Delay Limited Transmission of a Uniform Source over an AWGN Channel

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***Abstract*—Delay limited transmission of a uniform source over an additive white Gaussian noise (AWGN) channel under an average power constraint is considered. Assuming that the channel can be used only once, *mean squared error* (MSE) distortion is studied for both the bandwidth matched, and the 2:1 bandwidth compression cases. In the bandwidth matched scenario, simply scaling the source sample, i.e., analog transmission, performs better than transmitting the scalar quantized source samples. For the bandwidth compression scenario, a hybrid digital analog transmission scheme that quantizes the first source sample and superimposes the quantized sample with the scaled version of the second sample is studied. It is shown that, in this scheme, as opposed to the bandwidth matched case, a finite number of quantization indices minimizes the achievable distortion. The performance of this hybrid scheme is then compared with a numerically optimized encoder using the steepest decent algorithm iteratively. It is observed that the performance of the hybrid scheme is reasonably close to the numerically optimized scheme, while having a significantly lower computational complexity. The theoretical Ziv-Zakai (ZZ) bound on the average distortion is also considered to better understand the gap between the optimal performance and the proposed scheme.**

I. Introduction

We study the transmission of a uniform source over an additive white Gaussian noise (AWGN) channel under strict delay constraints. In this model a single use of the underlying channel is possible to transmit the source samples. In particular, we consider the transmission of one or two source samples. Most previous work on delay-limited joint source channel coding focus exclusively on Gaussian source and channel distributions. While Gaussian channel noise is a common assumption and accurate in most scenarios, the validity of Gaussian source assumption is harder to argue for many practical applications. In many sensing applications, the underlying system parameters that are being sensed, such as temperature, pressure, humidity, etc., are bounded within a certain finite interval. When there is no prior information on the distribution of the parameter of interest, a uniform distribution assumption is arguably the most appropriate one following the maximum entropy principle [1].

In the case of a Gaussian source and an AWGN channel, the *minimum mean squared error* (MMSE) distortion is trivially achieved by uncoded transmission in the bandwidth matching scenario [2]. However, this is not true for non-Gaussian distributions, and the optimal transmission scheme is not known in general. Many techniques have been developed for the Gaussian problem for both bandwidth compression and expansion scenarios [3]–[9] with varying performances and complexities to implement.

Lower bounds for this problem are harder to come by. Shannon lower bound is commonly used, although it is based on the infinite block-length source and channel coding assumptions. Apart from the ideally matched case of Gaussian source and channel distributions, Shannon lower bound is not tight in general. Ziv and Zakai provided an alternative lower bound in [10] exploiting the data processing inequality for general information measures, in particular the Rényi information of order α. Ziv-Zakai (ZZ) lower bound is used in [11] to derive tighter bounds on the MSE distortion for the Gaussian problem in the high SNR/ high resolution regime for both bandwidth compression and expansion. It is shown in [11] that as the mismatch between the source and channel bandwidths increases, the improvement of the ZZ bound over the Shannon bound also increases.

The rest of the paper is organized as follows: In Section II we introduce the system model. In Section III we study the bandwidth matched scenario. In Section IV 2:1 bandwidth compression is considered, and a hybrid transmission scheme is proposed. In Section IV-B we study the numerically optimized encoder structure using the steepest decent algorithm. In Section V the ZZ bound for this scenario is considered. In Section VI, numerical results and comparisons are presented, and finally Section VII concludes the paper.

II. system model

We consider the transmission of a uniform source V over an AWGN channel. The channel output is given by $Y = X + W$, where X is the channel input, and W is the AWGN with zero mean and variance σ_n^2, i.e., $W \sim \mathcal{N}(0, \sigma_n^2)$. The encoder $h : \mathbb{R}^m \to \mathbb{R}$ maps m source samples to the channel input X, and must satisfy the average power constraint P, that is, $E[|X|^2] \leq P$. The source V is uniformly distributed over $[-\frac{1}{2}, \frac{1}{2}]$. In the bandwidth matching scenario we have $m = 1$, whereas in the 2:1 bandwidth compression scenario we have $m = 2$, and assume that the source samples V_1 and V_2 are independent. The reconstruction function $g : \mathbb{R} \to \mathbb{R}^m$ maps the channel output to the source estimates, $\hat{V}^m = g(Y)$. The goal is to characterize the encoder $h(\cdot)$ and the decoder $g(\cdot)$ such that the average MSE, $D^* = \min_{h,g} \frac{1}{m} \sum_{i=1}^m \mathbb{E}[|V_i - \hat{V}_i|^2]$ is minimized, where the expectation is over the source and channel noise distributions.

