Correlated Sources over an Asymmetric Multiple Access Channel with One Distortion Criterion

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***Abstract*— We consider transmission of two arbitrarily correlated sources over a discrete memoryless asymmetric multiple access channel, where one of the sources is available at both transmitters. We want to transmit the common source lossless in the Shannon sense, while there is a distortion requirement on the other source. For a given bandwidth ratio between the channel and source bandwidths and a distortion requirement, we derive the necessary and sufficient conditions, and show that separation of source and channel coding is optimal. Finally, we extend our results to the case where perfect casual feedback is available at either one or both of the transmitters.**

I. Introduction

The discrete memoryless (DM) asymmetric multiple access channel (AMAC) with two encoders, introduced by Haroutunian [1], is a multiple access channel with two transmitters and one receiver, where one of the transmitters has access to the other's message in addition to its own private message. Haroutunian characterized the capacity region of DM AMAC with independent sources. Later, Prelov [2]-[4] showed that for a DM AMAC with independent sources, feedback cannot increase the capacity region. Note that, this model of AMAC is different from the MAC with cribbing encoders setup of Willems [5], where the channel inputs of one of the transmitters (not the message itself) is assumed to be available to the other in a noncausal fashion.

Transmission of correlated sources over a DM multiple access channel (MAC) was first investigated by Cover et al. [6] who showed that Shannon's separation principle, i.e., first applying distributed source coding and then using channel codes for the underlying MAC, is not optimal contrary to its optimality in the point-to-point setting. Sufficient conditions for reliable transmission of correlated sources over DM MAC were provided in [6] which were later shown by Dueck [7] not to be necessary. The problem of finding the necessary and sufficient conditions in the most general setting remains open to this day.

In [8], De Bruyn et al. consider lossless transmission of arbitrarily correlated sources over a DM AMAC and find necessary and sufficient conditions. Surprisingly, they show that Shannon's separation principle is optimal for this setting, i.e., Slepian-Wolf source coding [9] followed by capacity achieving channel coding for the DM AMAC is optimal. They also show that the same necessary and sufficient conditions hold when casual perfect feedback is available at either or both of the transmitters.

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In this paper, we extend the results of [8] to the case where the common source is transmitted losslessly in the Shannon sense, while we have a distortion constraint on the other source. We show that, first applying distributed source compression with one distortion criterion, and then using the optimal channel code for the given DM AMAC is optimal. Similar to [8], our results can also be extended to one- or two-sided perfect feedback cases.

The scenario considered in this paper can model a situation where two sensors communicate a common information reliably to a destination through a MAC, while some additional correlated information is transmitted with certain distortion criterion by only one of the sensors. Another application would be the cognitive radio channel as modeled in [10], where the authors derive achievable rates of an interference channel in which the message of one of the transmitters is known by the other transmitter non-casually. Although their model focuses on separate destinations, cognition in the multiple access setting is also important, and optimality of separate source and channel coding reinforces modular design for future cognitive multiple access radio networks.

The paper is organized as follows: in Section II, we introduce the problem, and then state our main results. In Section III, we present some background and derive results that will be necessary to prove our main theorems. We prove the separation theorem for no-feedback case in Section IV, and for two-sided perfect feedback case in Section V. Then Conclusion and Appendix follow.

Throughout the paper, we will denote random variables by capital letters, sample values by the respective lower case letters, and the alphabets by the respective calligraphic letters. The random vector $(X_1, \dots, X_n)$ will be denoted by X^n, and we denote the complement of a certain element in a vector by $X_i^c \triangleq (X_1, \dots, X_{i-1}, X_{i+1}, \dots, X_n)$.

II. Problem Setup and the Main Result

We consider two i.i.d. sources which are arbitrarily correlated according to a joint distribution $p(s,t) \in \mathcal{P}(\mathcal{S} \times \mathcal{T})$, that is, $\{S_i, T_i\}_{i=1}^{\infty}$ are generated i.i.d according to $p(s,t)$ over a finite alphabet $\mathcal{S} \times \mathcal{T}$. We denote this source pair by $(\mathcal{S}, p(s,t), \mathcal{T})$.

