheme is capacity-achieving for this class of interference channels [13] when the receivers have message side information, and we obtain a single-letter characterization of the capacity region. Hence, we conclude that, for this class of interference channels, when the side information is a deterministic function of the interfering source, the sufficient conditions provided in Theorem 2 are also necessary, yielding a single-letter characterization of the necessary and sufficient conditions for reliable transmission. This means that the achievability result presented in Theorem 2 is tight in some special cases.

The special class of interference channels we focus on in this subsection is a class of Z-interference channels. For the Z-interference channels, $p(y_1, y_2|x_1, x_2)$ can be written as $p(y_2|x_1, x_2) \cdot p(y_1|x_1)$, i.e., the channel between X_1 and Y_1 is a single user channel characterized by $p(y_1|x_1)$. This corresponds to an interference channel in which only the second transmitter-receiver pair faces interference. In particular, the members of the class of Z-interference channels we consider satisfy the following conditions:

1) For any positive integer n, $H(Y_2^n|X_2^n = x_2^n)$, when evaluated with the distribution $\sum_{x_1^n} p(x_1^n)$ $p(y_2^n|x_1^n, x_2^n)$, is independent of x_2^n for any $p(x_1^n)$.
2) Define τ as

$$\tau = \max_{p(x_1)p(x_2)} H(Y_2). \tag{8}$$

Then there exists a $p^*(x_2)$ such that $H(Y_2)$, when evaluated with the distribution $\sum_{x_1,x_2} p(x_1)$ $p^*(x_2)p(y_2|x_1, x_2)$, is equal to τ for any $p(x_1)$.

Please refer to [13] for intuition behind these conditions and examples of Z-interference channels that satisfy these two conditions.

In the next lemma, we provide a single-letter characterization of $\mathcal{C}_I$, i.e., the capacity region of this class of Z-interference channels with message side information. Since Receiver 1 does not face interference, there is no benefit to having access to the side information W_{2s}. Hence, without loss of generality, we assume $R_{2s} = 0$.

Lemma 2: The capacity region of Z-interference channels satisfying Conditions 1 and 2, with message side information W_{1s} at Receiver 2, is characterized by

$$R_{1p} + R_{1s} \leq I(X_1; Y_1), \tag{9}$$

$$R_{2p} \leq I(W, X_2; Y_2) \text{ and} \tag{10}$$

$$R_{1p} + R_{2p} \leq I(X_1; Y_1|W) + I(W, X_2; Y_2) \tag{11}$$

for some $p(w)p(x_1|w)$, where the mutual informations and entropies are evaluated with the joint distribution of the form $p(w, x_1, x_2, y_1, y_2) = p(w)p(x_1|w)p^*(x_2)p(y_1|x_1)$ $p(y_2|x_1, x_2)$.

Proof: A proof of Lemma 2 can be found in [27]. ■

The proof of Lemma 2 indicates that superposition encoding and partial decoding is capacity-achieving. More specifically, the codebook at Transmitter 1 is such that the inner codebook carries the side information at Receiver 2, i.e., W_{1s}, and part of W_{1p}, and the outer codebook carries the remaining part of W_{1p}.

Comparing these results in the case of side information at the receiver with the traditional Z-interference channel [13], the rate of W_{1p} takes the place of W_1, which means that the message that causes interference is reduced from W_1 to W_{1p}. Due to the fact that W_{1s} is available at Receiver 2, W_{1s} does not cause any interference and therefore its rate can be made as large as possible within the constraint of the capacity of the channel $p(y_1|x_1)$ depicted by (9).

Having established the capacity region of this special class of Z-interference channels with message side information at the receiver, we next consider the joint source-channel coding problem for this channel model with the assumption that each side information sample $V_{1,i}$ is a deterministic function of the corresponding source sample $U_{1,i}$, i.e., $V_{1,i} = h_1(U_{1,i})$, for $i = 1, 2, \cdots$ for some deterministic function h_1. Since the first transmitter-receiver pair is interference-free, without loss of generality, we assume $V_2 = \emptyset$.

Since source-channel separation is shown to be optimal in Theorem 3 for the source and side information structure under consideration, we are able to characterize necessary and sufficient conditions for the reliable transmission of the sources in the single-letter form using the capacity region characterization given in Lemma 2.