III. MATCHED BANDWIDTH

We first consider the bandwidth matching scenario, i.e., $m = 1$. We consider a transmission scheme, in which the source sample is first scalar quantized, then the quantization index is scaled and transmitted over the channel. The channel input is given by $X = \beta \cdot \mathcal{Q}(V)$, where β is a scaling factor, and $\mathcal{Q}$ is the uniform quantizer with quantization indices $q_i = \frac{2i-N-1}{2N}$, $i = 1, ..., N$. Due to the uniform source assumption, each quantization index is equally likely, i.e., $p(q_i) = \frac{1}{N}$. We denote the variance of the quantized source sample by σ_q^2. Then the average input power is given by

$$\begin{aligned} E[|X|^2] &= \beta^2\sigma_q^2 = \beta^2 \sum_{i=1}^{N} \frac{1}{N} \cdot \left(\frac{2i-1}{2N} - \frac{1}{2}\right)^2 \\ &= \frac{\beta^2}{4N^3} \sum_{i=1}^{N} \left[4i^2 - 4i(N+1) + (N+1)^2\right] \\ &= \frac{\beta^2}{12} \cdot \left(1 - \frac{1}{N^2}\right). \end{aligned}$$

This encoder maps each interval of the uniform quantizer to an interval on the real line, whose length is chosen based on the power constraint. To satisfy the average power constraint, β is chosen such that $\beta^2 \leq 12P\left(1 - \frac{1}{N^2}\right)^{-1}$. With an optimal MMSE estimator at the receiver, the estimated sample and the corresponding average distortion are found as:

$$\begin{aligned} \hat{V}(y) &= \mathbb{E}[V|y] = \frac{\int_{-\frac{1}{2}}^{\frac{1}{2}} v f_V(v) f_W(y - \beta\mathcal{Q}(v)) dv}{\int_{-\frac{1}{2}}^{\frac{1}{2}} f_V(v) f_W(y - \beta\mathcal{Q}(v)) dv} \\ &= \frac{\sum_{i=1}^{N} f_W(y - \beta q_i) \int_{q_i - \frac{1}{2N}}^{q_i + \frac{1}{2N}} v dv}{\sum_{i=1}^{N} f_W(y - \beta q_i) \int_{q_i - \frac{1}{2N}}^{q_i + \frac{1}{2N}} dv} \\ &= \frac{N \sum_{i=1}^{N} f_W(y - \beta q_i)\left((q_i + \frac{1}{2N})^2 - (q_i - \frac{1}{2N})^2\right)}{2\sum_{i=1}^{N} f_W(y - \beta q_i)} \\ &= \frac{\sum_{i=1}^{N} q_i f_W(y - \beta q_i)}{\sum_{i=1}^{N} f_W(y - \beta q_i)} \triangleq g_{w,\beta}^N(y), \quad (1) \\ D^N &= \mathbb{E}[|V - \hat{V}|^2] = \sigma_v^2 - E[V\hat{V}] \\ &= \frac{1}{12} - \sum_{j=1}^{N} \int_{q_i - \frac{1}{2N}}^{q_i + \frac{1}{2N}} v \int_{-\infty}^{\infty} g_{w,\beta}^N(\beta q_j + w) f_W(w) dw dv \\ &= \frac{1}{12} - \frac{1}{N} \sum_{j=1}^{N} q_j \int_{-\infty}^{\infty} g_{w,\beta}^N(\beta q_j + w) f_W(w) dw. \quad (2) \end{aligned}$$