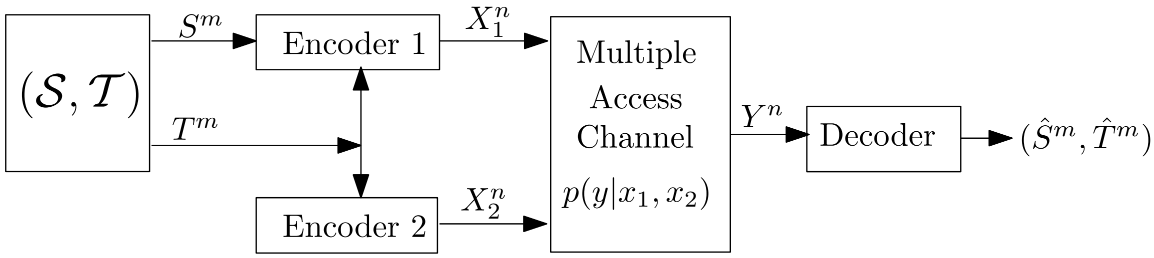

Fig. 1. Asymmetric multiple access channel with arbitrarily correlated sources.

We assume that source S is only available to encoder 1, while source T is available to both encoders of a multiple access channel (see Fig. 1). Encoders transmit these length-m sequences of the sources over the DM AMAC denoted by $(\mathcal{X}_1 \times \mathcal{X}_2, p(y|x_1,x_2), \mathcal{Y})$. Encoder 1 maps its observation (S^m, T^m) to a channel codeword of length-n $X_1^n = f(S^m,T^m) \in \mathcal{X}_1^n$ by the encoding function

$$f : \mathcal{S}^m \times \mathcal{T}^m \to \mathcal{X}^n. \tag{1}$$

On the other hand, encoder 2 mapsT^m to a channel codeword $X_2^n = g(T^m) \in \mathcal{Y}^n$ by the encoding function

$$g : \mathcal{T}^m \to \mathcal{X}_2^n. \tag{2}$$

The output of the DM AMAC is $Y^n \in \mathcal{Y}^n$. The channel is characterized by the conditional distribution

$$p(y^n|x_1^n, x_2^n) = \prod_{i=1}^{n} p_{Y|X_1,X_2}(y_i|x_{1,i}, x_{2,i}). \tag{3}$$

The decoder maps the channel output Y^n to the estimate $(\hat{S}^m, \hat{T}^m) = h(Y^n)$ by the decoding function

$$h : \mathcal{Y}^n \to \hat{\mathcal{S}}^m \times \hat{\mathcal{T}}^m. \tag{4}$$

We define the per letter distortion measures for sources S and T, respectively, as

$$d_s : \mathcal{S} \times \hat{\mathcal{S}} \to [0, \infty) \tag{5}$$

$$d_t : \mathcal{T} \times \hat{\mathcal{T}} \to \{0, 1\} \qquad d_t(t, \hat{t}) = 1 - \delta_{t\hat{t}} \tag{6}$$

Here the distortion function d_s is arbitrary. Let

$$\Delta_s^m = E\left[\frac{1}{m}\sum_{i=1}^{m} d_s(S_k; \hat{S}_k)\right], \tag{7}$$

and

$$\Delta_t^m = E\left[\frac{1}{m}\sum_{i=1}^{m} d_t(T_k; \hat{T}_k)\right]. \tag{8}$$

Then, with Δ^m defined as

$$\begin{aligned} \Delta^m &= \Delta^m(S^m, T^m; \hat{S}^m, \hat{T}^m), \\ &\triangleq \left(\frac{1}{m}\sum_{i=1}^{m} d_s(S_k; \hat{S}_k), \frac{1}{m}\sum_{i=1}^{m} d_t(T_k; \hat{T}_k)\right), \end{aligned} \tag{9}$$

we have $E[\Delta^m] = (\Delta_s^m, \Delta_t^m)$.

Definition 2.1: We say that the pair (b, D) is *achievable* for the source pair $(\mathcal{S}, p(s,t), \mathcal{T})$ over the DM AMAC $(\mathcal{X}_1 \times \mathcal{X}_2, p(y|x_1,x_2), \mathcal{Y})$, if for every $\epsilon > 0$, there exist sufficiently large n and m with $b = n/m$, and encoders f and g and decoder h such that

$$E[\Delta^m] \le (D + \epsilon, \epsilon).$$

The main problem of this paper is to find the necessary and sufficient conditions for the achievability of the pair (b, D) while transmitting the source pair $(\mathcal{S}, p(s,t), \mathcal{T})$ over the DM AMAC $(\mathcal{X}_1 \times \mathcal{X}_2, p(y|x_1,x_2), \mathcal{Y})$. As in [8], we also consider a DM AMAC with arbitrarily correlated sources and two-sided casual perfect feedback (see Fig. 2), where the channel output is casually available to both encoders. Encoder 1, in addition to the source sequences S^m and T^m, also observes the previous channel outputs $Y^{i-1} \triangleq (Y_1, \ldots, Y_{i-1})$ before deciding the i-th channel input $X_{1,i}$. Then the encoding function of encoder 1, f is now defined as an n-sequence of functions as follows.