Corollary 2: For Z-interference channels satisfying Conditions 1 and 2, and side information $V_1 = h_1(U_1)$ at Receiver 2, necessary and sufficient conditions for reliable transmission are

$$H(U_1) < I(X_1; Y_1) \tag{12}$$

$$H(U_2) < I(W, X_2; Y_2) \text{ and} \tag{13}$$

$$H(U_1|V_1) + H(U_2) < I(W, X_2; Y_2) + I(X_1; Y_1|W) \tag{14}$$

for some $p(w)p(x_1|w)$, where the mutual informations and entropies are evaluated with $p(u_1, v_1, u_2, w, x_1, x_2, y_1, y_2) = p(u_1, v_1)p(u_2)p(w)p(x_1|w)p^*(x_2)p(y_1|x_1)p(y_2|x_1, x_2)$.

Proof: Corollary 2 follows directly from combining Theorem 3 and Lemma 2. ■

In Corollary 1, specify $V_2 = \emptyset$, choose $\overline{W}_2 = \widetilde{W}_2 = \emptyset$, $\overline{W}_1 = V_1$, $Q = \emptyset$ and $p(x_2) = p^*(x_2)$. Renaming $\widetilde{W}_1$ as W_1 and using Condition 2 and the fact that $H(U_1) - H(V_1) = H(U_1|V_1)$, we obtain a sufficient condition which is the same as the necessary and sufficient condition specified in Corollary 2. Hence, we conclude that in this special case, the sufficient conditions described in Corollary 1 based on separate source and channel coding are also necessary. This shows that the conditions presented in Theorem 2 are also necessary at least in certain scenarios.

Corollary 2 shows how the side information $V_1 = h_1(U_1)$ about the interference U_1 helps in reliable transmission, and determines the most efficient way of using this side information: Transmitter 1 performs a separation-based encoding scheme. It first splits its source U_1^n into V_1^n and a remaining part using entropy-achieving data compression techniques, and thus obtains two messages $W_{V_1^n}$ and $W_{U_1^n|V_1^n}$. Then, it further splits message $W_{U_1^n|V_1^n}$ into two parts W_{inner} and W_{outer}, at rates γ and $H(U_1|V_1) - \gamma$, respectively. Next, it performs superposition encoding, transmitting $W_{V_1^n}$ and W_{inner} through the inner code at rate $H(V_1) + \gamma$, and W_{outer} through the outer code at rate $H(U_1|V_1) - \gamma$. Transmitter 2 performs separation-based source-channel coding, first mapping U_2^n into a message W_2 and then mapping W_2 into a codeword of an i.i.d. codebook generated with distribution $p^*(x_2)$. Receiver 1 decodes both the inner and the outer codes. Receiver 2 knows the side information V_1^n and hence sees an inner codebook at an effective rate of γ only. It decodes the inner codeword and the codeword of Transmitter 2 jointly using the received signal and the available side information about the interference.

The intuition obtained from the special case derived in this subsection is that one should put as much information as possible about the side information within the inner codebook, in order to minimize the impact of interference when the side information about the interference is available at the receiver.

V. Z-Channel with Degraded Message Sets

The result in (9)-(11) is directly related to the capacity region of the Z-channel with degraded message sets, based on the intuition gained from the proof of Theorem 3 in [19]. The intuition in [19] is that when the receiver has some side information about the undesired message, we can set up a new scenario in which the receiver does not have access to the side information, and is required to decode it. Then, when we remove the rate constraint associated with decoding of the side information at the receiver in the capacity region of the new scenario, we get the capacity results of the original scenario. Therefore, the solution given in (9)-(11) resembles the solution of the following problem.

The channel is described by two transition probabilities $p(y_1|x_1)$ and $p(y_2|x_1, x_2)$, and satisfies both Conditions 1 and 2. There are three independent messages W_{1c}, W_{1p} and W_2. Transmitter 1 has messages W_{1c} and W_{1p} and Transmitter 2 has message W_2. W_{1c} needs to be decoded at both receivers, while W_{1p} and W_2 need to be decoded only at Receiver 1 and Receiver 2, respectively.

This channel model includes the Z-interference channel as a special case, when the rate of W_{1c} is zero. Compared to the definition of the Z-channel in [29], W_{1c} is not only intended for Receiver 2, but also for Receiver 1. Therefore, we call this channel model as the *Z-channel with degraded message sets*.