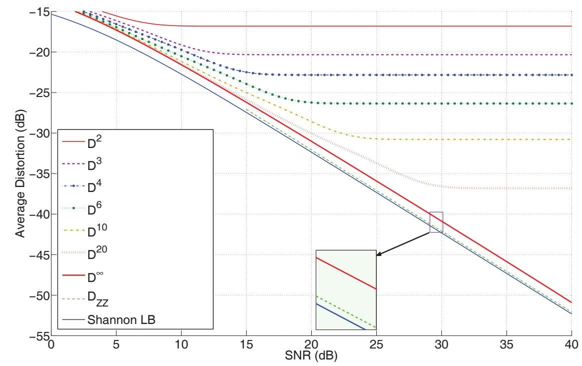

Figure 1. Average MSE (dB) vs. average SNR (dB) in the bandwidth matched scenario.

In Fig. (1) we plot the achievable MSE distortion as a function of the channel SNR for different values of N. It is clear that the distortion can be reduced by increasing the number of quantization points. While the MSE saturates for any finite N, we get the best performance by letting $N \to \infty$.

In this asymptotic limit the proposed digital scheme becomes analog transmission with $X = \alpha \cdot V$, where $\alpha = \sqrt{12P}$. With MMSE estimation at the receiver, the reconstructed sample and the average distortion for analog transmission is found as

$$\mathbb{E}[V|y] = \frac{\frac{\sigma_n}{\sqrt{2\pi}\alpha}\left(e^{-\frac{(2y+\alpha)^2}{8\sigma_n^2}} - e^{-\frac{(2y-\alpha)^2}{8\sigma_n^2}}\right) + \frac{y}{\alpha}\left[Q\left(\frac{2y-\alpha}{2\sigma_n}\right) - Q\left(\frac{2y+\alpha}{2\sigma_n}\right)\right]}{Q\left(\frac{2y-\alpha}{2\sigma_n}\right) - Q\left(\frac{2y+\alpha}{2\sigma_n}\right)},$$

$$D^{\infty} = \frac{1}{12} - \int_{-\infty}^{\infty} \int_{-\frac{1}{2}}^{\frac{1}{2}} v\mathbb{E}[V|\alpha v + w] f_W(w) dv dw,$$

where $Q(\cdot)$ is the Q-function defined as $Q(t) \triangleq \frac{1}{\sqrt{2\pi}} \int_t^{\infty} e^{-\frac{t^2}{2}}$.

Although analog transmission outperforms scalar quantization, unlike in the Gaussian scenario, it does not meet the Shannon lower bound. The comparison of the achievable MSE performance with the lower bound will be presented in Section VI.

IV. 2:1 BANDWIDTH COMPRESSION

A. Hybrid Digital- Analog Transmission

Our goal is to introduce an achievable scheme for the transmission of two source samples over one channel use with reasonable computational complexity. We use a scalar uniform quantizer for the first source sample, while the second sample is transmitted in an uncoded fashion. The channel input X is generated as the superposition of the scaled versions of the quantized first sample and the uncoded second sample:

$$X = \beta \cdot \mathcal{Q}(V_1) + \alpha \cdot V_2. \quad (3)$$

The coefficients α and β that are introduced to satisfy the power constraint. Once N is fixed we choose α such that V_2 is mapped to an interval of length at most $\frac{\beta}{N}$. This guarantees that the channel input intervals for different quantization indices do not overlap. In order to create some resistance against the channel noise we allow a gap of d between adjacent intervals, and set $\alpha = \frac{\beta}{N} - d$, where $0 \leq d \leq \frac{\beta}{N}$. The average power constraint for the hybrid scheme is found to be

$$\mathbb{E}[|X|^2] = \frac{1}{12}\left(\beta^2 - \frac{2\beta d}{N} + d^2\right) \leq P$$