$$X_{1,1} = f_1(S^m, T^m), \tag{10}$$

$$X_{1,i} = f_i(S^m, T^m, Y^{i-1}), \text{ for } i = 2, \ldots, n. \tag{11}$$

Similarly, the encoding function g of encoder 2 is defined as

$$X_{2,1} = g_1(T^m), \tag{12}$$

$$X_{2,i} = g_i(T^m, Y^{i-1}), \text{ for } i = 2, \ldots, n. \tag{13}$$

In the case of one-sided casual feedback, only the encoder function observing the feedback signal needs to be changed correspondingly. The decoder function, average distortion and achievability definitions remain unchanged.

We define $R_{S|T}(D)$ as the conditional rate-distortion function for source S with side-information T, whose definition will be given later. Following theorem is the main result of the paper.

Theorem 2.1: For the source pair $(\mathcal{S}, p(s,t), \mathcal{T})$ and the DM AMAC $(\mathcal{X}_1 \times \mathcal{X}_2, p(y|x_1,x_2), \mathcal{Y})$, (b, D) pair is *achievable* if,

$$R_{S|T}(D) < b \cdot I(X_1; Y|X_2), \tag{14}$$

$$H(T) + R_{S|T}(D) < b \cdot I(X_1, X_2; Y), \tag{15}$$

where the joint probability distribution on $\mathcal{X}_1 \times \mathcal{X}_2 \times \mathcal{Y}$ satisfies

$$p(x_1, x_2, y) = p(x_1, x_2)p(y|x_1, x_2). \tag{16}$$

Conversely, if the pair (b, D) is *achievable* for the source pair $(\mathcal{S}, p(s,t), \mathcal{T})$ and the DM AMAC $(\mathcal{X}_1 \times \mathcal{X}_2, p(y|x_1,x_2), \mathcal{Y})$, then there exists a probability distribution $p(x_1, x_2)$ on $\mathcal{X}_1 \times \mathcal{X}_2$

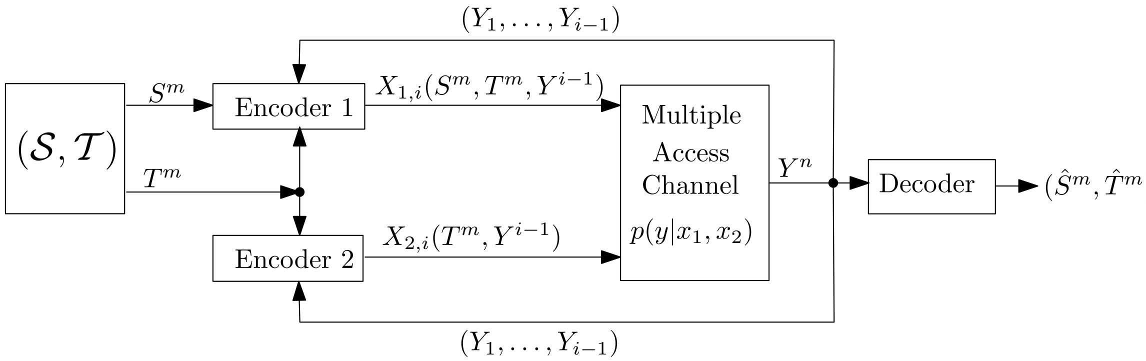

Fig. 2. Asymmetric multiple access channel with arbitrarily correlated sources and two-sided casual perfect feedback.

such that (14) and (15) hold with $<$ replaced by $\leq$, where the joint distribution on $\mathcal{X}_1 \times \mathcal{X}_2 \times \mathcal{Y}$ is given by (16).

Proof: Proof of the theorem is given in Section IV. ■

Similar to the lossless case in [8], this result can also be extended to the two-sided casual perfect feedback case as follows.