Then the capacity region for the Z-channel satisfying Conditions 1 and 2, with degraded message sets can be characterized as follows:

$$R_{1p} \leq I(X_1; Y_1|W) + \gamma, \tag{15}$$

$$R_{1c} + R_{1p} \leq I(X_1; Y_1), \tag{16}$$

$$R_{1c} \leq I(W; Y_2|X_2) - \gamma \text{ and} \tag{17}$$

$$R_2 + R_{1c} \leq \tau - H(Y_2|W, X_2) - \gamma, \tag{18}$$

for some $p(w)p(x_1|w)$ and $\gamma \geq 0$ where the mutual informations and entropies are evaluated using $p(w, x_1, x_2, y_1, y_2) = p(w)p(x_1|w)p^*(x_2)p(y_1|x_1)p(y_2|x_1, x_2)$. The proof of this result follows from arguments very similar to those used in the scenario of message side information at the receiver considered in Lemma 2.

VI. Conclusions

We have studied the problem of joint source-channel coding in interference channels with correlated receiver side information. In the case when the receiver side information is correlated with its desired source, we have shown that separate design of source and channel codes is optimal. In order to minimize the interference to the other transmitter-receiver pair, the transmitters should transmit only the part of their sources that is not already known by their corresponding receivers.

For the case in which the receiver side information is correlated with the interfering source, we have provided sufficient conditions for reliable transmission by proposing a joint source-channel coding scheme based on the idea of superposition encoding and partial decoding of Han and Kobayashi. As a special case, we have focused on the scenario in which the side information at the receiver is a deterministic function of the interfering source, and we have shown that source-channel separation is optimal for this situation as well. In both cases for which the optimality of source-channel separation is established, we have used the n-letter expression for the capacity region as a single-letter expression is not available in general.

Finally, for a class of Z-interference channels for which superposition encoding and partial decoding is optimal in the absence of receiver side information, when the receiver facing interference has access to a deterministic function of the interfering source, we have shown that the provided sufficient conditions are also necessary. Hence, the sufficient conditions are tight at least in some special cases.

APPENDIX A
ACHIEVABILITY SCHEME OF THEOREM 2

Fix a joint distribution as in (2).

Codebook generation: First, generate one random n-sequence q^n in an i.i.d. fashion according to $p(q)$.

Next, for Transmitter 1, generate a codebook of size L_1 with $\frac{1}{n}\log L_1 > I(U_1; W_1|Q)$, in which the codewords are generated i.i.d. with distribution $p(w_1|q)$. This codebook is denoted by $\mathcal{C}_w^1$.

For each possible source output u_1^n, choose one among all sequences in $\mathcal{C}_w^1$ jointly typical with u_1^n, uniformly at random, and call it $w_1^n(u_1^n)$. If there are no codewords of $\mathcal{C}_w^1$ jointly typical with u_1^n, randomly choose one codeword from $\mathcal{C}_w^1$ to be $w_1^n(u_1^n)$. In a similar fashion, we generate $\mathcal{C}_w^2$.

Codebook generation: For each possible u_1^n sequence, generate one x_1^n sequence in an i.i.d. fashion, conditioned on $w_1^n(u_1^n)$, u_1^n and q^n, according to $p(x_1|u_1, w_1, q)$. This x_1^n sequence is denoted by $x_1^n(u_1^n, w_1^n(u_1^n))$. The collection of all x_1^n sequences will be denoted as the codebook $\mathcal{C}_x^1$. Similarly, we generate the codebook $\mathcal{C}_x^2$.

Encoding: When Transmitter 1 observes the sequence u_1^n, it transmits $x_1^n(u_1^n, w_1^n(u_1^n))$. Similarly for Transmitter 2.

Decoding: Receiver 1 finds the unique pair (u_1^n, w_2^n), $u_1^n \in \mathcal{U}_1^n$, $w_2^n \in \mathcal{C}_w^2$, such that $(u_1^n, w_1^n(u_1^n)$, $x_1^n(u_1^n, w_1^n(u_1^n))$, w_2^n, $y_1^n, v_2^n)$ are jointly typical and declares the first component of the pair as the transmitted source. If there are more than one pair, and the first component of the pairs are the same, then the decoder declares the transmitted source to be the first component. If there are more than one pair, and the first component of the pairs are not the same, an error is declared. Also, if no such pair exists, an error is declared. Similarly for Receiver 2.

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