Since $0 \leq d \leq \frac{\beta}{N}$, we have

$$\sqrt{12P} \leq \beta \leq \sqrt{\frac{12P}{1-(\frac{1}{N})^2}}.$$

The goal is to characterize the parameters (N, d, β) that minimize the average MSE distortion. At the receiver we use MMSE estimation is applied to reconstruct the transmitted source samples. For the first quantized sample, similarly to (1), we have

$$\hat{V}_1(y) = g^N_{z,\beta}(y),$$

where $Z = W + \alpha V_2$ is the equivalent noise when estimating the second sample. $f_Z(z)$ can be characterized as

$$f_Z(z) = \frac{1}{\alpha}\left(Q\left(\frac{z-\frac{\alpha}{2}}{\sigma_n}\right) - Q\left(\frac{z+\frac{\alpha}{2}}{\sigma_n}\right)\right).$$

The expanded version of (4) is given in (5). For the analog second sample, we have

$$\hat{V}_2(y) = \frac{\sum_{i=1}^{N}\int_{-\frac{1}{2}}^{\frac{1}{2}} v f_W(y - \alpha v - \beta q_i)dv}{\sum_{i=1}^{N}\int_{-\frac{1}{2}}^{\frac{1}{2}} f_W(y - \alpha v - \beta q_i)dv}. \quad (4)$$

The resulting distortion under MMSE estimation can be obtained by calculating $\mathbb{E}[|V_i - \hat{V}_i|^2]$ for $i = 1, 2$. For the first sample, similarly to (2), the distortion is given by

$$D_1 = \frac{1}{12} - \frac{1}{N}\sum_{j=1}^{N} q_j \int_{-\infty}^{\infty} g^N_{z,\beta}(\beta q_j + z) f_Z(z)dz,$$

while for the second sample, we have

$$D_2 = \frac{1}{12} - \frac{1}{N}\sum_{j=1}^{N}\int_v\int_w v\hat{V}_2(\alpha v + \beta q_j + w) f_W(w)dwdv. \quad (6)$$

The final average distortion is obtained as $D = \frac{D_1+D_2}{2}$. The numerical results and comparisons for the hybrid scheme can be found in Section VI.

Remark IV.1. *We have observed in Section III that, in the bandwidth matched case, increasing N reduces the MMSE distortion, and analog transmission achieves the lowest distortion among all schemes of this type. On the other hand, in the bandwidth compression case N has conflicting effects on the distortion of the first and second samples. While increasing N decreases the distortion of the first sample, it has a negative effect on the distortion of the second sample. Since the quantized first sample acts as noise for the second sample, increasing the number of the quantization points increases the entropy of the noise, and makes it harder for the receiver to estimate the second source sample. As we will see in Section VI through numerical results, for a given power constraint P, there is a specific N^* which minimizes the average distortion.*

Remark IV.2. *Since the entropy of the quantizer output for the first source sample can be seen as a proxy for the severity of its impact as noise on the transmission of the second source sample, we can argue that minimizing the entropy of the scalar quantizer output will reduce the overall distortion. In [12], authors present optimal scalar quantization for uniform sources subject to an entropy constraint. They prove that for entropy R, the optimal quantizer has $N = \lceil e^R \rceil$ [1] quantization cells, with $N-1$ cells of equal length and the remaining cell of length less than the others. While we have considered this optimal quantizer in our simulations, we have not observed any significant gains by considering an additional cell smaller than the others; and hence, we limit the presentation in Section VI to uniform quantizers. However, we expect that entropy-constrained quantizers will be useful for applying our hybrid scheme to sources with non-uniform distributions.*

B. Numerically Optimized Encoder

In this section we study the encoding structure numerically optimized given that the receiver performs MMSE decoding. Numerical techniques have been previously used for joint source channel mappings in various scenarios [7], [13]. In the following for simplicity we use h, for the encoder $h(v_1, v_2)$, and g_1, g_2 for separate decoders, respectively. By writing the Lagrangian cost function for this system model we have

$$J_{h,g_1,g_2} = \frac{1}{12} - \frac{1}{2}(\mathbb{E}[|V_1\hat{V}_1|] + \mathbb{E}[|V_2\hat{V}_2|]) + \lambda E[|h(V_1, V_2)|^2],$$

where λ is the Lagrangian multiplier.