Theorem 2.2: For the source pair $(\mathcal{S}, p(s,t), \mathcal{T})$ and the DM AMAC $(\mathcal{X}_1 \times \mathcal{X}_2, p(y|x_1, x_2), \mathcal{Y})$ with two-sided casual perfect feedback, the (b, D) pair is *achievable* if (14) and (15) hold for a joint probability distribution on $\mathcal{X}_1 \times \mathcal{X}_2 \times \mathcal{Y}$ satisfying (16). Conversely, if the pair (b, D) is *achievable* for the source pair $(\mathcal{S}, p(s,t), \mathcal{T})$ and the DM AMAC $(\mathcal{X}_1 \times \mathcal{X}_2, p(y|x_1, x_2), \mathcal{Y})$ with two-sided casual perfect feedback, then (14) and (15) hold with $<$ replaced by $\leq$ for joint distribution on $\mathcal{X}_1 \times \mathcal{X}_2 \times \mathcal{Y}$ of the form (16).

Proof: Proof of the theorem is given in Section V. ■

Remark 2.1: Note that, since the same sufficient conditions are also necessary for both no-feedback and two-sided feedback cases, we can conclude that, same conditions are necessary and sufficient for one-sided casual perfect feedback case as well.

Remark 2.2: Although the proof we provide here is for a discrete source alphabet $\mathcal{S}$, following the arguments in [15], the results can be extended to a continuous source alphabet.

Note that, these two theorems state that conditions (14)-(15) are necessary and sufficient for the achievability of a (b, D) pair, unless they hold with equality. As it will be clear from the proof, this can be interpreted as the optimality of distributed source coding followed by AMAC channel coding. Intuitively, the reason behind the optimality of separation in this setting is that, the structure of the message sets already enables the encoders to achieve all possible joint distributions for the channel inputs. Hence, there is no further need to exploit correlation induced by the sources. However, in the usual MAC scenario, since the channel input distributions are independent, the correlation among the sources would help the encoders to achieve certain joint distributions through joint source-channel coding, which would not be possible to achieve through separate source and channel coding.

III. Background

In this section, we provide some background that will be used in the following section to prove the main result of the paper. We start with the capacity region of a DM AMAC with independent messages given in [1]. Consider two independent messages $W_1 \in \{1, 2, \ldots, M_1\}$ and $W_2 \in \{1, 2, \ldots, M_2\}$, where each pair (w_1, w_2) occurs with probability $1/M_1 M_2$. We assume that W_1 is available to encoder 1, while W_2 is available to both encoders. While encoder and decoder functions are similar to the joint source-channel setting with S^m and T^m replaced by W_1 and W_2, respectively; the goal of the receiver is to reconstruct $(\hat{W}_1, \hat{W}_2)$. An (M_1, M_2, n, P_e)-code for the DM AMAC $(\mathcal{X}_1 \times \mathcal{X}_2, p(y|x_1, x_2), \mathcal{Y})$ with independent messages is defined as the set of encoding and decoding functions with error probability $P_e \triangleq Pr\{(\hat{W}_1, \hat{W}_2) \neq (W_1, W_2)\}$.

Definition 3.1: A rate pair (R_1, R_2) is said to be achievable for the DM AMAC with independent messages if, for any $\epsilon > 0$ there exists for all sufficiently large n, an (M_1, M_2, n, P_e)-code, such that $0 \leq R_1 \leq \log(M_1)/n$, $0 \leq R_2 \leq \log(M_2)/n$, and $P_e \leq \epsilon$.

Capacity region of a DM AMAC is defined as the closure of all the achievable rate pairs (R_1, R_2) and is given in [1].

Theorem 3.1: The capacity region of a DM AMAC is given by

$$\mathcal{C}_{AMAC} = \bigcup_{p(x_1,x_2,y)\in\mathcal{P}} \left\{(R_1, R_2) \in \mathbb{R}_+^2 | R_1 \leq I(X_1; Y|X_2), \right.$$
$$\left. R_1 + R_2 \leq I(X_1, X_2; Y)\right\},$$

where $\mathcal{P}$ is the set of all probability distributions in the form of $p(x_1, x_2, y) = p(x_1, x_2)p(y|x_1, x_2)$.

