Transition to turbulence of axisymmetric flow in cyclindrical geometry

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(i) Motivation;

The ability to accurately and efficiently predict transition to turbulence is fundamental to all attempts at laminar flow control. An improved model for flow stability in three-dimensional cylindrical geometry is presented here, showing that is possible to get solutions unconstrained by the parallel flow assumption. The ideas are developed from the 'Parabolised Stability Equations' (PSE) which were initially developed in Cartesian coordinates by Herbert & Bertolotti, who built on work by Hall on Görtler Vortices. A model in cylindrical coordinates is desirable because it is suitable for many real-world applications as well as generating interesting academic results.

(ii) Research;

As is common the model begins with non-dimensionalised Navier-Stokes equations, which are then perturbed with three-dimensional infinitesimal disturbances. The disturbances are modelled to have wave-like nature and a slowly changing profile in the streamwise direction. To get a solution at an initial streamwise position, traditional hydrodynamic stability theory is followed and the streamwise derivatives are considered small enough to ignore. The equations of this planar problem can be reordered and solved as a generalized eigenvalue problem. In the second step the first derivatives of the disturbances in the streamwise direction are reintroduced to problem, by virtue of the perturbations being only weakly elliptic in this direction. The equations have been 'parabolized' and the results from the initial point can be marched downstream with a simple Euler method, resulting in a full picture of the three-dimensional development of the base flow and the disturbances downstream. This method is solved numerically via a Chebyshev-Fourier spectral discretisation and it has a huge computational efficiency advantage over using Direct Numerical Simulation (DNS). This approach also is much more accurate than traditional methods at high frequencies as it only requires the parallel flow assumption at the first step.

(iii) Application for industry

Results for the test cases of the unbounded circular jet and circular pipe flow with suction the disturbances show that these flows can become unstable under certain conditions, while circular pipe flow without suction always remains stable to infinitesimal disturbances. Neutral curves and other results will be presented in academic journals. Now that the model has demonstrated its reliability, accuracy and efficiency it can be used to provide more accurate predictions for transition to turbulence for several real-world situations. The Aerospace industry is already starting to use PSE models for flat or curved surfaces but a model in cylindrical geometry will increase the range of problems that can be analysed by this method, such flow around aircraft fuselages and through turbine engines. This research could also have an impact in the marine industry and for modelling blood flow through the cardiovascular system.