Applying the standard method in variational calculus to the cost function above, we have

$$\nabla_h J_{h,g_1,g_2} = -\frac{f_V(v_1, v_2)}{2} \cdot \left(\int_w (v_1 \acute{g_1}(h+w) + v_2\acute{g_2}(h+w)) f_W(w)dw - 4\lambda h\right),$$

where $\acute{g_i}$ is the derivative of $g_i(\cdot)$, which is the optimal MMSE decoder for the sample $i = 1, 2$ given by

$$g_i(y) = \frac{\int\int v_i e^{-\frac{(y-h)^2}{2\sigma_w^2}} dv_1 dv_2}{\int\int e^{-\frac{(y-h)^2}{2\sigma_w^2}} dv_1 dv_2}.$$

The necessary condition for the optimality of h is $\nabla_h J_{h,g_1,g_2} = 0$. Then, the optimal encoder for an individual pair of (v_1, v_2) is obtained as below

$$h = \frac{1}{4\lambda}\int_w (v_1\acute{g_1}(h+w) + v_2\acute{g_2}(h+w)) f_W(w)dw. \quad (7)$$

Since it is not possible to solve (7) explicitly, we use an iterative algorithm to find the optimal encoding function.

[1] $\lceil z \rceil$ is the minimum integer greater than z.

$$\hat{V}_2(y) = \frac{\sum_{i=1}^{N}\left[\frac{\sigma_n}{\sqrt{2\pi}}\left(\exp\left\{-\frac{(y+\frac{\alpha}{2}-\beta q_i)^2}{2\sigma_n^2}\right\} - \exp\left\{-\frac{(y-\frac{\alpha}{2}-\beta q_i)^2}{2\sigma_n^2}\right\}\right) + (y-\beta q_i)\left(Q(\frac{y-\frac{\alpha}{2}-\beta q_i}{\sigma_n}) - Q(\frac{y+\frac{\alpha}{2}-\beta q_i}{\sigma_n})\right)\right]}{\sum_{i=1}^{N}\left(\alpha\left(Q(\frac{y-\frac{\alpha}{2}-\beta q_i}{\sigma_n}) - Q(\frac{y+\frac{\alpha}{2}-\beta q_i}{\sigma_n})\right)\right)} \tag{5}$$

However, due to the non-convexity of the problem there is no guarantee that the algorithm will converge to the global optimum, and the performance of the resultant encoder will be sensitive to the initial mapping. In our simulations we use a linear encoder as the initial mapping, and use the steepest decent algorithm to update the encoder. At every step the encoder is updated as follows:

$$h_{j+1}(v_1, v_2) = h_j(v_1, v_2) - \mu \cdot \nabla_h J_{h,g_1,g_2}. \tag{8}$$

where μ is the step size. The updating process is terminated once $\nabla_h J_{h,g_1,g_2}$ is less than a preset threshold value (we have considered 10^{-10} in our simulations).

V. MSE LOWER BOUND

A. Shannon Lower Bound

The trivial lower bound is obtained by relaxing the delay-constraint, and using Shannon's classical result on the optimality of source-channel separation as the block-length goes to infinity. For a uniform source spread over an interval of length Δ, the rate-distortion function is not known, but it can be lower bounded by $\frac{1}{2}\log\frac{\Delta^2}{2\pi e D}$. Hence, the Shannon lower bound for our problem is $D_{\text{SLB}} = \frac{1}{2\pi e}(1+\frac{P}{\sigma_n^2})^{-1/2}$.