Next, we consider the asymmetric source coding problem with one distortion criterion, where encoder 1 has access to both correlated sources S and T, while encoder 2 has only access to T. For the source coding problem, we assume error free direct links from the encoders to the decoder and define an $(m, M_1, M_2, \Delta_s^m, \Delta_t^m)$ code by two encoders

$$f_1: \quad \mathcal{S}^m \times \mathcal{T}^m \to \{1, \ldots, M_1\},$$
$$f_2: \quad \mathcal{T}^m \to \{1, \ldots, M_2\},$$

a decoder

$$g: \quad \{1,\ldots,M_1\}\times\{1,\ldots,M_2\}\rightarrow \hat{\mathcal{S}}^m\times\hat{\mathcal{T}}^m$$

with

$$(\hat{S}^m,\hat{T}^m)=g(f_1(S^m,T^m),f_2(T^m)). \tag{17}$$

and per letter distortion measures and average distortion $E[\Delta_m]$ defined as in (5),(6) and (9), respectively.

Definition 3.2: For the above source coding problem, a rate pair (R_1,R_2) is said to be D-achievable for the source pair $(\mathcal{S},p(s,t),\mathcal{T})$, if for any $\epsilon>0$, and for sufficiently large m, there exists an $(m,M_1,M_2,\Delta_s^m,\Delta_t^m)$ code such that

$$\begin{aligned} M_1 &\le 2^{n(R_1+\epsilon)}, & \Delta_s^m &\le D+\epsilon,\\ M_2 &\le 2^{n(R_2+\epsilon)}, & \Delta_t^m &\le \epsilon. \end{aligned}$$

We define

$$\mathfrak{R}(D)=\{(R_1,R_2):(R_1,R_2)\text{ is }D\text{-achievable}\}.$$

This asymmetric source coding problem with one distortion criterion is one of the many cases considered by Kaspi and Berger in [14]. The achievable rate region is fully characterized in [14], however, for our proof in the next section, we obtain a slightly different characterization, whose equivalence to the Kaspi-Berger region is proven in the Appendix.

For our alternative characterization of $\mathfrak{R}(D)$, we need the following definitions and results. As in [12], we define the functional $\mathcal{E}(\cdot)$ on a joint distribution p_{SQ}, as the minimum possible estimation error of S given Q:

$$\mathcal{E}(S|Q)=\min_{f:\mathcal{Q}\rightarrow\hat{\mathcal{S}}} E[d(S,f(Q))], \tag{18}$$

Now we can write the conditional rate distortion function $R_{S|T}(D)$, defined in [11], using the functional $\mathcal{E}(\cdot)$ with distortion measure $d_s(\cdot,\cdot)$ as

$$R_{S|T}(D)=\min_{U:\mathcal{E}(S|U,T)\le D} I(S;U|T), \tag{19}$$

over all joint distributions $p(u|s,t)$.

Let $\mathfrak{R}^*(D)$ be the set of (R_1,R_2) pairs satisfying

$$\begin{aligned} R_1 &\ge R_{S|T}(D), && (20)\\ R_1+R_2 &\ge H(T)+R_{S|T}(D). && (21) \end{aligned}$$

Theorem 3.2: $\mathfrak{R}(D)=\mathfrak{R}^*(D)$.

Proof: Proof of the theorem is given in the Appendix. ■

IV. Proof of Theorem 2.1

Proof: We start with the direct part. Assume for some positive R_1 and R_2, we have

$$\frac{1}{b}R_{S|T}(D)<\frac{1}{b}R_1<I(X_1;Y|X_2),$$
$$\frac{1}{b}\left[H(T)+R_{S|T}(D)\right]<\frac{1}{b}\left(R_1+R_2\right)<I(X_1,X_2;Y)$$

From Theorem 3.2, we observe that the left hand side of the above inequalities form the sufficient conditions for compressing T losslessly and S with distortion D, where R_1 and R_2 are the compression rates of encoder 1 and encoder 2, respectively. On the other hand, from the capacity region of DM AMAC with independent messages given in Theorem 3.1, the right hand side form the sufficient conditions to transmit the compressed sources reliably over the channel.