B. Ziv-Zakai (ZZ) Lower Bound

The ZZ bound on the distortion is obtained by using an information measure different from the mutual information which still satisfies the data processing inequality. Tighter than Shannon bounds have been obtained for the bandwidth mismatch Gaussian problem in [11] using Rènyi information of order γ. While the maximum Rènyi information of order γ is not known in general for the AWGN channel, in [11] a high SNR approximation is derived as follows:

$$C_\gamma \cong \frac{1}{2}\log(P c_{1,2-\gamma}^{-2}) - H_\gamma(W), \tag{9}$$

where $0 < \gamma < \frac{5}{3}$, and $H_\gamma(Z)$ is the Renyi entropy of the channel noise, where

$$\begin{aligned} H_\gamma(W) &= \frac{1}{1-\gamma}\log\int f_W(w)^\gamma dw, \\ c_{n,\gamma} &= a_{n,\gamma}^{1/n}[\gamma(1+\frac{2}{n})-1]^{-1/2} b_{n,\gamma}, \end{aligned} \tag{10}$$

$$b_{n,\gamma} = \begin{cases} (1-\frac{n(1-\gamma)}{2\gamma})^{\frac{1}{n(1-\gamma)}}, & \gamma \neq 1, \\ e^{-1/2}, & \gamma = 1, \end{cases}$$

$$a_{n,\gamma} = \begin{cases} \frac{(1-\gamma)^{\frac{n}{2}+1}\Gamma(\frac{n}{2}+1)}{\pi^{\frac{n}{2}}\beta(\frac{n}{2}+1, \frac{1}{1-\gamma}-\frac{n}{2})}, & \gamma < 1, \\ \frac{1}{\pi^{\frac{n}{2}}}, & \gamma = 1, \\ \frac{(\gamma-1)^{\frac{n}{2}+1}\Gamma(\frac{n}{2}+1)}{\pi^{\frac{n}{2}}\beta(\frac{n}{2}+1, \frac{1}{\gamma-1})}, & \gamma > 1. \end{cases}$$

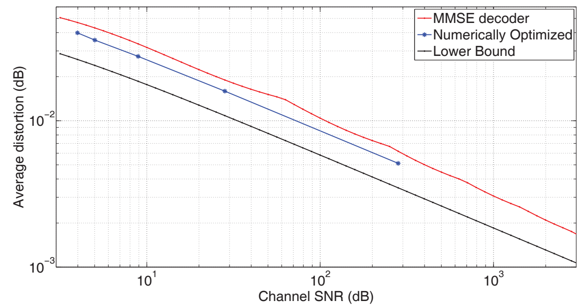

Figure 2. Average MSE distortion (dB) vs. the average SNR (dB) under bandwidth compression for a source variance of $\frac{1}{12}$ and $\sigma_n^2 = 1$.

Similarly, in [14] the authors derive an approximation for the Renyi rate distortion function of order γ at high resolution. They show that for a k-dimensional source with distribution f, Rènyi rate-distortion function of order γ under MSE distortion measure can be approximated as follows:

$$R_\gamma(D) \cong \frac{k}{2}\log\left(\frac{1}{k}c_{k,\gamma}^2 ||f^{\frac{(2-\gamma)2+k(\gamma-1)}{k(\gamma-1)}}||_{\frac{k(\gamma-1)}{k(\gamma-1)+2}}\right), \tag{11}$$

where $||f||_p \triangleq \left(\int_{\text{supp}(u)} f(u)^p du\right)^{1/p}$.

Combining (9) and (11) with the data processing inequality, we obtain the bound on the MSE distortion for $k:1$ bandwidth compression in the high SNR regime:

$$D_{ZZ} = c_{k,\gamma}^2 \cdot \left(\int_{-\infty}^{\infty} f_W(w)^\gamma dw\right)^{\frac{2}{k(1-\gamma)}} \left(k\left(\frac{P}{c_{1,2-\gamma}^2}\right)^{\frac{1}{k}}\right)^{-1}.$$

We remark here that the ZZ lower bound reduces to the Shannon lower bound when $\gamma \to 1$; and hence, the distortion lower bound obtained by minimizing D_{ZZ} over γ is going to be at least as tight as the Shannon lower bound.

VI. NUMERICAL RESULTS

In this section, we numerically analyze the transmission schemes and compare their performances with each other and the analytical lower bounds. For the numerical analysis, integrals in the distortion and encoder expressions in Section IV are calculated by discretizing the domain of the source and noise random variables. The tails of the random variables with unbounded support, e.g., the channel noise, are ignored.