Next, we prove the converse. We first define the following random variables for $i=1,\ldots,n$:

$$U_i\triangleq(Y^n,S^{i-1},T_i^c). \tag{22}$$

We can obtain the following set of inequalities:

$$\begin{aligned}
\frac{1}{n}I(X_1^n;Y^n|X_2^n) &\ge \frac{1}{n}I(X_1^n;Y^n|X_2^n,T^m), && (23)\\
&\ge \frac{1}{n}I(S^m;Y^n|X_2^n,T^m), && (24)\\
&= \frac{1}{n}I(S^m;Y^n,X_2^n|T^m), && (25)\\
&\ge \frac{1}{n}I(S^m;Y^n|T^m), && (26)\\
&= \frac{1}{n}\sum_{i=1}^m I(S_i;Y^n|S^{i-1},T^m), && (27)\\
&= \frac{1}{n}\sum_{i=1}^m I(S_i;Y^n,S^{i-1},T_i^c|T_i)\\
&\quad -I(S_i;S^{i-1},T_i^c|T_i), && (28)\\
&= \frac{1}{n}\sum_{i=1}^m I(S_i;U_i|T_i), && (29)\\
&\ge \frac{1}{n}\sum_{i=1}^m R_{S|T}(\mathcal{E}(S_i|U_i,T_i)), && (30)\\
&\ge \frac{1}{n}\sum_{i=1}^m R_{S|T}(\mathcal{E}(S_i|T^n,Y^n)), && (31)\\
&\ge \frac{1}{n}\sum_{i=1}^m R_{S|T}(E[d_s(S_i,\hat{S}_i)]), && (32)\\
&\ge \frac{1}{b}R_{S|T}(D+\epsilon), && (33)
\end{aligned}$$

where (23) follows since $T^m-X_1^n-Y^n$ form a Markov chain given X_2^n; (24) follows from the data processing inequality and the fact that $S^m-X_1^n-Y^n$ form a Markov chain given (X_2^n,T^m); (25) follows from the chain rule and the fact that $S^m-T^m-X_2^n$ form a Markov chain; (26) follows from the chain rule and non-negativity of the mutual information; (27) and (28) follow from the chain rule; (29) follows from the memoryless assumption on the sources and the definition of U_i; (30) follows from (19); (31) follows from the monotonicity of $R_{S|T}(\cdot)$ and the definition of $\mathcal{E}(\cdot)$ in (18); (32) follows from (18) and the fact that $\hat{S}_i$ is a function of Y^n; (33) follows from the convexity and monotonicity of $R_{S|T}(\cdot)$, and the assumption we made in the beginning that distortion D is achievable.

We will find a similar bound for the joint mutual information. For this we will need Fano's inequality which states

$$\begin{aligned} H(T^m|\hat{T}^m) &\le 1+\Delta_t^m m\log(|\mathcal{T}|),\\ &\triangleq m\delta(\Delta_t^m), && (34) \end{aligned}$$

where $\delta(x)$ is a non-negative function that goes to zero as $x \to 0$

We have

$$
\begin{aligned}
\frac{1}{n}I(X_1^n, X_2^n; Y^n) &\geq \frac{1}{n}I(S^m, T^m; Y^n), && (35)\\
&= \frac{1}{n}I(T^m; Y^n) \\
&\quad + \frac{1}{n}I(S^m; Y^n|T^m), && (36)\\
&\geq \frac{1}{n}I(T^m; Y^n) \\
&\quad + \frac{1}{b}R_{S|T}(D+\epsilon), && (37)\\
&= \frac{1}{n}\left[H(T^m) - H(T^m|Y^n)\right] \\
&\quad + \frac{1}{b}R_{S|T}(D+\epsilon), && (38)\\
&\geq \frac{1}{b}\left[H(T) + R_{S|T}(D+\epsilon)\right] \\
&\quad - \frac{1}{n}H(T^m|\hat{T}^m), && (39)\\
&\geq \frac{1}{b}\left[H(T) + R_{S|T}(D+\epsilon)\right. \\
&\quad \left. -\delta(\Delta_t^m)\right], && (40)
\end{aligned}
$$

where (35) follows since $(T^m, S^m) - (X_1^n, X_2^n) - Y^n$ form a Markov chain; (36) follows from the chain rule; (37) follows from (26)-(33); (39) follows from the memoryless source assumption and the fact that $T^m - Y^n - \hat{T}^m$ form a Markov chain; and finally (40) follows from Fano's inequality and the definition (34).

By choosing a large enough m, we let $\epsilon \to 0$ and $\Delta_t^m \to 0$ and using the continuity of $R_{S|T}(\cdot)$, we obtain the following set of inequalities.

$$
\begin{aligned}
R_{S|T}(D) &\leq \frac{b}{n}I(X_1^n; Y^n|X_2^n), \\
H(T) + R_{S|T}(D) &\leq \frac{b}{n}I(X_1^n, X_2^n; Y^n),
\end{aligned}
$$

Now, notice that the mutual information expressions on the right hand side of the above inequalities are concave functions of the joint probabilities $p(x_{1,i}, x_{2,i})$. Hence, from Jensen's inequality, we get

$$
\begin{aligned}
R_{S|T}(D) &\leq b \cdot I(X; Z|Y), \\
H(T) + R_{S|T}(D) &\leq b \cdot I(X, Y; Z),
\end{aligned}
$$

for some joint probability distribution of the form (16), completing the proof of the converse. ■

V. Proof of Theorem 2.2

Proof: We only prove the converse as the direct part simply follows form Theorem 2.1. The converse proof also resembles the converse proof of Theorem 2.1. We can follow the same steps from (23)-(33) since all the arguments about the conditional rate distortion function and the facts that $T^m - X_1^n - Y^n$ form a Markov chain given X_2^n; $S^m - X_1^n - Y^n$ form a Markov chain given (X_2^n, T^m); and that $S^m - T^m - X_2^n$ form a Markov chain still hold. Similarly, the Fano's inequality in (34), and the Markov relation $(T^m, S^m) - (X_1^n, X_2^n) - Y^n$ continue to hold, which allow us to follow the same steps in (35-40). Thus we can prove the converse as in the converse of Theorem 2.1. ■

VI. Conclusion

We consider transmission of arbitrarily correlated sources over a discrete memoryless asymmetric multiple access channel. While we require the common source to be transmitted losslessly in the Shannon sense, we allow a certain distortion on the private source. Our results provide a source-channel separation theorem which states that optimal distributed source compression followed by optimal channel coding achieves the optimal overall performance. In order to prove this result, we gave a new characterization of the achievable rate region for asymmetric source coding problem with one distortion criterion.

Appendix

In this Appendix we show that the achievable region for the asymmetric source coding problem with one distortion criterion given by Kaspi and Berger in [14] is equivalent to the region $\mathfrak{R}^*(D)$.

Proof: The Kaspi-Berger region, $\mathfrak{R}_{KB}(D)$ can be written as the set of all (R_1, R_2) pairs for which there exist auxiliary random variables $U \in \mathcal{U}$ and $V \in \mathcal{V}$, jointly distributed with S and T, $|\mathcal{U}| < \infty$, $|\mathcal{V}| < \infty$, which satisfy

i) $S - T - V$ form a Markov chain;
ii) there exist $\hat{S}(U)$ such that $E[d_s(S, \hat{S})] \leq D$ and $H(T|U) = 0$;
iii) $R_1 \geq I(S, T; U|V)$ and $R_1 + R_2 \geq I(S, T; U, V)$.

Using (19), we can express the region $\mathfrak{R}^*(D)$ as the set of rate pairs (R_1, R_2) for which there exists an auxiliary random variable $W \in \mathcal{W}$, $|\mathcal{W}| < \infty$, jointly distributed with S and T such that:

i) there exists $\hat{S}(W, T)$ satisfying $E[d_s(S, \hat{S})] \leq D$;
ii) $R_1 \geq I(S; W|T)$ and $R_1 + R_2 \geq H(T) + I(S; W|T)$.

We will prove that $\mathfrak{R}^*(D) = \mathfrak{R}_{KB}(D)$. We start by showing that $\mathfrak{R}^*(D) \subseteq \mathfrak{R}_{KB}(D)$. Assume that $(R_1, R_2) \in \mathfrak{R}^*(D)$. We take $V = T$ and $U = (T, W)$. Then it is easy to see that conditions i) and and ii) of Kaspi-Berger are satisfied. We also have

$$
\begin{aligned}
I(S, T; U|V) &= I(S, T; T, W|T), \\
&= I(S; T, W|T), \\
&= I(S; W|T), \\
&\leq R_1,
\end{aligned}
$$

and

$$
\begin{aligned}
I(S, T; U, V) &= I(S, T; T, W), \\
&= I(S, T; T) + I(S, T; W|T), \\
&= H(T) + I(S; W|T), \\
&\leq R_1 + R_2,
\end{aligned}
$$