In Fig. 1 we plot the results for the bandwidth matching case. The ZZ bound improves slightly (approximately 0.22 dB) compared to the Shannon lower bound. The gap between the lower bound and the analog transmission is 1.22 dB.

In Fig. 2, 2:1 bandwidth compression is considered, and the performance of the numerically optimized encoder using (8) and the proposed hybrid transmission scheme are compared with the lower bound. We have fewer results for the numerically optimized encoder since the algorithm needs long time to converge, especially at high SNR values. We observe that the proposed hybrid scheme performs reasonably close to the numerically optimized encoder. This performance is remarkable considering the significantly low complexity of the hybrid encoder. As opposed to the bandwidth matching case, there is no visible gain from the ZZ lower bound compared to the Shannon bound in this case. We also observe that the optimal N in the hybrid scheme, that is, the number of quantization indices for the scalar quantized sample, increases with SNR. This is in line with our argument that the optimal quantizer balances the distortion of the first sample with the noise effect on the second sample. The size of the optimal noise gap, d, for the hybrid scheme is plotted for different channel SNRs in Fig. 3. We observe that d also increases with SNR, but the rate of increase for d decreases with SNR. Also, the optimal value of d is not continuous at the points where N is increased by one. Finally, in Fig. 4, the numerically optimized encoder mapping is shown for $\lambda = 0.01$. It is worth to mention that for higher SNRs (equivalently, for lower λ) we need to increase the discretization accuracy. The slow convergence of the encoder mapping at higher SNRs makes this algorithm hard to implement in practice. For higher SNRs it is observed that the encoder has a linear structure.

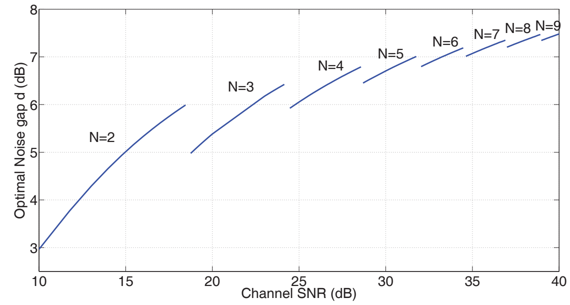

Figure 3. Optimal noise gap d (dB) vs. channel SNR (dB) for the hybrid digital analog scheme.

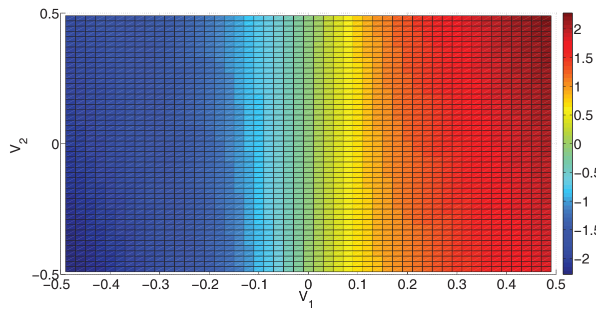

Figure 4. Encoder mapping for $\lambda = 0.01$, $P = 2.0701$, $D = 0.05457$. Each discretized pair of (v_1, v_2) is assigned a value between $[-2.3, 2.3]$. The color code indicates the channel input value for each (V_1, V_2) pair.

VII. Conclusion

We have considered the delay limited transmission of a uniform source over an AWGN channel under an average power constraint. A considerably simple hybrid transmission scheme is studied, which quantizes one source sample to a very low number of quantization indices (2 quantization bins are sufficient up to channel SNR of 18 dB, while only 5 bins are required below 32 dB), and superposes the second sample in an uncoded fashion. This hybrid scheme is considered with MMSE estimation at the receiver. Comparison with a computationally demanding numerically optimized scheme, using an iterative steepest descent algorithm, is also considered. Finally, we have also studied the Ziv- Zakai lower bound for this problem in the high SNR regime, and have seen that it improves upon the Shannon lower bound in the bandwidth matched scenario, reducing the gap between the lower bound and the uncoded transmission to 1.22 dB. The improvement in the 2:1 bandwidth compression case is negligible.

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