Thus $(R_1, R_2) \in \mathfrak{R}_{KB}(D)$. To show that $\mathfrak{R}_{KB}(D) \subseteq \mathfrak{R}^*(D)$, assume that $(R_1, R_2) \in \mathfrak{R}_{KB}(D)$ and take $W = U$. Obviously, condition i) of $\mathfrak{R}^*(D)$ is satisfied for this choice of W. We have

$$\begin{aligned}
I(S;W|T) &= I(S;U|T), \\
&\leq I(S;U|T) + I(T;U|V) \\
&\quad +I(T;V) - I(T;V), \\
&= I(S;U|T) + I(T;U,V) - I(T;V), \\
&= I(S;U|T) + I(T;U) \\
&\quad +I(T;V|U) - I(T;V), \\
&\leq I(S,T;U) + I(S,T;V|U) - I(T;V), \\
&\leq I(S,T;U,V) - I(S,T;V), \\
&= I(S,T;U|V), \\
&\leq R_1,
\end{aligned}$$

and

$$\begin{aligned}
H(T) + I(S;W|T) &= H(T) + I(S;U|T), \\
&\leq H(T) - H(T|U,V) + I(S;U,V|T), \\
&= I(T;U,V) + I(S;U,V|T), \\
&= I(S,T;U,V), \\
&\leq R_1 + R_2,
\end{aligned}$$

This concludes the proof of the fact that $\mathfrak{R}_{KB}(D) \subseteq \mathfrak{R}^*(D)$, hence we get $\mathfrak{R}_{KB}(D) = \mathfrak{R}^*(D)$. Since [14] proves that $\mathfrak{R}_{KB}(D) = \mathfrak{R}(D)$, we have $\mathfrak{R}^*(D) = \mathfrak{R}(D)$. ■

REFERENCES

[1] E. A. Haroutunian, “Lower bound for the error probability of multipleaccess channels,” Probl. Peredrrch. Inform, vol. 11, pp. 23-36, Apr.-June 1975.

[2] V. V. Prelov, “On the capacity and the zero-error capacity of certain multiple-access channels,” in Abstracts, 9th Prague Conf. Inform. Theory, Stat. Dec. Fns., Rand., Processes, Prague, Czechoslovakia, 1982.

[3] V. V. Prelov, “Feedback for some multiple access channels,” in Abstracts of Papers, Int. Workshop Convolutional Codes and Multi-User Communication, Sochi, USSR, 1983, pp. 112-116.

[4] V. V. Prelov, “Transmission over a multiple-access channel with a special source hierarchy,” Probl. Peredach. Inform., vol. 20, pp. 3-10, Oct.-Dec. 1984.

[5] F. M. J. Willems, E. C. van der Meulen, “The discrete memoryless multiple access channel with cribbing encoders,” IEEE Trans. Inform. Theory, vol. 31, No. 3, pp. 313327, May 1985.

[6] T. M. Cover, A. El Gamal, and M. Salehi, “Multiple access channels with arbitrarily correlated sources,” IEEE Truns. Injorm. Theory, vol. IT-26, pp. 648-657, Nov. 1980.

[7] G. Dueck, “A note on the multiple access channel with correlated sources,” IEEE Trans. Inform. Theory, vol. IT-27, pp. 232-235, Mar. 1981.

[8] K. de Bruyn, V. V. Prelov, E. van der Meulen, “Reliable transmission of two correlated sources over an asymmetric multiple-access channel,” IEEE Transactions on Information Theory, vol. 33, pp. 716 - 718, Sep. 1987.

[9] D. Slepian and J. K. Wolf, “Noiseless coding of correlated information sources,” IEEE Truns. Inform. Theory, vol. IT-19, pp. 471-480, July 1973.

[10] N. Devroye, P. Mitran, V. Tarokh, “Achievable Rates in Cognitive Radio Channels,” IEEE Trans. Inform. Theory, vol.52, no.5, pp. 1813-1827, May 2006.

[11] R. M. Gray, “Conditional rate distortion theory,” Information Systems Laboratory Technical Report 6502-2, October 1972.

[12] S. Shamai, S. Verdù, R. Zamir, “Systematic lossy source-channel coding”, IEEE Trans. Inform. Theory, vol. IT-44, no. 2, pp. 564-579, March 1998.

[13] T. Cover, J. Thomas, *Elements of Information Theory,* Wiley and Sons, New York, 1991.

[14] A. H. Kaspi and T. Berger, “Rate-distortion for correlated sources with partially separated encoders,” IEEE Trans. Inform. Theory, IT-28(6):828840, 1982.

[15] A. Wyner, “The rate-distortion function for source coding with side information at the decoderII: General sources,” Inform., Contr., vol. 38, no. 1, pp. 6080, July 1978.